VideoText Interactive

HomeSchool and Independent Study Sampler

Print Materials for “Algebra: A Complete Course”

Unit II, Part A, Lesson 4 – “Combinations”

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COMBINATIONS

\[3w + 4 = 10\]

Add \(-4\)
\[3w + 4 + (-4) = 10 + (-4)\]
\[3w = 6\]

Multiply \(\frac{1}{3}\)
\[\frac{1}{3}(3w) = \frac{1}{3}(6)\]
\[w = \frac{6}{3}\] or \(2\)

Check
\[3(2) + 4 = 10\]
\[6 + 4 = 10\]
\[10 = 10\] True

\[S = \{2\}\]

Or ...

\[3w + 4 = 10\]
Multiply \(\frac{1}{3}\)
\[\frac{1}{3}(3w + 4) = \frac{1}{3}(10)\]
\[w + \frac{4}{3} = \frac{10}{3}\]
Add \(-\frac{4}{3}\)
\[w + \frac{4}{3} + \left(-\frac{4}{3}\right) = \frac{10}{3} + \left(-\frac{4}{3}\right)\]
\[w = \frac{6}{3}\] or \(2\) ✔
\[-6x - 2 \geq 40\]

Add 2

\[-6x - 2 + 2 \geq 40 + 2\]

Multiply \(\frac{1}{-6}\)

\[-\frac{1}{6}(-6x) \leq \frac{1}{-6}(42)\]

\[1x \leq \frac{42}{-6}\text{ or } -7 \checkmark\]

Check

\[-6(-10) - 2 \geq 40\]

\[60 - 2 \geq 40\]

\[58 \geq 40\text{ True}\]

\[S = \{x \mid x \leq -7\}\]
LESSON 4  Combinations

Objective: To be able to solve simple equations or inequalities by making 0’s and 1’s appropriately.

Important Terms:

"The Opposite Of" – a real number which has the same absolute value as a given number, but the opposite sign, so that the sum of the two numbers is 0. For example, the opposite of +3 is -3, because +3 + (-3) = 0.

Reciprocal – a real number (not equal to 0) which has the same sign as a given number but which, in fraction form, has the numerator and denominator interchanged, so that the product of the two numbers is 1. For example, the reciprocal of \( \frac{1}{3} \) is \( \frac{3}{1} \) because \( \frac{1}{3} \times \frac{3}{1} = \frac{3}{3} = 1 \) or 1.

Example 1: Find the solution set for the following open sentence by making the appropriate 0’s and 1’s.

\[ 3x - 8 = 34 \]

Solution: In this equation, we are trying to find appropriate values for “1” of the placeholder. That means we want only 1x, so we must make a “1” out of the 3 and a “0” out of the -8.
Example 1 cont'd:

**Method 1**
We make the "1" first by multiplying by $\frac{1}{3}$ (the reciprocal of 3 or $\frac{3}{1}$).

\[
3x - 8 = 34
\]

**Mult. $\frac{1}{3}$**

\[
\frac{1}{3}(3x - 8) = \frac{1}{3}(34)
\]

\[
x - \frac{8}{3} = \frac{34}{3}
\]

Now make a zero.

\[
\text{Add } \frac{8}{3}
\]

\[
x - \frac{8}{3} + \frac{8}{3} = \frac{3}{3} + \frac{8}{3}
\]

\[
x + 0 = \frac{42}{3}
\]

\[
x = 14
\]

We check this solution by substitution in the original equation.

\[
3(14) - 8 = 34
\]

\[
42 - 8 = 34
\]

\[
34 = 34 \quad \text{It checks.}
\]

The solution set is $S = \{14\}$.

**Method 2**
We make the "0" first by adding +8 (the opposite of -8).

\[
3x - 8 = 34
\]

**Add +8**

\[
3x - 8 + (+8) = 34 + (+8)
\]

\[
3x + 0 = 42
\]

\[
x = 42
\]

Now make a 1. Multiply by $\frac{1}{3}$ (the reciprocal of 3 or $\frac{3}{1}$).

**Mult. $\frac{1}{3}$**

\[
\frac{1}{3}(3x) = \frac{1}{3}(42)
\]

\[
\frac{3}{3}x = \frac{42}{3}
\]

\[
x = 14
\]
Example 1 cont’d:

We already know this is the correct answer.

Notice that it makes no difference whether we make the 1 or 0 first. Upon closer examination, however, you might prefer to make the 0 first as that may possibly eliminate some of the fractions which may occur in the solution process.

Example 2: Find the solution set for each of the following open sentences by making the appropriate 0’s and 1’s.

a. \(-3x + 1 < -26\)  
b. \(\frac{3n}{4} - 6 \geq 3\)

Solution: a. We want to make a “0” out of the +1, so we add -1 (its opposite).

\[-3x + 1 < -26\]
Add -1

\[-3x + 1 + (-1) < -26 + (-1)\]

\[-3x < -27\]

We want to make a “1” out of the -3, so we multiply by \(\frac{1}{-3}\) (its reciprocal).

Mult. \(\frac{1}{3}\)

\(\frac{1}{3}(-3x) > \frac{1}{-3}(-27)\)

\(-x > -9\)

\(x < 9\)

Notice we reverse the relation symbol when we multiply an inequality by a negative number.

We can partially check this range of solutions by a sample substitution in the original inequality. 11 > 9, so we will try that sample.

\[-3(11) + 1 < -26\]

\[-33 + 1 < -26\]

\[-32 < -26\] It checks.

The solution set is as follows:
Example 2 cont’d:

b. We want to make a “0” out of the –6 so we add +6 (its opposite).

\[
\frac{3n}{4} - 6 \geq 3
\]

Add +6

\[
\frac{3n}{4} - 6 + (+6) \geq 3 + (+6)
\]

\[
\frac{3n}{4} + 0 \geq 9
\]

\[
\frac{3n}{4} \geq 9
\]

We want to make a “1” out of the \(\frac{3}{4}\), so we multiply by \(\frac{4}{3}\) (its reciprocal).

Mult. \(\frac{4}{3}\)

\[
\frac{4}{3} \left( \frac{3n}{4} \right) \geq \frac{4}{3} (9)
\]

\[
\frac{12n}{12} \geq \frac{36}{3}
\]

\[
1n \geq 12
\]

Again we partially check this range of solutions by a sample substitution in the original inequality. \(16 \geq 12\), so we will try that sample.

\[
\frac{3(16)}{4} - 6 \geq 3
\]

\[
\frac{48}{4} - 6 \geq 3
\]

\[
12 - 6 \geq 3
\]

\[
6 \geq 3
\]

It checks.

The solution set is as follows:
Lesson 4 – Exercises:

Find the solution set for each of the following open sentences by making the appropriate 0’s and 1’s.

1. \(2n - 1 = 5\)  
2. \(\frac{x}{2} - 6 = 14\)  
3. \(\frac{r}{4} + 8 = 7\)  
4. \(3t + 5 = 29\)  
5. \(3t + 8 > 20\)  
6. \(4x - 12 < 16\)  
7. \(\frac{n}{8} + 16 > 15\)  
8. \(\frac{7x}{9} - 3 \geq 4\)  
9. \(5c + 7 < 18\)  
10. \(2w + 7 \leq 1\)  
11. \(-6z - 7 \geq 11\)  
12. \(5x + 4 \leq -6\)  
13. \(\frac{2}{3}x - 5 < 7\)  
14. \(\frac{3}{4}y - 2 < -8\)  
15. \(4x + 13 \geq 5\)  
16. \(2z - 1 > 7\)  
17. \(-5m - 10 < 25\)  
18. \(9y + 4 > -14\)  
19. \(\frac{2m}{3} - 5 \geq 1\)
Lesson 4 – Combinations

1. \(2n - 1 = 5\)
   \(2n - 1 + 1 = 5 + 1\)
   \(2n + 0 = 6\)
   \(\frac{1}{2}(2n) = \frac{1}{2} \cdot 6\)
   \(\frac{1}{2} \cdot n = \frac{1}{2} \cdot 3\)
   \(n = 3\)
   \(S = \{3\}\)

2. \(\frac{3}{4} - 6 = 14\)
   \(\frac{3}{4} - 6 + 6 = 14 + 6\)
   \(\frac{3}{4} + 0 = 20\)
   \(\frac{3}{4} = 20\)
   \(\frac{3}{4} \cdot x = 20\)
   \(\frac{3}{4} \cdot \frac{1}{4} = \frac{3}{4} \cdot 20\)
   \(\frac{3}{4} \cdot x = 40\)
   \(1 \cdot x = 40\)
   \(x = 40\)
   \(S = \{40\}\)

3. \(\frac{3}{4} + 8 = 7\)
   \(\frac{3}{4} + 8 + 8 = 7 + 8\)
   \(\frac{3}{4} + 0 = 1\)
   \(\frac{3}{4} = 1\)
   \(\frac{3}{4} \cdot r = \frac{3}{4} \cdot (-1)\)
   \(\frac{3}{4} \cdot r = -4\)
   \(1 \cdot r = -4\)
   \(r = -4\)
   \(S = \{-4\}\)

4. \(3t + 5 = 29\)
   \(3t + 5 + 5 = 29 + 5\)
   \(3t + 0 = 24\)
   \(3t = 24\)
   \(\frac{3}{4} \cdot 3t = \frac{3}{4} \cdot 24\)
   \(\frac{3}{4} \cdot t = 6\)
   \(1 \cdot t = 8\)
   \(t = 8\)
   \(S = \{8\}\)

5. \(3t + 8 > 20\)
   \(3t + 8 + 8 > 20 + 8\)
   \(3t + 0 > 12\)
   \(3t > 12\)
   \(\frac{3}{4} (3t) > \frac{3}{4} \cdot 12\)
   \(\frac{3}{4} \cdot r > \frac{3}{4} \cdot 7\)
   \(1 \cdot r > 4\)
   \(r > 4\)
   \(S = \{r \mid r > 4\}\)

6. \(4x - 12 < 16\)
   \(4x - 12 + 12 < 16 + 12\)
   \(4x < 28\)
   \(4x < 28\)
   \(\frac{1}{2} (4x) < \frac{1}{2} \cdot 28\)
   \(\frac{1}{2} \cdot x < \frac{1}{2} \cdot 7\)
   \(1 \cdot x < 7\)
   \(x < 7\)
   \(S = \{x \mid x < 7\}\)

7. \(\frac{3}{4} + 16 > 15\)
   \(\frac{3}{4} + 16 + 16 > 15 + 16\)
   \(\frac{3}{4} + 0 > 7\)
   \(\frac{3}{4} > 7\)
   \(\frac{3}{4} \cdot n > \frac{3}{4} \cdot (-1)\)
   \(\frac{3}{4} \cdot n > -8\)
   \(1 \cdot n > -8\)
   \(n > -8\)
   \(S = \{n \mid n > -8\}\)

8. \(\frac{3}{8} - 3 \geq 4\)
   \(\frac{3}{8} - 3 + 3 \geq 4 + 3\)
   \(\frac{3}{8} = 7\)
   \(\frac{3}{8} + 0 \geq 7\)
   \(\frac{3}{8} \geq 7\)
   \(\frac{3}{8} \cdot x \geq \frac{3}{8} \cdot 7\)
   \(1 \cdot x \geq 7\)
   \(x \geq 7\)
   \(S = \{x \mid x \geq 7\}\)

9. \(5c + 7 < 18\)
   \(5c + 7 + 7 < 18 + 7\)
   \(5c + 0 < 11\)
   \(5c < 11\)
   \(\frac{1}{5} \cdot 5c < \frac{1}{5} \cdot 11\)
   \(\frac{1}{5} \cdot c < \frac{1}{5} \cdot 22\)
   \(1 \cdot c < 4\)
   \(c < 4\)
   \(S = \{c \mid c < 4\}\)

10. \(2w + 7 \leq 1\)
    \(-2w + 7 + 1 \leq 1 + 1\)
    \(2w + 0 \leq 6\)
    \(2w \leq 6\)
    \(\frac{1}{2} \cdot 2w \leq \frac{1}{2} \cdot 6\)
    \(\frac{1}{2} \cdot w \leq \frac{1}{2} \cdot 3\)
    \(1w \leq 3\)
    \(w \leq 3\)
    \(S = \{w \mid w \leq 3\}\)

11. \(-6z - 7 \geq 11\)
    \(-6z - 7 + 7 \geq 11 + 7\)
    \(-6z + 0 \geq 18\)
    \(-6z \geq 18\)
    \(\frac{1}{-6} (-6z) \leq \frac{1}{-6} (18)\)
    \(z \leq -3\)
    \(1 \cdot z \leq -3\)
    \(z \leq -3\)
    \(S = \{z \mid z \leq -3\}\)

12. \(5x + 4 \leq -6\)
    \(5x + 4 + 6 \leq -6 + 6\)
    \(5x + 0 \leq 0\)
    \(5x \leq 10\)
    \(\frac{1}{5} (5x) \leq \frac{1}{5} (10)\)
    \(x \leq 2\)
    \(1 \cdot x \leq 2\)
    \(x \leq 2\)
    \(S = \{x \mid x \leq 2\}\)

13. \(\frac{3}{4} x - 5 < 7\)
    \(\frac{3}{4} x - 5 + 5 < 7 + 5\)
    \(\frac{3}{4} x + 0 < 12\)
    \(\frac{3}{4} x < 12\)
    \(\frac{1}{4} \cdot \frac{3}{4} x < \frac{1}{4} \cdot 12\)
    \(\frac{1}{4} \cdot x \leq \frac{3}{4} \cdot 12\)
    \(1 \cdot x \leq 18\)
    \(x \leq 18\)
    \(S = \{x \mid x \leq 18\}\)

14. \(\frac{3}{4} y - 2 < 8\)
    \(\frac{3}{4} y - 2 + 2 < 8 + 2\)
    \(\frac{3}{4} y + 0 < 6\)
    \(\frac{3}{4} y < 6\)
    \(\frac{1}{4} \cdot \frac{3}{4} y < \frac{1}{4} \cdot 6\)
    \(\frac{1}{4} \cdot y < \frac{3}{4} \cdot 6\)
    \(1 \cdot y < 8\)
    \(y < 8\)
    \(S = \{y \mid y < 8\}\)

15. \(4x + 13 \geq 5\)
    \(4x + 13 + 13 \geq 5 + 13\)
    \(4x + 0 \geq 8\)
    \(4x \geq 8\)
    \(\frac{1}{4} (4x) \geq \frac{1}{4} (8)\)
    \(x \geq 2\)
    \(1 \cdot x \geq 2\)
    \(x \geq 2\)
    \(S = \{x \mid x \geq 2\}\)

16. \(2z - 1 > 7\)
    \(2z - 1 + 1 > 7 + 1\)
    \(2z + 0 > 8\)
    \(2z > 8\)
    \(\frac{1}{2} (2z) > \frac{1}{2} \cdot 8\)
    \(z > 4\)
    \(1 \cdot z > 4\)
    \(z > 4\)
    \(S = \{z \mid z > 4\}\)
17.  
\[ -5m - 10 < 25 \]
\[ -5m + 10 < 25 + 10 \]
\[ -5m + 0 < 35 \]
\[ -5m < 35 \]
\[ \frac{1}{3}(-5) > \frac{1}{3}(35) \]
\[ \frac{1}{3} \cdot m > \frac{1}{3} \]
\[ 1 \cdot m > -7 \]
\[ m > -7 \]
\[ S = \{ m \mid m > -7 \} \]

18.  
\[ 9y + 4 > -14 \]
\[ 9y + 4 + -4 > -14 + -4 \]
\[ 9y + 0 > -18 \]
\[ 9y > -18 \]
\[ \frac{1}{9}(9y) > \frac{1}{9}(-18) \]
\[ \frac{1}{9} \cdot y > \frac{-18}{9} \]
\[ 1 \cdot y > -2 \]
\[ y > -2 \]
\[ S = \{ y \mid y > -2 \} \]

19.  
\[ \frac{1}{3}p - 5 \geq 1 \]
\[ \frac{1}{3}p - 5 + 5 \geq 1 + 5 \]
\[ \frac{1}{3}p + 0 \geq 6 \]
\[ \frac{1}{3}p \geq 6 \]
\[ \frac{1}{3} \cdot m \geq 6 \]
\[ \frac{1}{3} \cdot \frac{1}{3} \cdot m \leq \frac{1}{3} \cdot \frac{1}{3} \cdot 6 \]
\[ \frac{1}{9} \cdot m \leq \frac{1}{9} \]
\[ 1 \cdot m \leq -9 \]
\[ m \leq -9 \]
\[ S = \{ m \mid m \leq -9 \} \]
For each of the following solution statements, give the solution set, using the proper set notation - roster or rule, and using a number line.

1. \( w = -3 \)  
   \[ S = \{ -3 \} \]

2. \( x = 2 \)  
   \[ S = \{ 2 \} \]

3. \( a > 1 \)  
   \[ S = \{ a \in \mathbb{R} \mid a > 1 \} \]

4. \( m \leq -2 \)  
   \[ S = \{ m \in \mathbb{R} \mid m \leq -2 \} \]
Find the solution(s) for each of the following open sentences. Express your answer using set notation.

5. \( x + 7 = 15 \)

6. \( c - 7 = 12 \)

7. \( w - 4 > -7 \)

8. \( 4x = 12 \)

9. \( -3n = -15 \)

10. \( 3n < 30 \)

11. \( -4y \geq 12 \)

12. \( 3x - 1 = 17 \)

13. \( \frac{4}{3}m + 1 < 25 \)
Unit II – First Degree Relations with One Placeholder
Part A – Basic Equations and Inequalities

Lesson 1 – Solution Statements and Solution Sets
Lesson 2 – First Type – Making Zeros
Lesson 3 – Second Type – Making Ones
Lesson 4 – Combinations

For each of the following solution statements, give the solution set, using the proper set notation – roster or rule, and using a number line.

1. \( y \geq -4 \)  
   \( S = \{ \} \)

2. \( b < 0 \)  
   \( S = \{ \} \)

3. \( x = 3 \)  
   \( S = \{ \} \)

4. \( a = -1 \)  
   \( S = \{ \} \)
Find the solution(s) for each of the following open sentences. Express your answer using set notation.

5. \(5a = 30\)
6. \(x + 9 = 15\)
7. \(-3m < 15\)
8. \(m - 9 = 30\)
9. \(-4y = -28\)
10. \(5x + 2 = 32\)
11. \(f - 5 < -8\)
12. \(5x > 60\)
13. \(\frac{5}{2}n + 1 \geq 26\)
Unit II – First Degree Relations with One Placeholder
Part A – Basic Equations and Inequalities

Lesson 1 – Solution Statements and Solution Sets
Lesson 2 – First Type – Making Zeros
Lesson 3 – Second Type – Making Ones
Lesson 4 – Combinations

For each of the following solution statements, give the solution set, using the proper set notation – roster or rule, and using a number line.

1. \( w = -3 \) \( S = \{ -3 \} \)

2. \( x = 2 \) \( S = \{ 2 \} \)

3. \( a > 1 \) \( S = \{ a | a > 1 \} \)

4. \( m \leq -2 \) \( S = \{ m | m \leq -2 \} \)
Find the solution(s) for each of the following open sentences. Express your answer using set notation.

5. \[ x + 7 = 15 \]
   \[ x + 7 + \cdot 7 = 15 + \cdot 7 \]
   \[ x + 0 = 8 \]
   \[ x = 8 \]
   \[ S = \{8\} \]  

6. \[ c - 7 = 12 \]
   \[ c - 7 + \cdot 7 = 12 + \cdot 7 \]
   \[ c + 0 = 19 \]
   \[ c = 19 \]
   \[ S = \{19\} \]  

7. \[ w - 4 > -7 \]
   \[ w - 4 + 4 > -7 + 4 \]
   \[ w + 0 > -3 \]
   \[ w > -3 \]
   \[ S = \{w \mid w > -3\} \]  

8. \[ 4x = 12 \]
   \[ \frac{1}{4}(4x) = \frac{1}{4}(12) \]
   \[ \frac{4}{4}x = \frac{12}{4} \]
   \[ 1x = 3 \]
   \[ x = 3 \]
   \[ S = \{3\} \]  

9. \[ -3n = -15 \]
   \[ \frac{1}{-3}(-3n) = \frac{1}{-3}(-15) \]
   \[ n = 5 \]
   \[ S = \{5\} \]  

10. \[ 3n < 30 \]
    \[ \frac{1}{3}(3n) < \frac{1}{3}(30) \]
    \[ n < 10 \]
    \[ S = \{n \mid n < 10\} \]  

11. \[ -4y \geq 12 \]
    \[ \frac{1}{-4}(-4y) \leq \frac{1}{-4}(12) \]
    \[ \frac{-4}{-4}y \leq \frac{12}{-4} \]
    \[ y \leq -3 \]
    \[ S = \{y \mid y \leq -3\} \]  

12. \[ 3x - 1 = 17 \]
    \[ 3x - 1 + 1 = 17 + 1 \]
    \[ 3x + 0 = 18 \]
    \[ x = 6 \]
    \[ S = \{6\} \]  

13. \[ \frac{4}{3}m + 1 < 25 \]
    \[ \frac{4}{3}m + 1 - 1 < 25 - 1 \]
    \[ \frac{4}{3}m < 24 \]
    \[ m < 18 \]
    \[ S = \{m \mid m < 18\} \]
Unit II – First Degree Relations with One Placeholder
Part A – Basic Equations and Inequalities

Lesson 1 - Solution Statements and Solution Sets
Lesson 2 - First Type - Making Zeros
Lesson 3 - Second Type - Making Ones
Lesson 4 - Combinations

For each of the following solution statements, give the solution set, using the proper set notation - roster or rule, and using a number line.

1. \( y \geq -4 \) 
   \[ S = \{ y \mid y \geq -4 \} \]

2. \( b < 0 \) 
   \[ S = \{ b \mid b < 0 \} \]

3. \( x = 3 \) 
   \[ S = \{ 3 \} \]

4. \( a = -1 \) 
   \[ S = \{ -1 \} \]
Find the solution(s) for each of the following open sentences. Express your answer using set notation.

5. \[ 5a = 30 \]
   \[ \frac{1}{5}(5a) = \frac{1}{5}(30) \]
   \[ \frac{6}{5}a = \frac{30}{5} \]
   \[ 1a = 6 \]
   \[ a = 6 \]
   \[ S = \{ 6 \} \]

6. \[ x + 9 = 15 \]
   \[ x + 9 - 9 = 15 - 9 \]
   \[ x + 0 = 6 \]
   \[ x = 6 \]
   \[ S = \{ 6 \} \]

7. \[ -3m < 15 \]
   \[ \frac{1}{3}(-3m) > \frac{1}{3}(15) \]
   \[ -m > 5 \]
   \[ m < -5 \]
   \[ S = \{ m | m < -5 \} \]

8. \[ m - 9 = 30 \]
   \[ m - 9 + 9 = 30 + 9 \]
   \[ m + 0 = 39 \]
   \[ m = 39 \]
   \[ S = \{ 39 \} \]

9. \[ -4y = -28 \]
   \[ \frac{1}{-4}(-4y) = \frac{1}{-4}(-28) \]
   \[ -\frac{4}{4}y = -\frac{28}{-4} \]
   \[ y = 7 \]
   \[ S = \{ 7 \} \]

10. \[ 5x + 2 = 32 \]
    \[ 5x + 2 - 2 = 32 - 2 \]
    \[ 5x = 30 \]
    \[ x = 6 \]
    \[ S = \{ 6 \} \]

11. \[ f - 5 < -8 \]
    \[ f - 5 + 5 < -8 + 5 \]
    \[ f + 0 < -3 \]
    \[ f < -3 \]
    \[ S = \{ f | f < -3 \} \]

12. \[ 5x > 60 \]
    \[ \frac{1}{5}(5x) > \frac{1}{5}(60) \]
    \[ \frac{5}{5}x > \frac{60}{5} \]
    \[ x > 12 \]
    \[ S = \{ x | x > 12 \} \]

13. \[ \frac{5}{2}n + 1 \geq 26 \]
    \[ \frac{5}{2}n + 1 - 1 \geq 26 - 1 \]
    \[ \frac{5}{2}n \geq 25 \]
    \[ \frac{2}{5} \left( \frac{5}{2}n \right) \geq \frac{2}{5} \left( 25 \right) \]
    \[ \frac{10}{10}n \geq \frac{50}{5} \]
    \[ n \geq 10 \]
    \[ S = \{ n | n \geq 10 \} \]