Example: Tell which of the following are relation symbols, and state in words, what each means.

a. $\nless$ b. Δ c. $\perp$ d. $\sqrt{3}$ e. $\cong$

Solution:

a. $\nless$ is a relation symbol, meaning "is not less than."

b. $\triangle$ is not a relation symbol. It is the symbol for the word "triangle," when it is used in Geometry. In Arithmetic, it is sometimes used as a placeholder. For example:

$$5 - \triangle = 2$$

c. $\perp$ is a relation symbol, meaning "is perpendicular to," and is used to state that two lines meet to form a right angle.

d. $\sqrt{3}$ is not a relation symbol. It is a number symbol, read as "the second root of 3", or , more commonly, "the square root of 3." This is an irrational number, approximately equal to 1.732...

e. $\cong$ is a relation symbol. In Geometry, it states that two figures are "congruent." In other words, they have the same shape and are the same size.

Lesson 1 — Exercises:

For each of the following exercises, identify the part of the mathematical speech and state the meaning in words.

1. π
2. m
3. 15.318
4. $\sqrt{11}$
5. $\overline{)\quad}$
6. $\ngtr$
7. $\sqrt[3]{27}$
8. ( )
9. $17\frac{1}{3}$
10. $\cong$
11. e
12. $7.\overline{45}$
13. { }
14. $\frac{10}{7}$
15. 6^3
16. +
17. $\perp$
18. $\sqrt{16}$
19. [ ]
20. •
21. $\neq$

Example 1: List two examples of physical models of a point, a line and a plane.

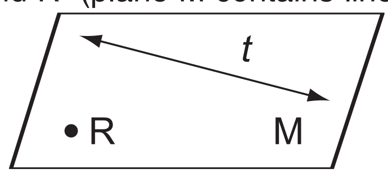
Solution: Point – possible answers include a pin hole, a tiny seed, a grain of sand, a grain of salt, and others.

Line – possible answers include a guy wire used to support a radio tower, a row of trees in a nursery, a string of pixels on a television screen, and others.

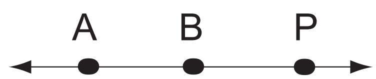
Plane – possible answers include the ceiling of a room, the surface of a tennis court, the side of a box, and others.

Example 2: Draw and label a figure for each of the following descriptions.

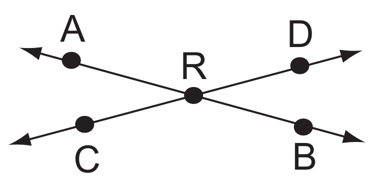
a. Plane M contains $\overleftrightarrow{t}$ and R (plane M contains line t, and point R)
Possible Solution:



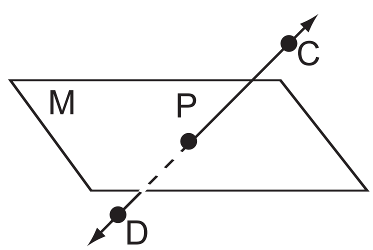
b. P is on $\overleftrightarrow{AB}$ (point P is on line AB)
Possible Solution:



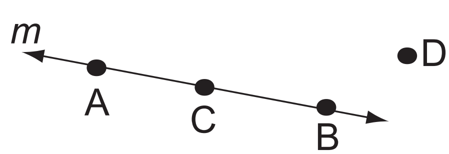
c. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at R (line AB and line CD intersect at point R)
Possible Solution:



d. Plane M and $\overleftrightarrow{CD}$ intersect at P (plane M and line CD intersect at point P)
Possible Solution:



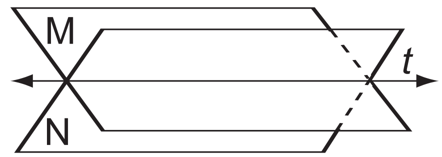
e. A, B, and C lie on $\overleftrightarrow{m}$ but D does not lie on $\overleftrightarrow{m}$ (point A, point B, and point C lie on line m, but point D does not lie on line m)
Possible Solution:



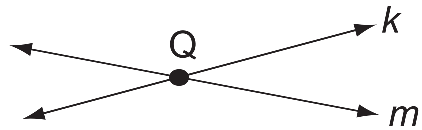
Example 2: (cont'd)

f. Plane M intersects plane N in $\overleftrightarrow{t}$ (plane M intersects plane N in line *t*)

Possible Solution:



Example 3: Write a description of the following diagram and represent that statement with geometric symbols.



Possible Solution: Point Q is the intersection of line *k* and line *m* (Q is the intersection of $\overleftrightarrow{k}$ and $\overleftrightarrow{m}$)

Lesson 2 — Exercises:

In exercises 1 through 3, replace each __?___ with a geometric term that makes the statement true.

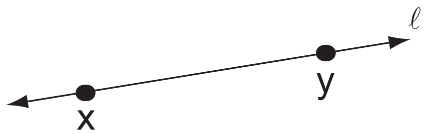
1. A __?___ has no thickness, but extends infinitely in two opposite directions.
2. A __?___ has no thickness, but extends infinitely in all directions along a flat surface.
3. A __?___ is a location. It has no thickness or length.

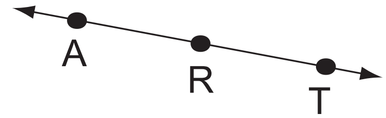
For each item in exercises 4 through 6, list three other examples of physical models which are different from the examples given in this lesson.

4. Point
5. Line
6. Plane

Lesson 2 — Exercises: (cont'd)

Name each of the lines in exercises 7 and 8, in two different ways.

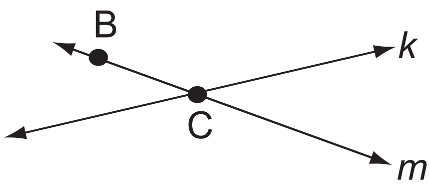
7. 

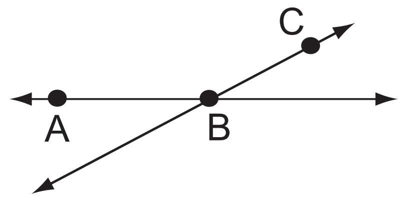
8. 

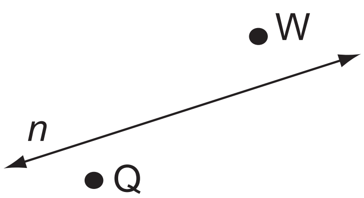
Use a ruler or straight edge to draw each line indicated in exercises 9-11, labeling each appropriately.

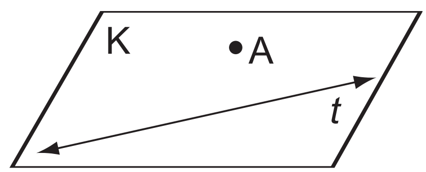
9. $\overleftrightarrow{AB}$ 10. $\overleftrightarrow{KL}$ 11. $\overleftrightarrow{PU}$

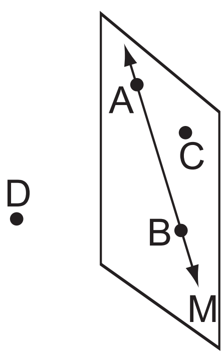
Write a complete description for each diagram in exercises 12-17.

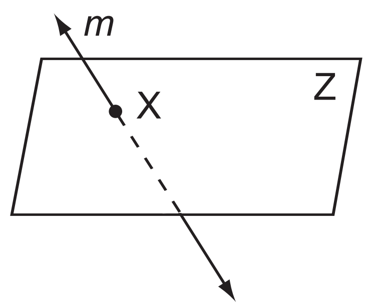
12. 

13. 

14. 

15. 

16. 

17. 

18. Sketch and label a diagram showing two planes that do not intersect.

19. Sketch and label a diagram showing a vertical plane intersected by a horizontal line. Draw hidden parts with dashes.

20. Sketch and label a diagram showing that points P and Q both lie in the intersection of planes A and B.

Unit I — The Structure of Geometry

Part A — What is Geometry?

Lesson 3 — *More on Operations*

Objective: To understand the basic operations, or "actions", of Geometry.

Important Terms:

Transformation – In Geometry, the movement of all of the points of a geometric figure, according to a specific set of rules, creating a new geometric figure. This movement is often called a "mapping", and establishes a correspondence between the points of the original figure, and the points of the new figure.

Pre-Image – The original geometric shape before a transformation is applied to it. The pre-image is sometimes called the "object."

Image – The geometric shape which appears after a transformation has been applied to the pre-image.

Translation – Sometimes called a "rigid" transformation, or a "slide," this is a size and shape preserving transformation, which maps every point in the pre-image, to its corresponding point in the image, by a set of straight line segments – all of which are parallel and equal in length. This transformation is defined by the direction and length of the movement, and is shown by a translation vector (→) which shows the distance and direction of the translation.

Rotation – This transformation "rotates" the pre-image about a fixed point, in such a way that every point in the pre-image turns through the same-sized angle, relative to that fixed point, preserving size and shape. A rotation is defined by the position of the fixed point about which the turn is made (called the center of rotation), the direction (usually counter-clockwise, and called the direction of rotation), and the angle of the turn (called the angle of rotation, in degrees). It is shown by the fixed point and a directional arc for a specific angle.

Reflection – A transformation in which each point of the pre-image moves across a fixed line, to a point in the image, which is the same distance from that fixed line (called the line of reflection or the "mirror" line) as the original point. A reflection is defined by the line of reflection, and is shown by that line, with arrowheads on each end. This transformation also preserves size and shape.

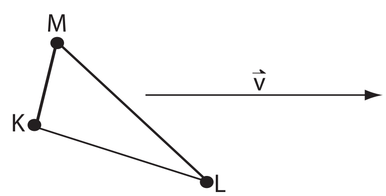
Important Terms: (cont'd)

Dilation – A transformation in which the distance between any point of the pre-image, and a specified point (called the center of dilation), is multiplied by some constant factor to produce the image. A dilation is defined by that center of dilation and the multiplier (called the scale factor), which will enlarge the pre-image if the multiplier has an absolute value greater than one, or reduce the pre-image if the multiplier has an absolute value strictly between zero and one.

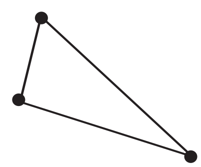
Isometry Transformation – A transformation or combination of transformations which results in the image being exactly the same shape and size as the pre-image.

Similarity Transformation – A transformation or combination of transformations which results in the image being exactly the same shape, but not the same size as, the the pre-image.

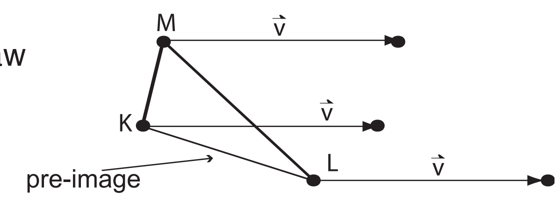
Example 1: Sketch the translation image of triangle KLM (△ KLM), under the translation $\vec{v}$.



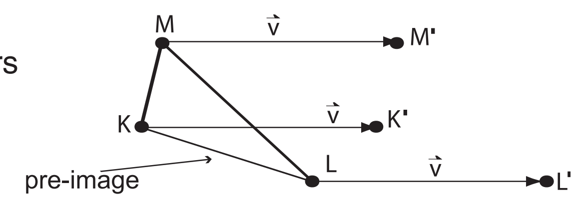
Solution: *Step 1:* Trace the triangle.



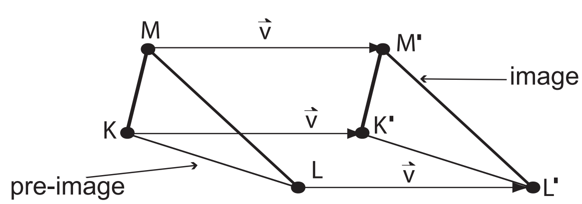
Step 2: Using points K, L, and M as the initial points, draw three vectors that are equal in length to $\vec{v}$, in both length and direction.



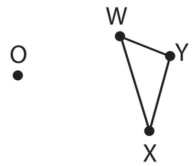
Step 3: Label the ending, or terminal points of these vectors K', L', and M' (read "K prime, L prime, and M prime") respectively.



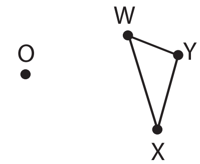
Step 4: Draw △K'L'M', which is the translation image along the vector v.



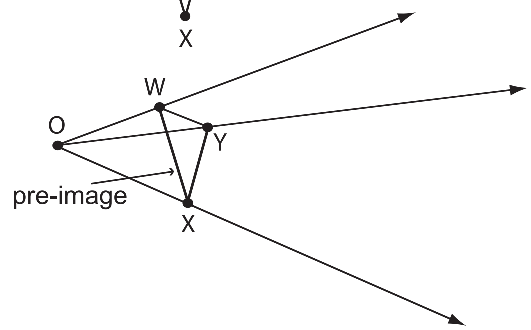
Example 4: In the diagram below, consider triangle WXY (△WXY) to be the pre-image of a dilation through a center of dilation O. Draw its dilation image using a scale factor of 1.5



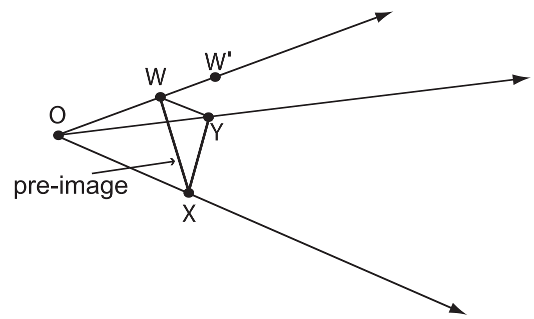
Solution: *Step 1:* Trace △WXY and O.



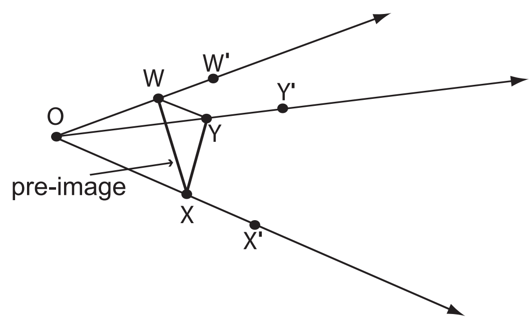
Step 2: Draw lines from point O through points W, X, and Y.



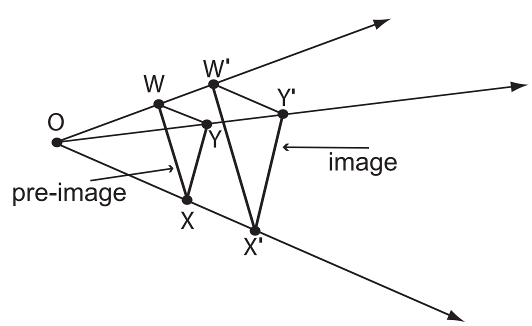
Step 3: Mark a point on $\overleftrightarrow{OW}$ which is a distance from point O, equal to 1.5 times the measure of segment OW, labeling it W'.



Step 4: Repeat this process on $\overleftrightarrow{OY}$ and $\overleftrightarrow{OX}$, labeling the endpoints of those measures as X', and Y'.



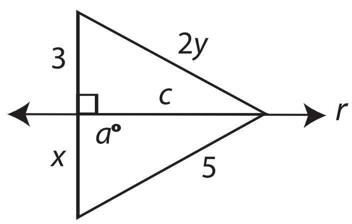
Step 5: Triangle W'X'Y' is the desired image of triangle WXY under a dilation through point O, with a scale factor of 1.5



Lesson 3 — Exercises: (cont'd)

Each diagram exercises 8 through10 below shows a polygon and its reflection image. Find the value of each placeholder (variable).

8.



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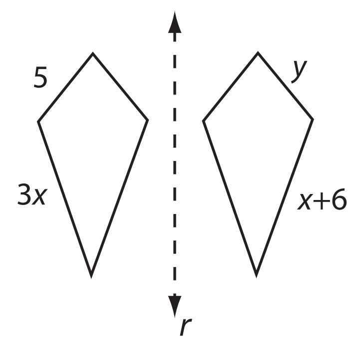
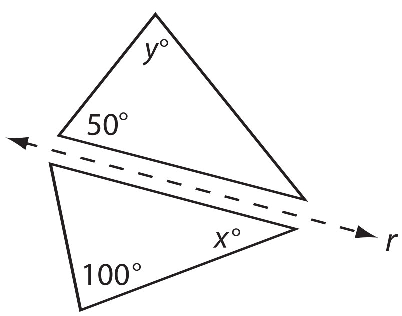
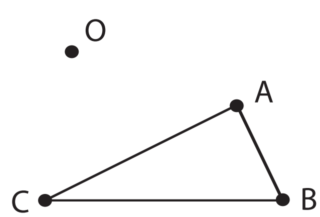


10.



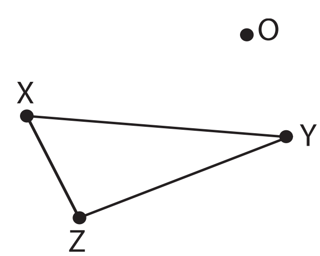
Draw the image of each figure in exercises 11and12 after a dilation, with center O and the given scale factor.

11.



Scale Factor 1.5

12.



Scale Factor 0.75

Unit I — The Structure of Geometry

Part A — What is Geometry?

Lesson 4 — *More on Relations*

Objective: To understand the basic relations, or "comparisons", of Geometry.

Important Terms:

Collinear Points – A set of points, all of which are contained in one straight line.

Non-Collinear Points – A set of points, for which there is no one straight line containing all of the points.

Coplanar Points – A set of points, all of which are contained in one plane.

Non-Coplanar Points – A set of points, for which there is no one plane containing all of the points.

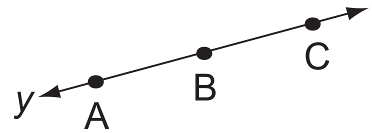
Coplanar Lines – A set of lines, all of which are contained in one plane.

Non-Coplanar Lines – A set of lines, for which there is no one plane containing all of the lines.

Parallel Lines – Two (or more) straight lines in the same plane which will never intersect. This relationship is symbolized by 2 vertical lines ($\parallel$). For example, the expression "*m* is parallel to *n*" is represented by $m \parallel n$.

Perpendicular Lines – Two straight lines which intersect to form a right angle. This relationship is symbolized by an inverted "T" ($\perp$). For example, the expression "*v* is perpendicular to *w*" is represented by $v \perp w$.

Example 1: Describe at least 3 relationships between points A, B, and C, and line *y*.



Solution: (answers will vary) Points A, B, and C are on line *y*. Points A, B, and C are contained in line *y*. Line *y* contains points A, B, and C. Line *y* passes through points A, B, and C.

Example 6: Using B = { a, b, c }, determine whether or not the following statements are true or false. Explain your answer.

a. $\{ b \} \in B$
b. $\{ a, c \} \subset B$
c. $c \in B$
d. $\{ \ \} \subset B$
e. $\{ a, b, c \} \subseteq B$

Solution:

a. False – While b itself is a member of the set B, the set itself is not a member of the set B. In fact, { b } is a subset of the set B.
b. True – The set whose members are a and c, is a subset of set B.
c. True – c is a member of the set B.
d. True – The empty set is a subset of every set. You cannot find anything in the empty set that is not in the set B.
e. True – The set whose members are a, b, and c, is an improper subset of set B, since it is equal to the set B. Every set is an improper subset of itself.

Example 7: Let A = { 1, 2, 3, 5, 7 }, B = { 0, 2, 3, 6, 9 }, C = { 4, 9 }, and find the following:

a. $A \cup B$
b. $A \cap B$
c. $A \cap C$
d. $B \cup C$
e. $B \cap C$

Solution:

a. { 0, 1, 2, 3, 5, 6, 7, 9 } – This is the union of the two sets. We have simply "combined" the two sets into one.
b. { 2,3 } – This is the intersection of the two sets. These are the only 2 elements which are common to the two sets.
c. { } – The intersection of the two sets is empty. The two sets have no elements in common.
d. { 0, 2, 3, 4, 6, 9 } – This is the union of the two sets.
e. { 9 } – This is the intersection of the two sets.

Lesson 5 — Exercises: (cont'd)

4. Tell which of the following could be paired as equal sets.

   A = { a, b, c, d }

   B = { x, y, z, w }

   C = { c, d, a, b }

   D = $\{ x \mid x = 1 \leq x \leq 4, x \in N \}$

   E = { }

   F = $\{ x \mid x = 2n - 1, n \in N \}$ Note: N represents the natural numbers

   G = { o }

   H = $\{ x \mid x = 2n + 1, n \in W \}$ Note: W represents the whole numbers

   I = $\emptyset$

5. Indicate which symbol $\{\in, \notin, \subset, \not\subset, \subseteq, \not\subseteq\}$ makes each of the following statements true.

   a) { 1, 2 } ______ { 1, 2 }

   b) 0 ______ { }

   c) { 1 } ______ { 1, 2 }

   d) 3 ______ { 1, 2, 3 }

   e) { 0, 1, 2 } ______ { even integers }

   f) { 0 } ______ { 0 }

   g) { 0 } ______ { }

6. Using A = { 1, 2, 3, 4 } and B = {3, 4, 5, 6 }, find $A \cap B$ and $A \cup B$.

7. Using A = { 0, 2, 4, 6, ... } and B = { 1, 3, 5, 7, ... }, find $A \cap B$ and $A \cup B$.

8. Using A = { 2, 4, 6, 8, ... } and B = { 1, 2, 3, 4, ...}, find $A \cap B$ and $A \cup B$.

Lesson 2 — Exercises:

For each of the figures in exercises 1 through12, determine which of the following classifications most specificially applies.

a) geometric figure

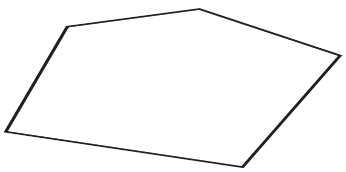
b) curve

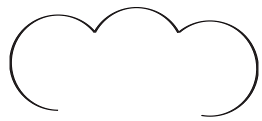
c) plane curve

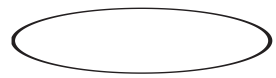
d) closed plane curve

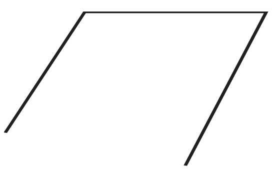
e) simple closed plane curve

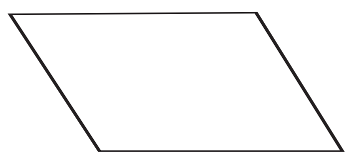
f) simple close plane curve made up only of straight line segments

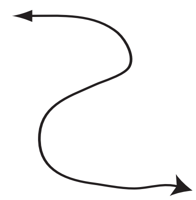
1. 

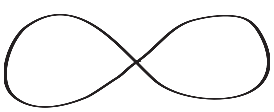
2. 

3. 

4. 

5. 

6. 

7. 

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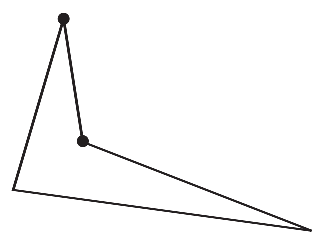
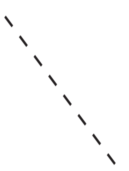
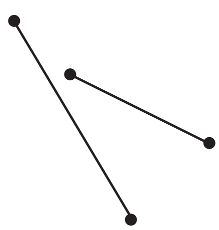
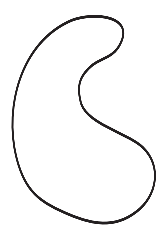
9. 

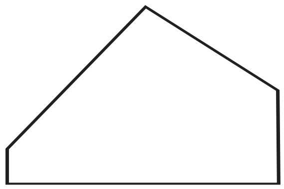
10. 

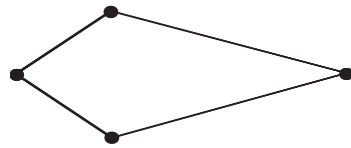
11. 

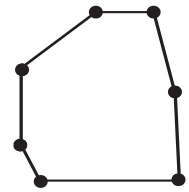
12. 

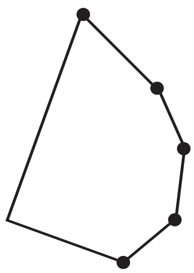
Lesson 3 — Exercises: (cont'd)

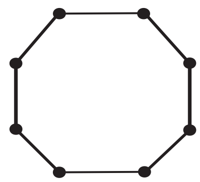
2. Name each of the following polygons, being as specific as possible. (Note: Remember that these exercises are designed to help you become familiar with all of the terminology related to polygons.You can never actually specify the measures or relationships between angles or sides, simply by observation. You must be given more information.)

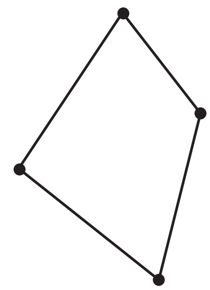
a. 

b. 

c. 

d. 

e. 

f. 

For each of the exercises 3 through 12, draw a rough sketch and then classify each triangle.

3. Sides of 6, 7, and 8 units

4. Sides of 6, 7, and 7 units

5. Sides of 7, 7, and 7 units

6. Sides of $10\sqrt{3}$, $5\sqrt{12}$, and $\sqrt{300}$

7. Sides of $\frac{1}{2}x$, $x - \frac{1}{2}x$, and $\frac{1}{4}(2x)$

8. Angles of 50, 60, and 70 degrees

9. Angles of 20, 90, and 70 degrees

10. Angles of 20, 120, and 40 degrees

11. Angles of 60, 60, and 60 degrees

12. Angles of x, $\sqrt{200}$, and 90 degrees

Lesson 3 — Exercises: (cont'd)

For the types of triangles listed in exercises 13 through 19, fill in the accompanying blanks.

Kind of triangle	Least number of
13. Scalene	Pairs of congruent sides _________
14. Isosceles	Pairs of congruent sides _________
15. Equilateral	Pairs of congruent sides _________
16. Equiangular	Pairs of congruent angles _________
17. Obtuse	Angles that are obtuse _________
18. Right	Angles that are right _________
19. Acute	Angles that are acute _________

Unit I — The Structure of Geometry

Part B — The Scope of Our Geometry

Lesson 4 — *Solids*

Objective: To briefly explore the connection between plane, or two-dimensional Geometry, and solid, or three-dimensional Geometry, by building the three basic three-dimensional figures.

Important Terms:

Prism – A three-dimensional geometric figure, created by "translating" a simple closed plane curve through space, and tracing the path of that translation. The pre-image and the image are generally called the bases of the prism and are used to name it.

Cylinder – A special prism, whose bases are circles.

Pyramid – A three-dimensional geometric figure, created by "connecting" all of the points on a simple closed plane curve to a point not in the plane of the curve. The simple closed plane curve is called the base of the pyramid, and is used to name it.

Important Terms: (cont'd)

Perimeter of a Rectangle – Formally, the perimeter P of a rectangle can be found by adding twice the measure of the length l of the rectangle, to twice the measure of the width w of the rectangle, as long as the length and width are measured in the same units. This is represented by the formula $P = 2l + 2w$. It is more generally represented as $P = 2a + 2b$, where a and b are 2 consecutive sides of the rectangle.

Perimeter of a Square – Formally, the perimeter P of a square can be found by multiplying the measure of one of the square's sides s by 4. This is represented by the formula $P = 4s$.

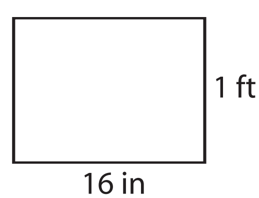
Area – Intuitively, the number of non-overlapping unit squares and parts of unit squares, which will exactly cover the interior of a simple closed plane curve.

Area of a Rectangle – Formally, the area A of a rectangle can be found by multiplying the measure of the base b, by the measure of the height h, as long as the base and height are measured in the same units. This is represented by the formula $A = b \bullet h$.

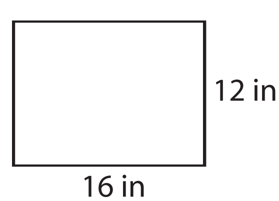
Area of a Square – Formally, the area A of a square, can be found by squaring the measure of a side s of the square. This is represented by the formula $A = s^2$.

Height (or Altitude) of a Rectangle – Using any side as the base, this is the measure of a line segment from a point on the base, drawn perpendicular to the line containing the opposite side.

Example 1: Find the perimeter, and the area, of the rectangle illustrated below, which is 1 foot high, and 16 inches wide.



Note: We cannot compute the perimeter and area of this rectangle in a meaningful way, until we have measured the rectangle using only one type of unit. In this case, it will be convenient to state all measurements in inches.



Solution: First, the perimeter of this rectangle can be found intuitively, by adding the measures of all of its sides.

$$P = 12 + 16 + 12 + 16 = 56 \text{ inches}$$

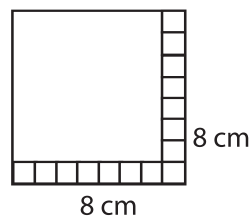
Example 2: (cont'd)

Formally, we could use the formula for the perimeter of a square.

$$
\begin{aligned}
P &= 4 \bullet s \\
&= 4 \bullet 8 \\
&= 32 \text{ centimeters}
\end{aligned}
$$

Second, the area of this square can be found intuitively, by placing 8 one-centimeter squares, in a row, along the base of the square. You can then place 7 more rows of these squares inside the rectangle, making a total of 64 square centimeters.

$$A = 8 \bullet 8 = 64 \text{ square centimeters}$$

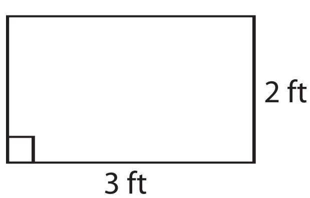


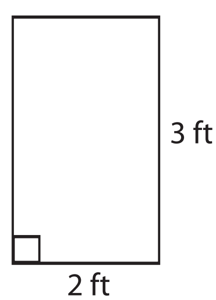
Formally, we could use the formula for the area of a square.

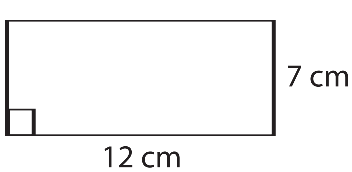
$$
\begin{aligned}
P &= s^2 \\
&= 8^2 \\
&= 64 \text{ centimeters}
\end{aligned}
$$

Lesson 1 — Exercises:

Find the area of each rectangle in exercises 1 through 6.

1. 

2. 

3. 

4. 

Lesson 1 — Exercises: (cont'd)

Complete the following table about squares. In exercises 23 and 24, express your answers in terms of *k* units.

	side	Perimeter	Area
19.		36′	
20.		20″	
21.			49 mm^2
22.			64 in^2
23.	2 *k* units		
24.	(*k* + 3) units		

Unit I — The Structure of Geometry

Part C — Measurement

Lesson 2 — *Parallelograms*

Objective: To understand, and demonstrate, the concepts of area and perimeter, as they relate to parallelograms.

Important Terms:

Parallelogram – A quadrilateral in which there are two pairs of parallel sides.

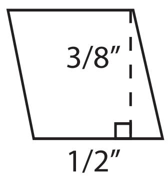
Rhombus – A parallelogram in which all four sides are of equal measure.

Perimeter – Intuitively, the measure of the distance around a simple closed plane curve. The perimeter of a polygon is simply the sum of the measures of its sides.

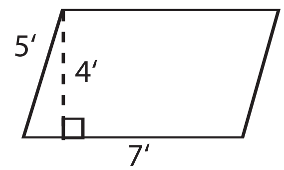
Height (or Altitude) of a Parallelogram – Using any side as the base, this is the measure of a line segment from a point on the base, drawn perpendicular to the line containing the opposite side.

Lesson 2 — Exercises: (cont'd.)

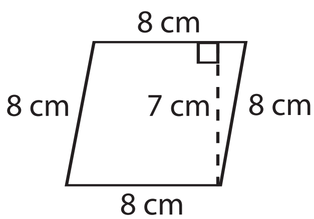
3.



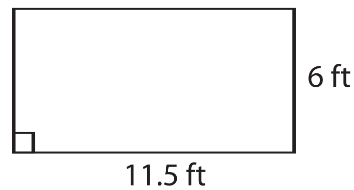
4.



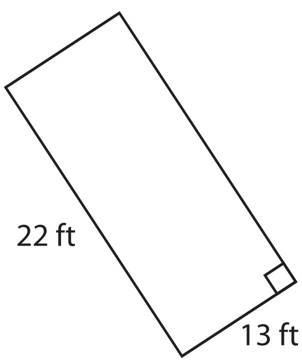
5.



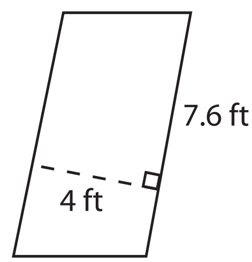
6.



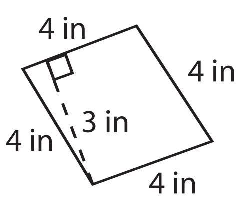
7.



8.



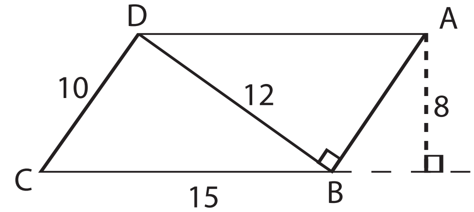
9.



10. Consider line segment CB as the base of parallelogram ABCD.

The area of the parallelogram is:

15 x ___________ = ___________ square units



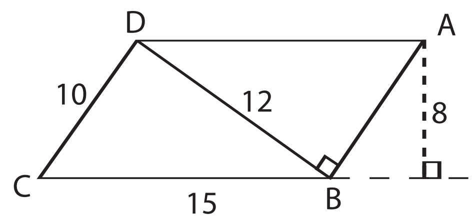
The perimeter of the parallelogram is:

15 + ___________ + ___________ + 10 = ___________ units

Lesson 2 — Exercises: (cont'd.)

11. Consider line segment AB as the base of parallelogram ABCD.

The area of the parallelogram is:



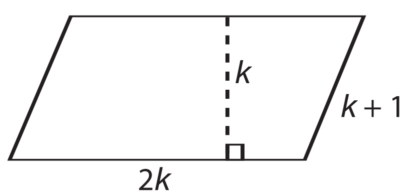
____________ x 12 = __________ square units

The perimeter of the parallelogram is:

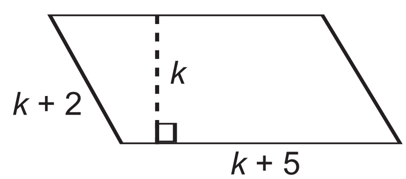
__________ + 10 + 15 + __________ = __________ units

Find the perimeter and the area of each parallelogram in exercises 12 through 14. Express your answers in terms of *k* units.

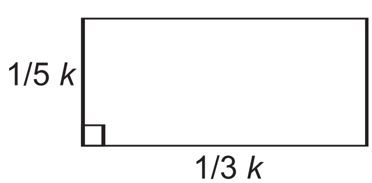
12.



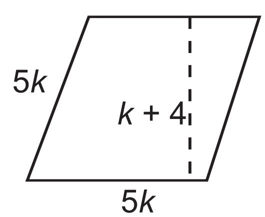
13.



14.



15.



Unit I — The Structure of Geometry

Part C — Measurement

Lesson 3 — *Triangles*

Objective: To understand, and demonstrate, the concepts of area and perimeter, as they relate to triangles.

Important Terms:

Triangle – A polygon made with three line segments.

Right Triangle – A triangle in which one of the angles is a right angle (90^{o}).

Perimeter – Intuitively, the measure of the distance around a simple closed plane curve. Formally, the perimeter of a polygon is simply the sum of the measures of its sides.

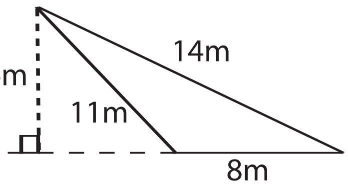
Perimeter of a Triangle – Formally, the perimeter of a triangle can be found by adding the measures of all three sides, as long as all of the sides are measured in the same units. There really is no standard formula for this relationship except to express it symbolically as $P = a + b + c$ where a, b, and c are the measures of the three sides.

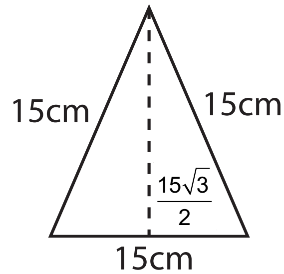
Area – Intuitively, the number of non-overlapping unit squares, and parts of unit squares, which can be fit into the interior of a simple closed plane curve.

Area of a Triangle – Formally, the area A of a triangle, can be found by multiplying the measure of the base b by one-half of the measure of the height h on that base, as long as the base and height are measured in the same units. This is represented by the formula $A = b \cdot \frac{1}{2} \cdot h$ or, more commonly, $A = \frac{1}{2} \cdot b \cdot h$.

Height (or Altitude) of a Triangle – Starting with any vertex, this is the measure of a line segment from that vertex, drawn perpendicular to the line containing the opposite side.

Lesson 3 — Exercises: (cont'd)

3.


4.


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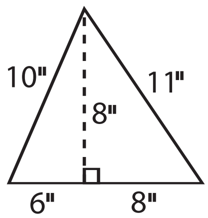

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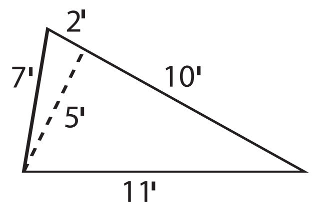
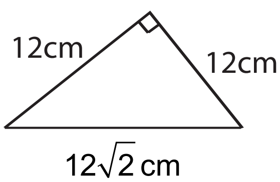
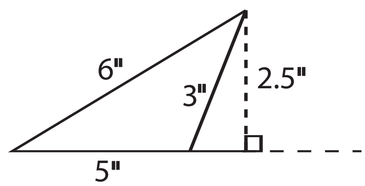
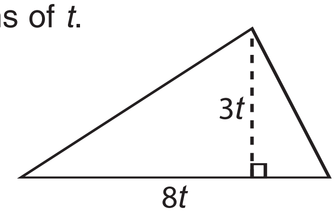
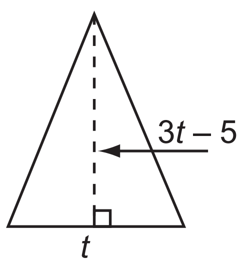


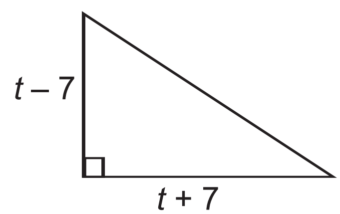
7.


8.


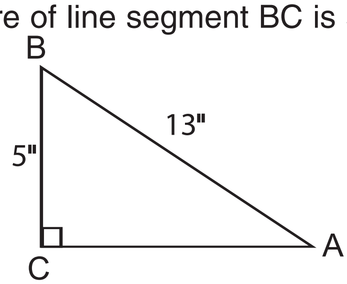
Find the area of each of the triangles in exercises 9 through 11. Express your answer in terms of *t*.

9.


10.


11.


12. Find the area of the given right triangle if the measure of the line segment AB is 13 inches and the measure of line segment BC is 5 inches.



Unit I — The Structure of Geometry

Part C — Measurement

Lesson 4 — *Trapezoids*

Objective: To understand, and demonstrate, the concepts of area and perimeter, as they relate to trapezoids.

Important Terms:

Trapezoid – A quadrilateral in which only one pair of opposite sides are parallel. Those parallel sides are called the bases of the trapezoid, and the height of the trapezoid is measured between those bases.

Perimeter – Intuitively, the measure of the distance around a simple closed plane curve. Formally, the perimeter of a polygon is simply the sum of the measures of its sides.

Perimeter of a Trapezoid – Formally, the perimeter of a trapezoid can be found by adding the measures of all four sides, as long as all of the sides are measured in the same units. There really is no standard formula for this relationship except to express it symbolically as P = a + b + c + d where a, b, c, and d are the measures of the four sides.

Area – Intuitively, the number of non-overlapping unit squares, and parts of unit squares, which can be fit into the interior of a simple closed plane curve.

Area of a Trapezoid – Formally, the area A of a trapezoid can be found by multiplying one-half the measure of the height h, by the sum of the measures of the bases b_1 and b_2 as long as the bases and the height are measured in the same units. This is represented by the formula $A = \frac{1}{2} \cdot h\left(b_1 + b_2\right)$.

Height (or Altitude) of a Trapezoid – Using either of the two parallel sides as the base, this is the measure of a line segment from a point on the base, drawn perpendicular to the line containing the opposite side.

Unit I — The Structure of Geometry

Part C — Measurement

Lesson 5 — *Regular Polygons*

Objective: To understand, and demonstrate, the concepts of area and perimeter, as they relate to regular polygons.

Important Terms:

Regular Polygon – A simple closed plane curve made only with straight line segments, in which all the sides are of equal measure, and all of the angles are of equal measure. In other words, the polygon is equilateral and equiangular.

Circumscribed Circle – A circle which completely encloses a polygon, and touches the polygon at all of its corners. In other words, the circle "contains" all of the corners of the polygon.

Inscribed Circle – A circle which is completely enclosed by a polygon, and barely touches all of the sides of the polygon.

Circumscribed Polygon – A polygon which completely encloses a circle, with all of its sides just touching the circle. This is equivalent to the polygon having an inscribed circle.

Inscribed Polygon – A polygon which is completely enclosed by a circle, and touches the circle at all of its corners. This is equivalent to the polygon having a circumscribed circle.

Center of a Regular Polygon – The point inside a regular polygon which is the same distance from all of the corners of the polygon. This is the same as the center of the circumscribed circle for that polygon.

Radius of a Regular Polygon – The distance from the center of the polygon to any one of the corners of the polygon. This is the same as the radius of the circumscribed circle for that polygon.

Apothem of a Regular Polygon – The shortest distance from the center of the polygon to any one of the sides of the polygon. This is the same as the radius of the inscribed circle for that polygon.

Important Terms: (cont'd)

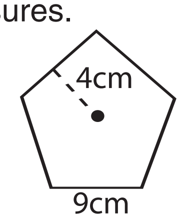
Perimeter – Intuitively, the measure of the distance around a simple closed plane curve. Formally, the perimeter of a polygon is simply the sum of the measures of its sides.

Perimeter of a Regular Polygon – Formally, the perimeter P of a regular polygon, can be found by multiplying the measure of one of its sides (call it s) by the total number of sides (call it n), since all of the sides of a regular polygon are of the same measure. As long as all of the sides are measured in the same unit, we can write this as a formula $P = n \cdot s$.

Area – Intuitively, the number of non-overlapping unit squares, and parts of unit squares, which can be fit into the interior of a simple closed plane curve.

Area of a Regular Polygon – Formally, the area A of a regular polygon, can be found by first taking one-half of the product, of the measures of a side s of the polygon, and the apothem a to that side. Then, we multiply that result by the number of sides n in the polygon. This can be represented by the formula $A = \frac{1}{2} \cdot a \cdot s \cdot n$. Further, noting that $s \cdot n$ has already been shown to be the perimeter of a regular polygon, this formula is often shortened to $A = \frac{1}{2} \cdot a \cdot P$, where P is the perimeter of the polygon.

Example : Find the perimeter and the area of the regular pentagon illustrated below, using the indicated measures.



Solution: First, the perimeter of this regular pentagon can be found intuitively, by simply adding the measures of all of its sides.

$$P = 9 + 9 + 9 + 9 + 9 = 45 \text{ cm}$$

Formally, we could use the formula for the perimeter of a regular polygon.

$$P = n \cdot s$$
$$= 5 \cdot 9$$
$$= 45 \text{ cm}$$

Example (cont'd)

Second, it is difficult, intuitively, to find the area of this regular pentagon without cutting it up in many pieces and rearranging, so we will find it formally, by using the formula for the area of a regular polygon. And, because we already know the perimeter of the polygon, we will use the shorter version of the formula.

$$A = \tfrac{1}{2} \cdot a \cdot P$$

$$= \tfrac{1}{2} \cdot 4 \cdot 45$$

$$= 90 \text{ square cm}$$

Lesson 5 — Exercises:

1. Complete each statement. (You might want to draw a sketch for each.)

   a) The apothem of a regular polygon is the radius of its _______?_______ circle.

   b) The radius of a regular polygon is the radius of its _______?_______ circle.

   c) The perimeter of a regular dodecagon with side length 8 is _______?_______ .

   d) The perimeter of a regular polygon with n sides, each of length s, is

      _______?_______ .

   e) The area of a regular octagon with perimeter 40 and apothem 6 is

      _______?_______ .

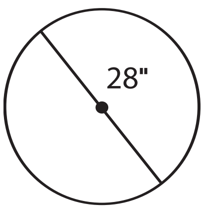
Lesson 6 — Exercises (cont'd)

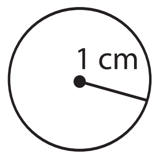
In exercises 4 through 6, the diameter of a circle is given. Find the radius.

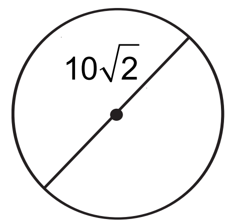
4. 41.6 inches
5. 8y units
6. $\sqrt{45}$ meters

In exercises 7 through 14, find the circumference and area of a circle with the given radius or diameter.

7. r = 6
8. d = 18.4 km
9. d = 3 cm

10. 

11. 

12. 

13. r = 9.8 cm
14. d = $2\frac{3}{5}$ ft

In exercises 15 through 17, find the radius of a circle with the given circumference.

15. $C = 61\pi$
16. $C = 4\pi\sqrt{3}$ cm
17. $C = \frac{\pi\sqrt{6}}{2}$ m

In exercises 18 through 20 find the radius of a circle with the given area.

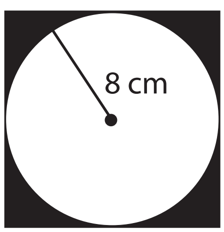
18. $4m^2$
19. $12\,in^2$
20. $8\ cm^2$

Lesson 6 — Exercises (cont'd)

In the following table, find the missing measures. Use 3.142 to approximate π and round all answers to the nearest hundredth.

	radius	diameter	Circumference	Area
21.	5 yds			
22.		8 m		
23.			28π in	
24.				$12\pi\ m^2$

25. Find the area of the shaded region. Give an exact answer.



Unit I — The Structure of Geometry

Part C — Measurement

Lesson 7 — *Prisms*

Objective: To understand, and demonstrate, the concepts of total area and volume, as they relate to prisms.

Important Terms:

Prism – A three-dimensional geometric figure created by "translating" a simple closed plane curve, through space, and tracing the path of that translation. The pre-image and the image are generally called the "bases" of the prism and are used to name it. All of the other faces are parallelograms, called the "lateral faces" of the prism, which are joined to each other at the "lateral edges".

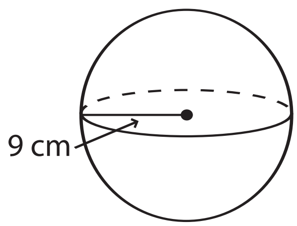
Lesson 9 — Exercises:

Evaluate each expression in exercises 1 through 8.

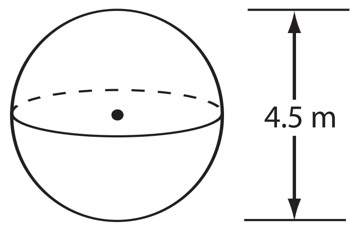
1. $\left(\frac{2}{3}\right)^3$
2. $\left(\frac{4}{5}\right)^3$
3. $\sqrt[3]{64}$
4. $\sqrt[3]{\frac{27}{125}}$
5. $\left(\sqrt{3}\right)^3$
6. $(.1)^3$
7. $(.01)^3$
8. $\left(\frac{3\sqrt{2}}{4}\right)^3$

Find the surface area and volume of each sphere in exercises 9 through 12. Give your answers in exact form, and also approximated form, using 3.14 for π, and rounding your result to the nearest tenth.

9.



10.



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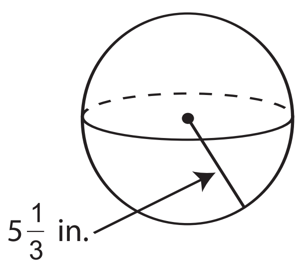
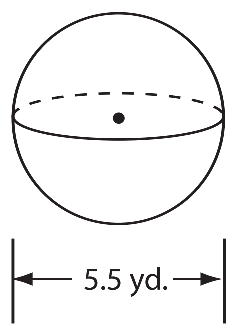


12.



13. If the surface area of a sphere is $9\pi\ m^2$, find the radius and the volume.

14. The volume of a sphere is 288π yds^3. Find the radius and the surface area.

15. The volume of a sphere is $4\pi\sqrt{3}$ inches^3. Find the radius and the surface area.

Example 2: Joyce has been shopping for a pet, and each of the three bulldogs she has seen, has barked loudly, and snapped at her. What conjecture might Joyce make, based on her experience? Can her conjecture be justified?

Solution: Joyce may decide never to own a bulldog, conjecturing that all bulldogs are vicious. Her conjecture is probably not valid, considering she has only seen three bulldogs. She may, in fact, have had the misfortune to see three bulldogs that were mistreated as pups, or were raised to be vicious. Notice, however, that we can only say that her conjecture is "probably" not valid, as we have no evidence otherwise, unless we have actually seen a bulldog that was friendly and gentle.

Lesson 1 — Exercises:

1. A farmer uses a method of weed control successfully in 12 different fields. What conclusion may the farmer reach?

2. After picking marigolds for the first time, Charlene began to sneeze. She also began sneezing the next five times she was near marigolds. What conclusion might she reach based on this experience?

3. Chauncy noticed that broccoli had been on the school lunch menu for the past six Thursdays. What conclusion might he reach based on this experience?

4. Because Susan finds that scarlet poppies, scarlet verbena, scarlet hawthorn, and scarlet honeysuckle have no odor, she says that scarlet flowers have no odor. Can this conclusion be justified? Explain your answer.

5. Can each of the following conclusions be justified? Give a reason for your answer.

   a. Mrs. Purdy will not patronize a store which two years ago sold her a dress that did not wear well.

   b. Mrs. Hopper will not patronize Joe's Market because a bag of their potatoes was underweight.

   c. Mr. Allen is planning a yard sale for May 30th this year. He knows it will not rain, since May 30th has been clear for the last three years.

6. For each group listed, give a possible use of inductive reasoning.

   a. Students
   b. Sports Fans
   c. Motorists
   d. Investors
   e. Employers
   f. Poll Takers

Unit I — The Structure of Geometry

Part D — Inductive Reasoning

Lesson 2 — *Applications in Mathematics*

Objective: To demonstrate the use of inductive reasoning in various areas of Mathematics.

Important Terms:

Analytical Thinking – This is the type of mental activity which separates a concept, as a whole, into its individual parts, so that each part can be studied independently.

Intuition – This is the type of mental activity which gives information or beliefs, based on hunches or insight.

Conjecture – Often called a guess, this is an opinion based on incomplete or inconclusive evidence, but supported by intuition.

Inductive Reasoning – This is the process of finding a general principle (called an induction), based upon the evidence of a finite number of specific cases.

Example 1: Look for a pattern in each of the following sequences of numbers, and use inductive reasoning to predict the next number in each sequence.

a) 3, 6, 12, 24, ___ **b)** 11, 15, 19, 23, ___ **c)** 5, 6, 8, 11, 15, ___

Solution: **a)** It is fairly easy to see that each number seems to be twice the preceding number. It therefore seems reasonable to predict that the next number in the sequence will be 48. Notice, however, that we cannot be absolutely certain. For example, the creator of this sequence may want us to simply repeat this sequence of four numbers again. In that case, the next number would be 3.

Lesson 2 — Exercises: (cont'd)

2. For each of the number patterns below, study the pattern, and make a conjecture about the number that makes the last statement true. Use a calculator to prove or disprove your conjecture.

a) 1 (9) + 2 = 11
12 (9) + 3 = 111
123 (9) + 4 = 1,111
1,234 (9) + 5 = 11,111
12,345,678 (9) + 9 = _______

b) 9 (9) +7 + 88
98 (9) + 6 = 888
987 (9) + 5 = 8,888
9,876 (9) + 4 = 88,888
9,876,543 (9) + 1 = _______

3. In doing his geometry assignment, Allan added the lengths of the sides of triangles to find the perimeters. Noticing the results for several equilateral triangles, he guesses that the perimeter of an equilateral triangle could be found another way. What did he mean?

4. Sarah observes that $(-1)^2 = +1$, $(-1)^4 = +1$, and $(-1)^6 = +1$. She concludes that every _______ power of _______ is +1.

5. For each number sequence below, look for a pattern and predict the next two numbers. Write a sentence describing how you found these numbers.

a) 1, 4, 16, 64, _____, _____, ...

b) 18, 15, 12, 9, _____, _____, ...

c) 1, $\frac{1}{3}$, $\frac{1}{9}$, $\frac{1}{27}$, _____, _____, ...

d) 1, 4, 9, 16, _____, _____, ...

e) 2, 3, 5, 8, 12, _____, _____, ...

f) 10, 12, 16, 22, 30, _____, _____, ...

g) 40, 39, 36, 31, 24, _____, _____, ...

6. In the following endless sequence of numbers, the difference is always 6. Do you think that all numbers in the sequence 5, 11, 17, 23, ... are prime numbers? Why or why not?

7. Use inductive reasoning to determine the total number of line segments determined by twenty collinear points. (Hint: Start with 2 labeled points on the line, which will generate 1 segment. Then, place a 3rd labeled point on the line, which will generate 2 additional segments. Now place a 4th labeled point on the line, which will generate 3 additional segments. Continue this process until you detect a pattern.)

8. Eighteen distinct coplanar lines intersect in one point. Use inductive reasoning to determine the number of rays in this figure.

Lesson 2 — Exercises: (cont'd)

2. When possible, draw valid conclusions from each of the following pairs of general and specific statements. If a valid conclusion is not possible, explain why it is not possible.

   a. General Statement – All rational numbers are real numbers.

      Specific Statement – An integer is a rational number.

   b. General Statement – All points have no size.

      Specific Statement – A, B, C, are points.

   c. General Statement – All line segments are sets of points.

      Specific Statement – A triangle is a set of points.

   d. General Statement – Distinct intersecting lines have exactly one point in common.

      Specific Statement – Lines p and q intersect.

   e. General Statement – A closed phrase is a mathematical expression which does not contain a relation symbol or a placeholder symbol.

      Specific Statement – 3 (7-5) is a closed phrase.

Lesson 1— Exercises: (cont'd)

3. Suppose the following statements are true.

Max is happy. (let that be p)

Sam is alive. (let that be q)

Write each of the following compound or negated statements symbolically, using only the symbols p, q, $\vee$, $\wedge$, and ~

a) Sam is alive and Max is happy.
b) Max is not happy.
c) Sam is alive and Max is sad.
d) It is false that Sam is alive.
e) Sam is not alive and Max is sad.
f) Max is happy or he is sad.
g) Sam is dead and Max is happy.
h) It is false that Max is sad.

4. Determine if each sentence is a negation of the given sentence. Write yes if it is a negation, and write no if it is not a negation.

-1 is a real number.

a) -1 is an imaginary number.
b) -1 is really a number.
c) -1 is a real number.
d) -1 is not really a number.
e) -1 is not a real number.
f) -1 is a rational number.

5. Determine if each sentence is a negation of the given sentence. Write yes if it is a negation, and write no if it is not a negation.

A triangle has four sides.

a) A triangle has three sides.
b) A triangle does not have sides.
c) A triangle has no sides.
d) A triangle does not have four sides.
e) Four triangles have sides.
f) A triangle has four sides.

Lesson 1— Exercises: (cont'd)

6. Tell whether each of the following statements are true or false.

a) All conjunctions are true.
b) If both p and q are false statements, then $p \wedge q$ is true.
c) If both p and q are false statements, then $p \vee q$ is false.
d) All disjunctions are true.
e) If at least p or q is a false statement, then $p \wedge q$ is false.
f) If at least p or q is a true statement, then $p \vee q$ is true.
g) A conjunction and its negation may both be true.
h) The conjunction of a statement p and its negation $\sim p$ is false.
i) The disjunction of a statement p and its negation $\sim p$ is true.

7. Suppose statement p is true and statement q is false. Determine if Julie would be telling the truth, making the following statements.

a) $p \wedge q$
b) $p \vee q$
c) $\sim p$
d) $\sim q$

8. Consider the following table.

Statement	p	q	r	s	t
Truth Value	T	T	T	F	F

Classify the following statements as true or false.

a) $p \wedge q$
b) $p \wedge t$
c) $r \vee p$
d) $r \vee t$
e) $p \wedge r$
f) $q \wedge r$
g) $p \wedge s$
h) $r \vee q$
i) $s \vee t$
j) $r \wedge s$
k) $r \vee s$
l) $t \vee p$
m) $t \vee q$
n) $p \vee t$

Unit I — The Structure of Geometry

Part F — Logic

Lesson 2 — *Conditionals*

Objective: To define conditional statements as they relate to mathematical deductive logic, and to demonstrate an understanding of their application in syllogisms, and the various laws of logic.

Important Terms:

Logic – A system of reasoning, in an orderly fashion, which draws conclusions from specific premises.

Simple Statement – A sentence in logic which declares that something is either true or false, but not both true and false, at the same time.

Symbolic Form of a Simple Statement – Used for convenience, this is the conventional way to represent simple statements in logic. For example, instead of having to repeatedly write, "X is a prime number", in an argument, we represent that entire statement as p.

Conditional Statement – A statement consisting of two phrases.The first is called the "hypothesis", often beginning with the word "if", "when", or some equivalent word. The second is called the "conclusion", and usually begins with the word "then". The purpose of writing statements in this form is to allow us to analyze them in an orderly fashion, and establish certain truths, "if" some specified condition is met. A conditional statement can also be represented symbolically by "if p, then q", or, even more simply, $p \rightarrow q$.

Syllogism – A logical argument consisting of three statements, two of which are accepted as being true, and a third which is to be drawn from the acceptance of those two. Particularly, the first statement is called the "major premise", and is a general statement, detailing the conditions to be satisfied, and the resulting conclusion. The second statement is called the "minor premise", and is a specific statement which satisfies the conditions of the major premise. Finally, the third statement is called the "conclusion" of the syllogism, and is considered to be the satisfactory end of the argument. For example, let the conditional $p \rightarrow q$ be the major premise, and let p be the minor premise. Considering both of these statements to be true, we can conclude q as a valid conclusion. This type of logical argument is foundational to the study of logic, and is also known by other names, including "the law of detachment", "confirming the hypothesis", and the Latin phrase, "modus ponens", meaning "method of consideration".

Lesson 2— Exercises:

Each of the exercises 1 through 4 below consists of two premises of a valid syllogism. In each case, state the conclusion.

1. All players with three strikes are out. David has three strikes.

2. All animal trainers are brave persons. Georgio is an animal trainer.

3. All pigs have four legs. A Yorkshire is a pig.

4. All geekums are gaggles. Slokum is a geekum.

In exercises 5 through 8 below, the first statement is one premise of a valid syllogism. The second statement is the conclusion. In each case, state the missing premise and tell if it is the major or minor premise.

5. All girls are pretty. Samantha is pretty.

6. Mike is a gentleman. Mike is a scholar.

7. James is tall. James is a Texan.

8. $\sqrt{11}$ is an irrational number. $\sqrt{11}$ is a non-terminating, non-repeating decimal.

In exercises 9 through 14 below, the last statement in each syllogism is the conclusion. State whether each syllogism is valid. If the syllogism is not valid, rewrite it to make it valid.

9. All Americans are free. A Floridian is an American. A Floridian is free.

10. All boys are clever. Josie is clever. Josie is a boy.

11. All class presidents are bright. Frankie is bright. Frankie is a class president.

12. All bipeds are quadrupeds. A horse is a biped. A horse is a quadruped.

13. Use Euler Circles (a special type of Venn Diagram using circles inside of circles) to pictorially show the syllogism in Exercise 7 to be valid.

14. Use Set Notation to illustrate the syllogism in Exercise 9. (Hint: You might draw a Venn Diagram showing the relationships.)

Lesson 2— Exercises: (cont'd)

For each of the statements in exercises 15 through 18, state the hypothesis and the conclusion.

15. If today is March 10, then yesterday was March 9.

16. If $x - 6 = 20$, then $x = 26$.

17. If you ride a bicycle, then you have strong legs.

18. If $x < 0$, then $5x > 6x$.

In exercises 19 through 22, write the statements in "if – then" form.

19. All residents of Dallas are residents of Texas.

20. A number is a real number if it is a rational number.

21. Two numbers are odd only if their sum is even.

22. Residence in Chicago implies residence in Illinois.

In exercises 23 and 24, statements p, q, and r are given. Write a valid argument using the law of syllogism (or chain rule).

23. p: Clayton is a 12th grader.
q: He is a senior.
r: He is in government class.

24. p: It is raining.
q: The sky is cloudy.
r: You cannot see the sun.

Assume the following statements in exercises 25 and 26 are true. Use the law of detachment to write another true statement.

25. If two lines are perpendicular, then they intersect. Lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are perpendicular.

26. If a figure is a square, then it has four sides. Figure ABCD has four sides.

Lesson 2— Exercises: (cont'd)

Assume the statements in exercises 27 and 28 are true. Explain why it is not possible to write another true statement using the law of detachment or the law of syllogism (chain rule).

27. If an animal is a mammal, then it is warm-blooded. A cat is warm-blooded.

28. If you vote for me, then your taxes will be lower. If you vote for me, then I will save you money.

State the converse of each of the conditionals in exercises 29 through 36, and tell whether the converse is always, sometimes, or never true.

29. If a man lives in Indianapolis, then he lives in Indiana.

30. If an animal is a normal horse, then it has four legs.

31. If it is raining, then the grass is wet.

32. If you are a native Californian, then you were born in California.

33. If a number is a multiple of 3, then the number is an odd number.

34. If 6 is a factor of an integer, then 6 is a factor of the square of an integer.

35. If a triangle is equilateral, then the triangle is isosceles.

36. If two numbers are both odd, then their sum is odd.

In exercises 37 through 40, make up an example, using words and/or numbers that illustrates each situation.

37. A conditional and its converse that are both true.

38. A conditional and its converse that are both false.

39. A true conditional statement whose converse is false.

40. A false conditional statement whose converse is true.

In exercises 41 and 42, write each of the given equivalences in two ways, using the "if and only if" phrase, and necessary and sufficient wording.

41. If x is an odd integer, then x^2 is an odd integer, and conversely.

42. If a geometric figure has four sides, then it is a quadrilateral, and if a geometric figure is a quadrilateral, then it has four sides.

Unit I — The Structure of Geometry

Part F — Logic

Lesson 3 — *Negations of Conditionals*

Objective: To define inverses, as negations of conditionals, and to demonstrate an understanding of their application to contrapositives.

Important Terms:

Logic – A system of reasoning, in an orderly fashion, which draws conclusions from specific premises.

Simple Statement – A sentence in logic which declares that something is either true or false, but not both true and false, at the same time.

Conditional Statement – A statement consisting of two phrases, one of which is called the "hypothesis", often beginning with the word "if", "when", or some equivalent word. The other statement is called the "conclusion", and usually begins with the word "then". The purpose of writing statements in this form is to allow us to analyze them in an orderly fashion, and establish certain truths, "if" some specified condition is met. A conditional statement can be represented symbolically by "if p, then q", or, even more simply, $p \rightarrow q$.

Converse – A conditional which results from interchanging the hypothesis and conclusion in a given conditional. For example, let $p \rightarrow q$ be a given conditional. Its converse would be $q \rightarrow p$. It is important to remember that a converse does not necessarily have the same truth value as the original conditional.

Inverse – A conditional which results from negating both the hypothesis and the conclusion in a given conditional. In other words, we are simply inserting the phrase, "It is not the case that", before the conditional. For example, let $p \rightarrow q$ be a given conditional. Its inverse would be, "It is not the case that $p \rightarrow q$". Symbolically, the inverse would be, $\sim p \rightarrow \sim q$. It is important to remember that an inverse does not necessarily have the same truth value as the original conditional.

Contrapositive – A conditional which results from negating the converse of a given conditional. For example, let $p \rightarrow q$ be a given conditional. Its converse would be $q \rightarrow p$. The contrapositive is then formed by negating that converse to get, $\sim q \rightarrow \sim p$. It is important to remember that a contrapositive has exactly the same truth value as the original conditional.

Lesson 3— Exercises:

Considering each of the conditionals in exercises 1 through 5 to be true, write the inverse, converse, and contrapositive for each, and determine which statements are true and which are false.

1. If $x^2 = 4$, then $x = 2$ or $x = {}^{-}2$.

2. If two angles are right angles, then they are congruent.

3. If you finish a marathon, then your physical condition is good.

4. If you are a zebra, then you do not know how to fly.

5. If you have a cold, then you are sick.

Using the given statements in exercises 6 through 8, show that the converse and the inverse of a statement are either both true or both false.

6. If an angle is a right angle, then the angle measures 90°.

7. If $m > 0$, $m^2 > 0$.

8. If an animal is a trout, then the animal is a fish.

9. Make up a conditional statement, written in "if – then" form. Then rewrite the inverse, of the converse, of its contrapositive. Explain the result and tell why it turns out as it does.

10. Dana is a lawyer. She wants to prove that her client, Jim, did not steal an expensive watch from a jewelry store show case. Obviously, if Jim stole the watch, then he was in the jewelry store. How can Dana prove Jim is innocent without ever saying Jim did not steal the watch? Why will this strategy work?

Unit I — The Structure of Geometry

Part F — Logic

Lesson 4 — *Fallacies*

Objective: To examine several types of false reasoning, in order to be aware of, and avoid, illogic, in our study of Geometry.

Important Terms:

Fallacy – An illogical argument, based on false assumptions, poorly defined terms, and/or facts which are not relevant, all leading to a false conclusion. In general, when a person uses a reasoning process that is not correct, we say that there is a "fallacy" in the process, and that the thinking is "fallacious". The word itself comes from a Latin word meaning, "to deceive".

Circular Reasoning – An illogical argument which attempts to "prove" a statement p, by assuming, at some point in the argument, that p is true. This is also called "arguing in a circle".

Reasoning on the Converse – An illogical argument which attempts to use the converse of a true conditional to prove a statement. In other words, if $p \rightarrow q$ is a true conditional, and, in fact, q is true, we still cannot proceed to the conclusion that p is true. This is also called "asserting the consequent".

Denying the Premise – An illogical argument which attempts to use the inverse (or negation) of a true conditional to prove a statement. In other words, if $p \rightarrow q$ is a true conditional, and, in fact, p is false (or ~p is true), we still cannot proceed to the conclusion that q is false (or ~q is true).

Faulty Analogy – An illogical argument which attempts to use similarities between two situations, to erroneously conclude that the two situations must be alike in some other way. In other words, just because something is true in one situation, does not mean that it must be true in the other situation.

Example 1: Brad and Chad are twins. They are alike in most physical characteristics. They both like guitar music. They both like to play golf, and both shoot about the same score. Intellectually, their abilities are about the same. Brad is a successful member of the debate team. If we reason by analogy, what conclusion might we make?