Geometry: A Complete Course

(with Trigonometry)

Module E - Student WorkText

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Larry E. Collins

ERRATA
3/2015

VideoText Interactive

Geometry: A Complete Course (with Trigonometry)
Module E–Student Worktext
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Theorem 66 -"If, in the same circle, or in congruent circles,two minor arcs
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LESSON 4 - Theorem 77 - "If two secant segments are drawn to a circle from a 88
single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment."
Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle, from a single point outside the circle, then the length of that tangent segment is the mean proportional between the length of the secant segment, and the length of its external segment."
LESSON 5 - Theorem 79 - "If a line is perpendicular to a diameter of a circle at one 92
of its endpoints, then the line must be tangent to the circle,at that endpoint,"
LESSON 6 - Theorem 80 - "If two tangent segments are drawn to a circle from the 94
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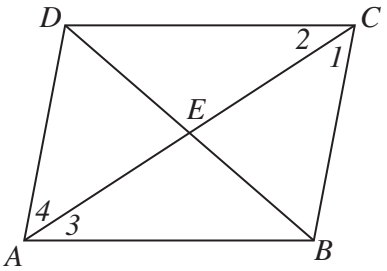
supplementary, then the quadrilateral is cyclic."

For Exercises 11 and 12, use the diagram to the right.



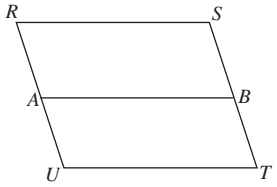
11. State the theorem that justifies each of the following conclusions.

a) $\angle DAB \cong \angle DCB$
b) $\overline{BE} \cong \overline{DE}$
c) $\overline{AD} \cong \overline{CB}$
d) $\overline{DC} \cong \overline{BA}$
e) $\angle DAB$ and $\angle CBA$ are supplementary
f) $\overline{AD} \parallel \overline{CB}$

12. Complete each of the following statements.

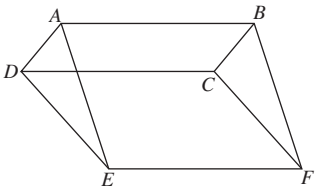
a) If AD = 18, BC = _____
b) If $\angle ADC = 105$, then $m\angle ABC$ = _____
c) If DB = 24, then DE = _____
d) If AE = 15, then AC = _____
e) If $m\angle DAB = 65$, then $m\angle ADC$ = _____
f) If $m\angle 2 = 30$, then $m\angle 3$ = _____
g) If BD = 8 and AE = 6, then AC = _____ and BE = _____
h) If $m\angle ABC = 2(m\angle BCD)$, then $m\angle ADC$ = _____
i) If $m\angle ADC = 130$, and $m\angle 1 = 35$, then $m\angle 2$ = _____
j) If $DC = 3x - 5$ and $AB = x + 9$, then DC = _____
k) If $m\angle ADC = 4x + 11$ and $m\angle DAB = 6x - 1$, then $m\angle ADC$ = _____

In Exercises 13–18, determine if each statement is true or false.
Give a reason for your answer, drawing a sketch if necessary.



Refer to the figure to the right for Exercise 13.

13. If RSTU is a parallelogram and $\overline{AB} \parallel \overline{UT}$, then ABTU is also a parallelogram.



Refer to the figure to the right for Exercises 14 and 15.

14. If CDEF is a parallelogram, and ABCD is a parallelogram, then ABFE is a parallelogram.
15. If $\overline{DC}$ is not in the same plane as $\overline{AB}$ and $\overline{EF}$, and if CDEF and ABCD are parallelograms, then ABFE is a parallelogram.

16. If a diagonal of a quadrilateral is drawn so that two congruent triangles are formed, then the quadrilateral is a parallelogram.
17. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.
18. If two sides of a quadrilateral are congruent and the other two sides are parallel, then the quadrilateral is a parallelogram.

Unit V — Other Polygons

Part A — Properties of Polygons

Lesson 3 — Special Parallelograms (Rectangle, Rhombus, Square)

Objective: To understand, and be able to prove, the relationships between special parallelograms, and to see the relationships exhibited by their parts.

Important Terms:

Quadrilateral – A polygon made with four line segments. Further, the segments are called the **sides** of the quadrilateral, and the endpoints of the segments are called the **vertices** of the quadrilateral.

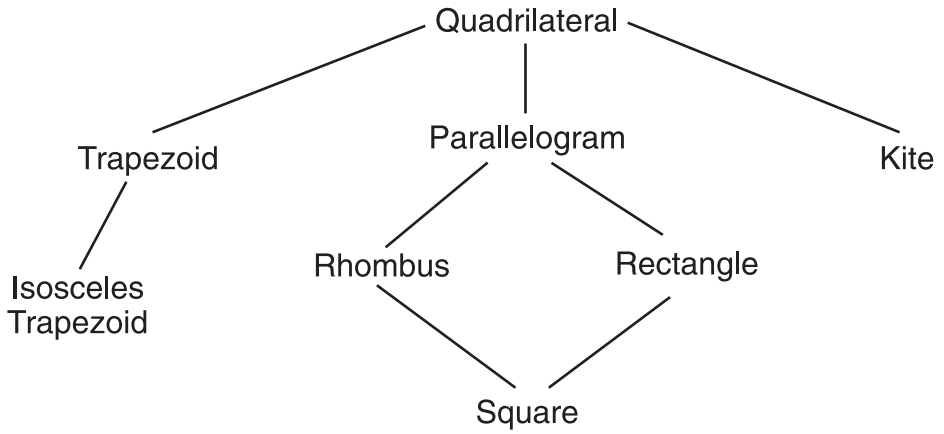
Rectangle – A parallelogram in which there are four right angles. (the standard symbol for a rectangle is ▭)

Rhombus – A quadrilateral in which all four sides are congruent.

Square– A quadrilateral in which all four sides are congruent.

Diagonal of a Polygon – A line segment joining any two non-consecutive vertices of a polygon.

Quadrilateral Hierarchy Theorem – "If a polygon is one of the seven types of quadrilaterals, then it is related to all other quadrilaterals, using the diagram below."



Theorem 48 – "If a quadrilateral is a rectangle, then it is a parallelogram."

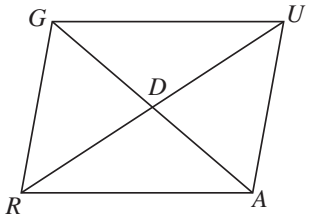
Theorem 49 – "If a quadrilateral is a rectangle, then its diagonals are congruent."

Theorem 50 – "If a quadrilateral is a rhombus, then it is a parallelogram"

Theorem 51 – "If a quadrilateral is a rhombus, then its diagonals are perpendicular."

Theorem 52 – "If a quadrilateral is a rhombus, then the diagonals bisect the interior angles of the rhombus."

For Exercises 20–22, use the diagram below.



20. Assume GUAR is a rectangle.
- **a)** If DG is 6, find DU.
- **b)** If DA is 9, find RU.
- **c)** If m∠RDG = 40, find m∠GRD.
- **d)** If m∠UAG = 65, find m∠UDG.

21. Assume GUAR is a rhombus.
- **a)** If m∠GRD = 53, find m∠DGR.
- **b)** If m∠GUA = 86, find m∠GUR.

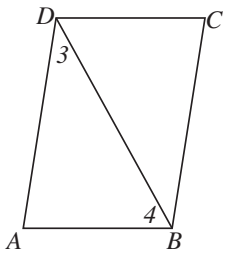
22. Assume GUAR is a square.
- **a)** If DG = $3x + 6$ and DU = $4x - 10$, find GA.
- **b)** Find m∠URA.

23. If quadrilateral ABCD is a rectangle, with coordinates A(2,5), B(2,1), C(7,1), and D(7,5), then find the lengths of the diagonals and verify that they are congruent.

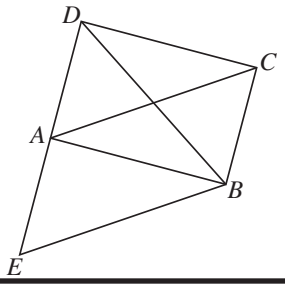
24. In quadrilateral ABCD, determine the coordinates of point D, so that ABCD is a square, given that A is (3,–1), B is (–1,3), C is (–5,–1), and D is (x,y).

25. Quadrilateral ABCD is a parallelogram, with AB = $2x + 4$, DC = $3x - 11$, and AD = $x + 19$. Show that ABCD is a rhombus.

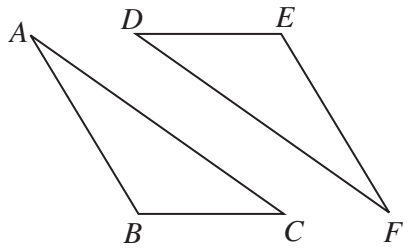
26. Given: ABCD is a parallelogram; m∠3 = m∠4
Prove: ABCD is a rhombus



27. Given: ABCD is a rectangle, ACBE is a parallelogram
Prove: △DBE is isosceles



Example 3: $\triangle ABC \cong \triangle FED$. The area of $\triangle FED$ is 17 square units. Find the area of $\triangle ABC$ and state the assumption(s) of Postulate 14 which apply.



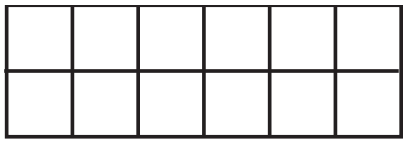
Solution: Since the two polygons are congruent and the area of $\triangle FED$ is 17 square units, the area of $\triangle ABC$ is 17 square units.

Postulate 14 - Second Assumption - Congruent polygons have the same area.

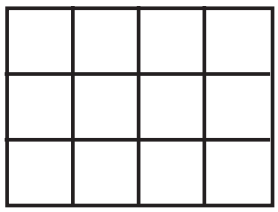
Example 4: Draw as many rectangles as you can that are separated into 12 one-centimeter squares. Name the dimensions for each rectangle.

Solution:

12 x 1



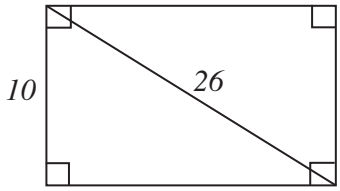
6 x 2



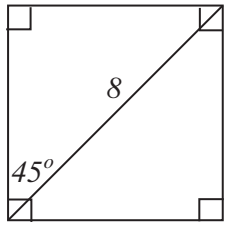
4 x 3

4. Find the area of each given rectangle.

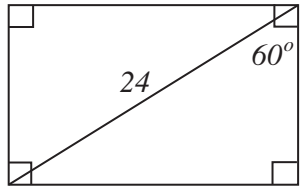
a)



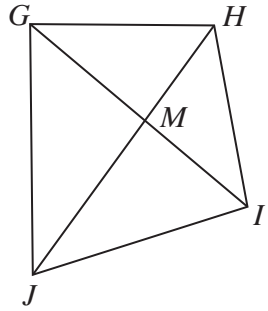
b)



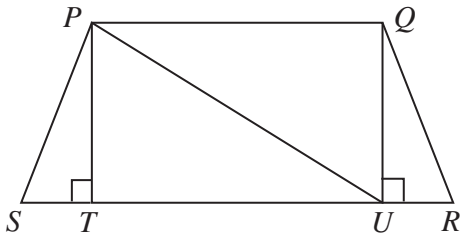
c)



5. In the figure at the right, quadrilateral GHIJ is divided into triangles by diagonals $\overline{GI}$ and $\overline{HJ}$ which intersect at point M. If you know that the area of △GHI = 7, the area of △GHJ = 9, and the area of △JMI = 8, then will the area of quadrilateral GHIJ = 9 + 7 + 8? Why or why not?

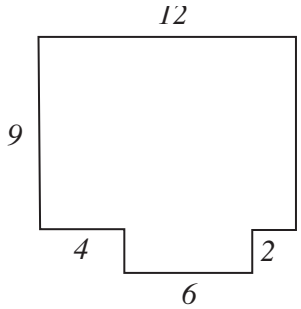


6. The area of △PST is 14, the area of △QRU is 15, and the area of △QPU is 26. △TUP and △QPU are congruent. What is the area of polygon PQRS? State the assumption(s) of Postulate 14 which justify your answer.

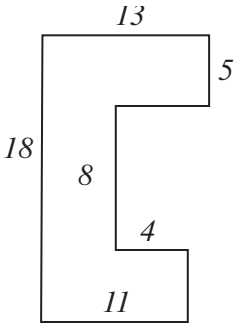


For Exercises 7 – 12, find the area of each figure. All adjacent sides are perpendicular.

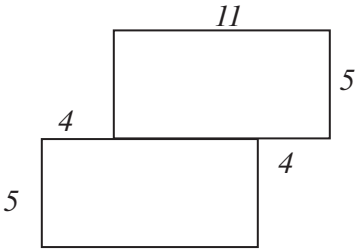
7.



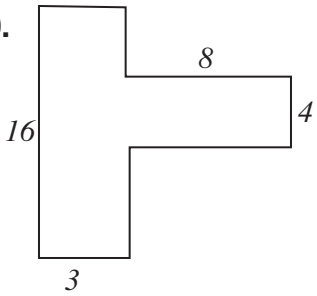
8.



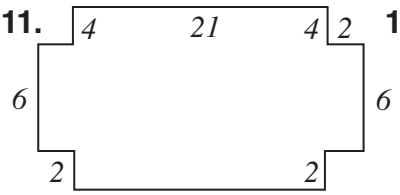
9.



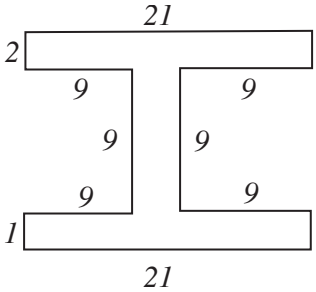
10.



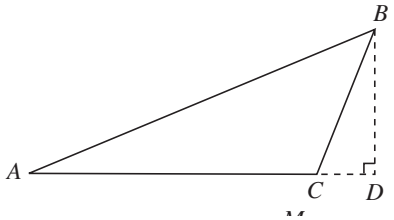
11.



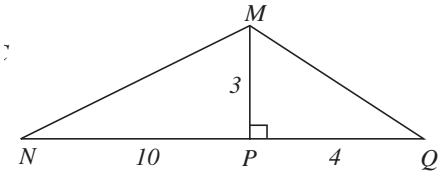
12.



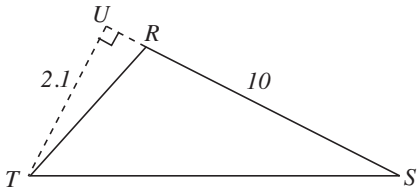
7. How is the area of △ABC related to the areas of two right riangles in the figure below? If BD = 7 inches, AC = 15 inches, and CD = 5 inches, what is the area of △ADB? What part(s) of Postulate 14 are involved in the process of finding the area of △ADB?



8. Find the area of △MNQ

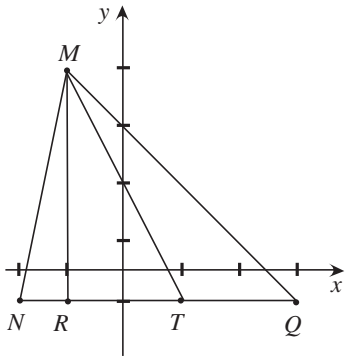


9. Find the area of △RST

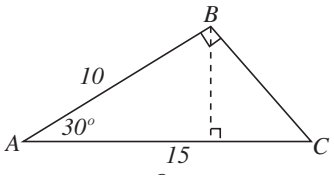


10. Using the figure shown at the right, find the area of:

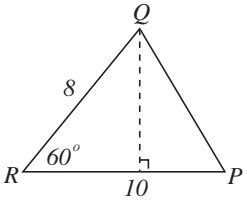
a) △MRN
b) △MQN
c) △MTQ
d) △MRQ



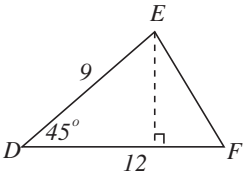
11. Find the area of △ABC



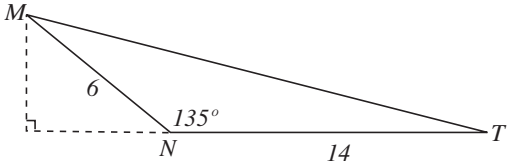
12. Find the area of △RQP



13. Find the area of △DEF



14. Find the area of △MNT

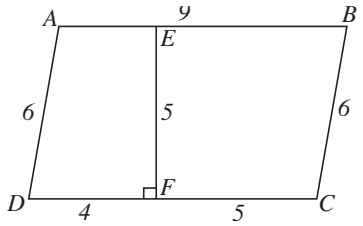


Lesson 3 — Exercises:

1. State Theorem 59 and sketch two figures, one using one side as the base and another using an adjacent side as the base, to illustrate the conditions of the theorem.

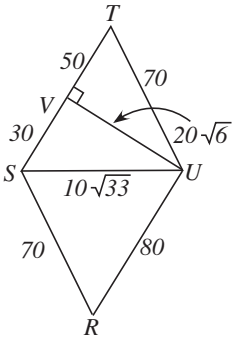
2. Find the area of the parallelogram in each part shown below

a)



List all the measures in the figure you did not need, to find the area of parallelogram ABCD.

b)



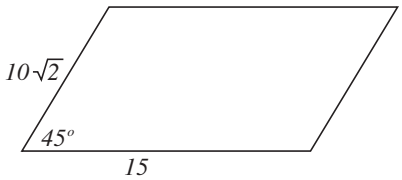
List all the measures in the figure you did not need, to find the area of parallelogram RSTU.

3. The base of a parallelogram is 18 units long and the area is 54 square units.
a) Find the measure of the height.
b) How many of the other three sides of the parallelogram can you determine?

4. In a parallelogram, the ratio of the altitude to its base is 4 to 5. If its area is 126 square units, what are the dimensions of the parallelogram?

In Exercises 5 – 8, find the area of the given parallelogram.

5.



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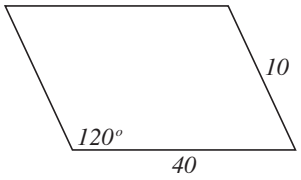

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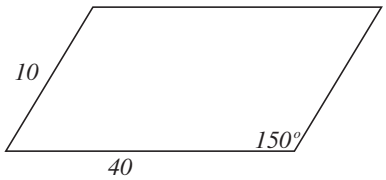

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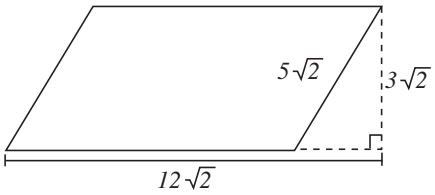
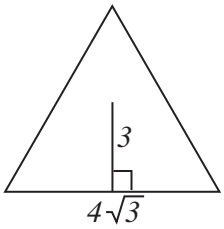


Lesson 5 — Exercises:

In exercises 1 – 6, find the area of each regular polygon. Note: the perpendicular segment shown is drawn to the center of the polygon.

1.



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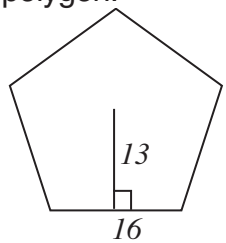

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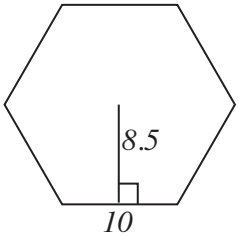

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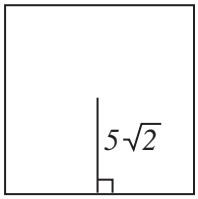

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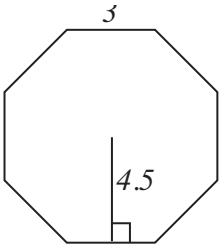

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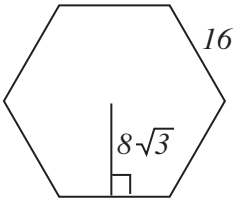
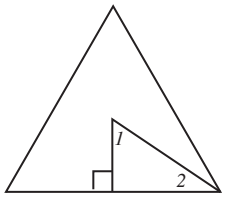


In exercises 7 – 9, find the measure of $\angle 1$ and $\angle 2$ in each regular polygon.
Note: the perpendicular segment is drawn to the center of the polygon.

7.



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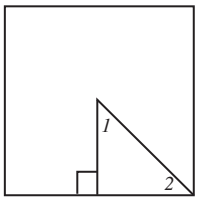

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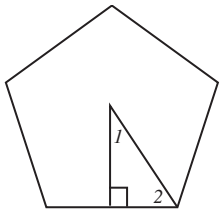
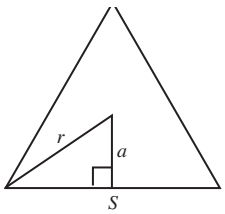
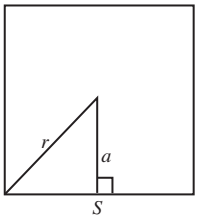


10. Refer to the equilateral triangle at the right for parts a – c.
The perpendicular segment is drawn to the center of the polygon.



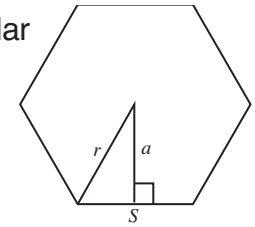
a) If $r = 12$, find s, P, a, and A.
b) If $s = 10\sqrt{3}$, find r, P, a, and A.
c) If $P = 24\sqrt{3}$, find s, r, a, and A.

11. Refer to the square at the right for parts a – c. The perpendicular segment is drawn to the center of the polygon.



a) If $s = 16$, find r, P, a, and A.
b) If $r = 8\sqrt{2}$, find s, P, a, and A.
c) If $a = 5\sqrt{2}$, find s, r, P, and A.

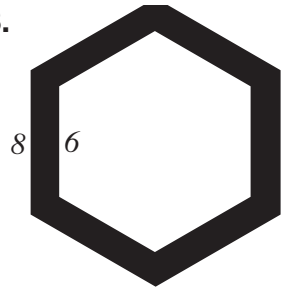
12. Refer to the hexagon at the right for parts a – c. The perpendicular segment is drawn to the center of the polygon.



a) If $r = 10$, find s, P, a, and A.
b) If $a = 9\sqrt{3}$ find r, P, s, and A.
c) If $s = 6$, find r, P, a, and A.

In Exercises 13 – 15, find the area of the shaded region.
All polygons shown are regular polygons.

13.



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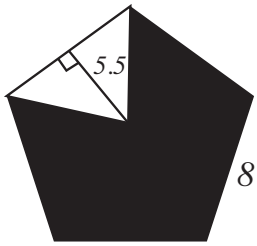

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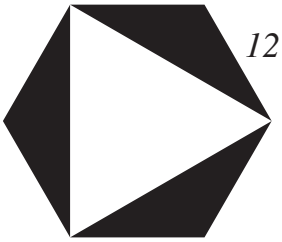


Unit V — Other Polygons

Part C — Applications

Lesson 1 — Using Areas in Proofs

Objective: To understand how the concept of the area of a polygon can be used to demonstrate theorems.

Important Terms:

Theorem 62 – "If you have a median of a triangle, then that median separates the points inside the triangle into two polygonal regions with the same area."

Theorem 63 – "If you have a rhombus, then the area enclosed by that rhombus, is equal to one-half the product of the measures of the diagonals of the rhombus."

Theorem 64 – "If you have two similar polygons, then the ratio of the areas of the two polygons is equal to the square of the ratio of any pair of corresponding sides."

Lesson 1 — Exercises:

1. State Theorem 62. Then sketch and label a triangle to show the conditions of the theorem.

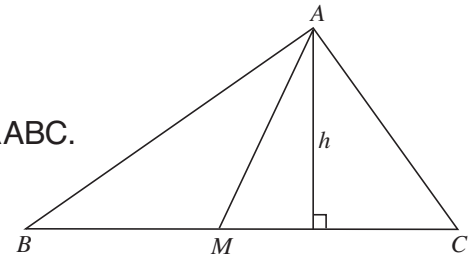
2. State Theorem 63. Then sketch and label a rhombus to show the conditions of the theorem.

3. State Theorem 64. Then sketch and label two triangles which demonstrate the conditions of the theorem.

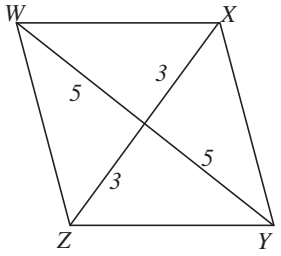
4. The lengths of the sides of two squares are 4 and 8. Find the ratio of the areas of the two squares.

5. Two regular pentagons have side lengths in the ratio of 13 to 20. Find the ratio of the areas of the two pentagons.

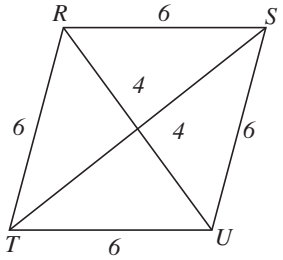
6. If BC = 16 and h = 5, find the area of △ABC and the area of △AMB. Note: $\overline{AM}$ is a median of △ABC.



7. Find the area of rhombus WXYZ.



8. Find the area of rhombus RSTU.

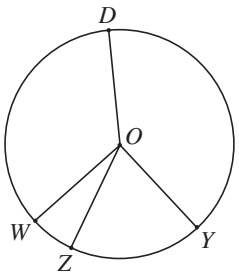


9. FInd the area of a rhombus with a 120° interior angle and sides of length 6”.

10. Find the area of a rhombus with a perimeter of 68” and one diagonal of length 30”.

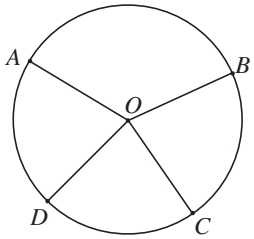
13. Use the given ⊙O to find each measure, if $m\overset{\frown}{DW} = 3x + 10$, $m\overset{\frown}{WZ} = x$, $m\overset{\frown}{ZY} = 2x + 10$, and $m\overset{\frown}{YD} = 4x$.

a) $m\angle WOZ$
b) $m\overset{\frown}{DYZ}$
c) $m\angle WOD$
d) $m\angle YOZ$



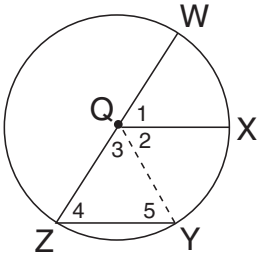
14. Use the given ⊙O to name a major arc for each minor arc named below.

a) $\overset{\frown}{AB}$
b) $\overset{\frown}{CD}$
c) $\overset{\frown}{AD}$
d) $\overset{\frown}{BC}$



15. Given: $\overline{WZ}$ is a diameter of ⊙Q
$m\overset{\frown}{WX} = m\overset{\frown}{XY} = n$

Prove: $m\angle Z = n$



Unit VI — Circles

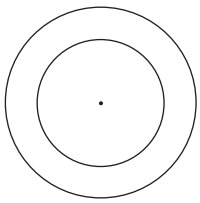
Part A — Fundamental Terms

Lesson 3 — Circle Relationships

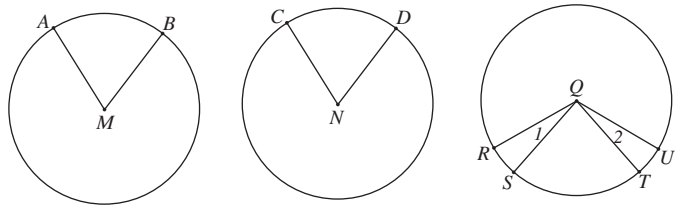
Objective: To recognize and understand the relationships between circles according to their relative positions, and to see how they may be related to other polygons.

Important Terms:

Concentric Circles – From two Latin words meaning, "with the same center", concentric circles share the same center point. Formally, in our Geometry, two circles are concentric, if and only if, they lie in the same plane, and share the same center. See the illustration below.



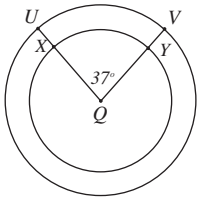
Example 2:



Theorem 66 states that, if ⊙M ≅ ⊙N and $\overset{\frown}{AB} \cong \overset{\frown}{CD}$,
then _____ ≅ _____. It further states that in ⊙Q,
if $\overset{\frown}{RS} \cong \overset{\frown}{UT}$, then _____ ≅ _____.

Solution: ∠M ≅ ∠N; ∠1 ≅ ∠2.

Example 3: In the given pair of concentric circles, find m$\overset{\frown}{XY}$ and m$\overset{\frown}{UV}$. Does m$\overset{\frown}{XY}$ = m$\overset{\frown}{UV}$? Are the arcs congruent? Explain.



Solution: m$\overset{\frown}{XY}$ = mUV = 37°
$\overset{\frown}{XY} \not\cong \overset{\frown}{UV}$

If you walked from point U to point V, you would walk a longer distance than if you walked from point X to point Y. The arcs UV and XY have different lengths. Arc length is not the same as arc measure. Arc length indicates a distance. Arc measure indicates an amount of turn. The length of $\overset{\frown}{UV}$ (or $\overset{\frown}{XY}$) is a distance that is measured in units such as inches or feet, but m$\overset{\frown}{UV}$ (or m$\overset{\frown}{XY}$) is measured in degrees. The units of measure are different.

Lesson 1 — Exercises:

1. Prove Theorem 65 - "In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent."
(Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your Course Notes to check.)
a) State the theorem.
b) Draw and label a diagram to accurately show the conditions of the theorem.
c) List the given information.
d) Write the statement we wish to prove.
e) Demonstrate the direct proof using the two column format.

2. Using the outline given in Exercise 1, prove Theorem 66.

Inscribed Angle of a Circle – An angle formed by any two chords with a common endpoint. Formally, in our Geometry, an angle is an inscribed angle of a circle, if and only if, the vertex of the angle is on the circle, and the sides are chords of the circle.

Tangent Line of a Circle – From the Latin word, "tangere", meaning, "to touch", a tangent line of a circle is a line which intersects the circle in exactly one point, called the "point of tangency", or the "point of contact". Note: We sometimes refer to a line segment as a tangent segment, if that segment is contained in a tangent line and intersects the circle in such a way that the point of tangency is one of its endpoints.

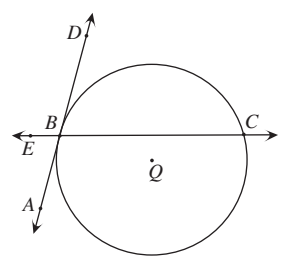
Intercepted Arc of a Circle – An angle of a cirlce intercepts an arc of a circle, if and only if, each of the following conditions hold: 1) The endpoints of the arc lie on the sides of the angles. 2) Each side of the angle contains one endpoint of the arc. 3) All points on the arc, except the endpoints, lie in the interior of the angle.

Measure of an Arc of a Circle – Based on its relationship with the central angles of a circle, this is defined as the measure of the central angle which intercepts the arc.

Measure of a Semicircle – Because we can consider a semicircle to be the intercepted arc of a central angle of 180°, (its rays are radii of the same diameter), we say that the measure of a semicircle is 180°.

Corollary 68a – "If, in a circle, a diameter is drawn to a tangent line, at the point of tangency, then that diameter is perpendicular to the tangent line, at that point."

Example 1: In the figure at the right, $\overleftrightarrow{AD}$ is a tangent line to circle Q at point B. If $\overleftrightarrow{EC}$ is a secant line intersecting $\overleftrightarrow{AD}$ at point B, and $m\angle DBC$ is 84°, find $m\overset{\frown}{BC}$.



Solution:

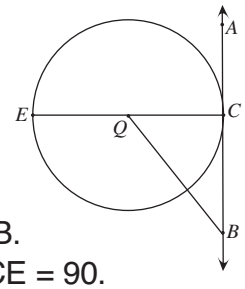
$$m\angle DBC = \frac{1}{2}\cdot m\overset{\frown}{BC} \qquad \text{(Theorem 68)}$$

$$84 = \frac{1}{2}\cdot m\overset{\frown}{BC}$$

$$2\cdot 84 = 2\cdot\frac{1}{2}\cdot m\overset{\frown}{BC}$$

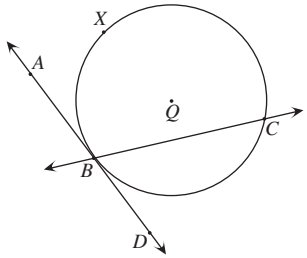
$$168 = m\overset{\frown}{BC}$$

Example 2: In the figure at the right, $\overleftrightarrow{AB}$ is a tangent line to ⊙Q at point C. $\overline{EC}$ is a diameter of circle Q. Find m∠BQC, m∠QBC, and m∠BCE if the radius is 6 units and QB = 12.



Solution: △QCB is a 30-60-90 right triangle since QC = ½ · QB. Therefore, m∠BQC = 60, m∠QBC = 30, and m∠BCE = 90.

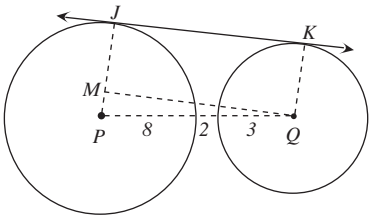
Example 3: In the figure at the right, $\overleftrightarrow{AD}$ is a tangent line to ⊙Q at point B. If $\overleftrightarrow{BC}$ is a secant line intersecting $\overleftrightarrow{AD}$ at point B and $m\overset{\frown}{BXC} = 226$, find m∠ABC.



Solution:

$$\begin{aligned} m\angle ABC &= \frac{1}{2}\cdot m\overset{\frown}{BXC} \\ &= \frac{1}{2}\cdot 226 \\ &= \frac{1\cdot 113\cdot \cancel{2}}{\cancel{2}} \\ &= 113 \end{aligned}$$

Example 4: $\overleftrightarrow{JK}$ is tangent to ⊙P and ⊙Q. Find JK.

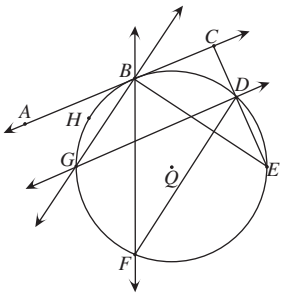


Solution: Using Postulate 2 (For any two points, there is exactly one line containing them.), we can draw $\overline{PJ}$ and $\overline{QK}$.
Using Postulate 9 (In a plane, through a point not on a given line, there is exactly one line parallel to the given line.), we can draw $\overline{QM}$ parallel to $\overleftrightarrow{JK}$.
By Corollary 68a, $\overline{JP} \perp \overleftrightarrow{JK}$ and $\overline{QK} \perp \overleftrightarrow{JK}$. So JKQM is a rectangle and △QMP is a right triangle. Use the pythagorean theorem to find QM. $\overline{QM} \cong \overline{JK}$.

$$\begin{aligned} (MP)^2 + (MQ)^2 &= (PQ)^2 \\ (JP - JM)^2 + (MQ)^2 &= (PQ)^2 \\ (8-3)^2 + (MQ)^2 &= (8+2+3)^2 \\ (5)^2 + (MQ)^2 &= (13)^2 \\ 25 + (MQ)^2 &= 169 \\ (MQ)^2 &= 144 \\ MQ &= 12 \end{aligned}$$

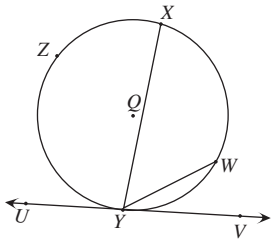
Therefore, JK = 12

Lesson 3 — Exercises:

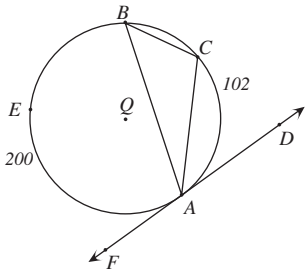


1. In ⊙Q shown at the right, $\overleftrightarrow{AC}$ is a tangent line to ⊙Q at point B. Name the angles that are formed by a tangent line and a secant, or by a tangent line and a chord, intersecting on the circle.

2. Using ⊙Q in exercise 1, name the arcs intercepted by each angle named in exercise 1.

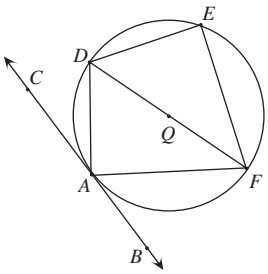


3. In ⊙Q shown at the right, $\overleftrightarrow{UV}$ is a tangent line to ⊙Q at point Y. $\overline{WY}$ and $\overline{XY}$ are chords of ⊙Q such that m$\overset{\frown}{WY}$ = 82 and m$\overset{\frown}{XW}$ = 96. Find m∠VYW, m∠VYX, m∠WYX, and m∠UYX.



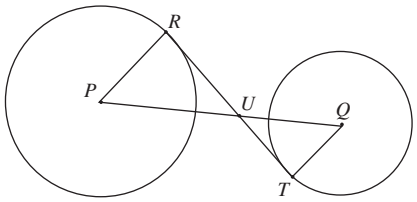
4. In the figure to the right, chord $\overline{AB}$ intersects tangent line $\overleftrightarrow{AD}$ at point A, m$\overset{\frown}{AC}$ = 102, and m$\overset{\frown}{AEB}$ = 200. Use this information to find the following:

a) m∠CAD
b) m$\overset{\frown}{BC}$
c) m∠ABC
d) m∠BAF
e) m∠BAC
f) m∠ACB



5. In the figure to the right, $\overline{DF}$ is a diameter of ⊙Q, $\overleftrightarrow{AB}$ is a tangent to ⊙Q at point A, m$\overset{\frown}{AD}$ = 80, and m$\overset{\frown}{DE}$ = 70. Use this information to find the following:

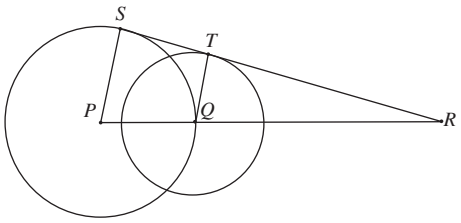
a) m∠DAC
b) m∠AFD
c) m∠DEF
d) m∠DFE
e) m$\overset{\frown}{AF}$
f) m∠BAF
g) m∠EDF



6. Use the figure to the right for the following:
Given: $\overline{RT}$ is a common internal tangent to ⊙P and ⊙Q

Prove: $\frac{PU}{QU} = \frac{RU}{TU}$

7. In the figure to the right, $\overline{SR}$ is tangent to ⊙P and ⊙Q at points S and T respectively. Also, QT = 6, TR = 8, and PR = 30. Find PQ, PS, and ST. (Hint: Use the Pythagorean Theorem)



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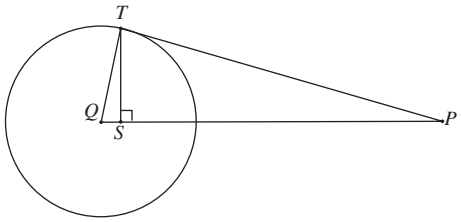
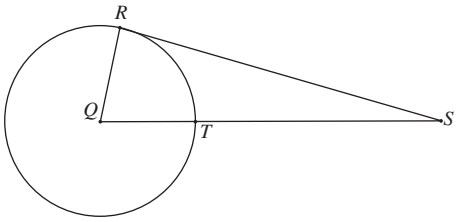


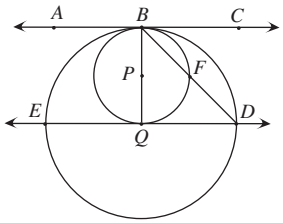
In the figure above, $\overline{PT}$ is tangent to ⊙Q at point T and $\overline{TS} \perp \overline{QP}$. Complete the following statements.

a) TS is the geometric mean between _____ and _____.
b) TQ is the geometric mean between _____ and _____.
c) If QS = 6 and SP = 24, TS = _____ and TP = _____.

9. In the figure to the right, $\overline{RS}$ is a tangent segment to ⊙Q, and $\overline{QR}$ is a radius of ⊙Q. If RS = 17 and ST = 7, find the length of the radius of ⊙Q.



Use the figure to the right for exercises 10 and 11.



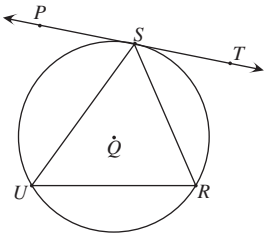
10. Given: $\overline{BQ} \perp \overleftrightarrow{ED}$ at center Q.

Prove: $m\overset{\frown}{BD} = 90$

11. Given: $\overleftrightarrow{AC}$ is tangent to ⊙P and ⊙Q at point B

Prove: $m\angle DBC = \frac{1}{2} \cdot m\angle DQB$

12. In ⊙Q shown to the right, $\overleftrightarrow{PT}$ is a tangent line at point S. $m\angle RST = 45$ and $m\angle USP = 85$. Find $m\overset{\frown}{SU}$, $m\overset{\frown}{SR}$, and the measure of each angle of △URS.

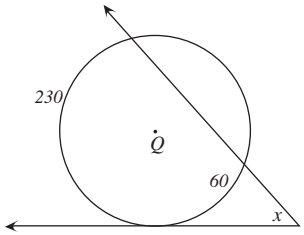


Lesson 5 — Exercises:

1. Prove Theorem 71
2. Prove Theorem 72

In exercises 3 through 12, find the value of *x* for each indicated angle or arc of ⊙Q.

3.



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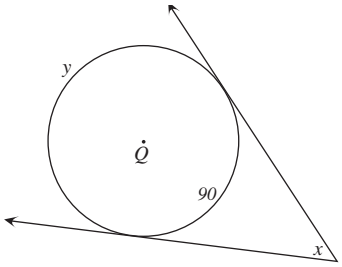

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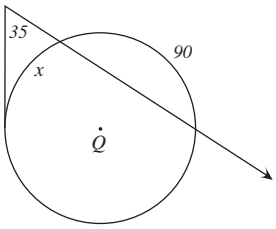

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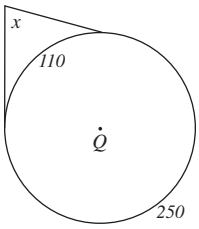

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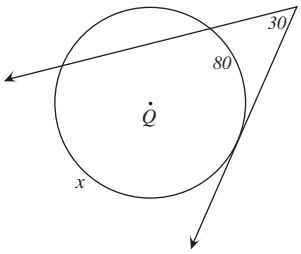

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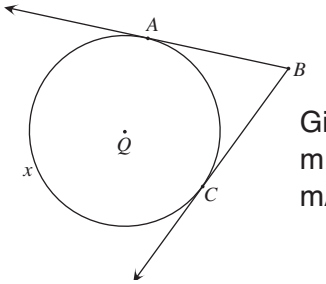
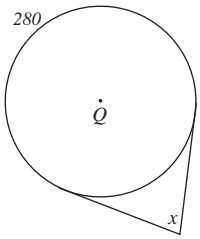


Given:
$m\angle B = x - 180$
$m\widehat{AC} = 90 + a$

9



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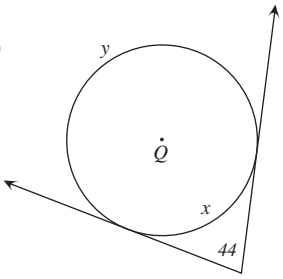
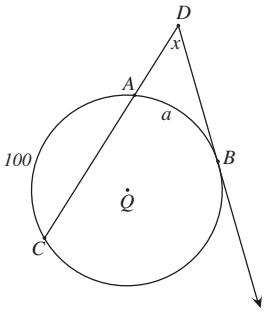


Note: for this exercse, find the values of *x* and *y*.

11.



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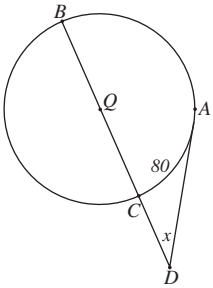
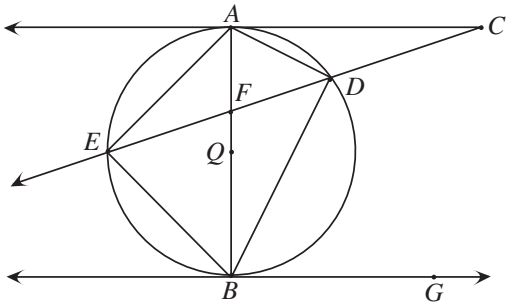


Use the figure to the right, and the given information for exercises 13 through 25.

$\overrightarrow{CA}$ is tangent to ⊙Q at point A. $\overrightarrow{CD}$ is a secant of ⊙Q intersecting the circle at points D and E. m∠C = 18, $m\widehat{AE} = 92$ (Note: $m\widehat{AE} > m\widehat{AD}$)



13. Find $m\widehat{AD}$

14. Find $m\widehat{BD}$

15. Find m∠EFB

16. Find m∠DAC

17. Find m∠ADC

18. Find m∠ADE

19. Find m∠AEC

20. $\overleftrightarrow{GB}$ and $\overleftrightarrow{CA}$ are _____.

21. Find m∠ADB

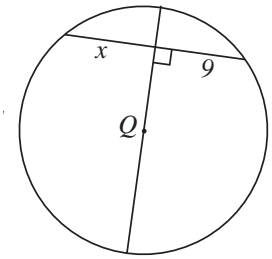
22. Find m∠CAB

23. Find m∠ABE

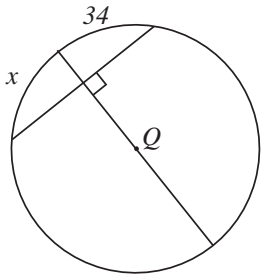
24. Find m∠BEA

25. Find m∠ABD

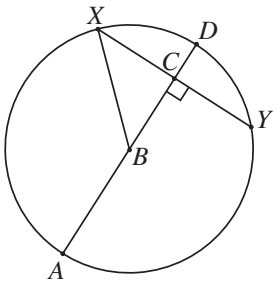
14. Find the value of x in ⊙Q.



15. Find the value of x on ⊙Q.

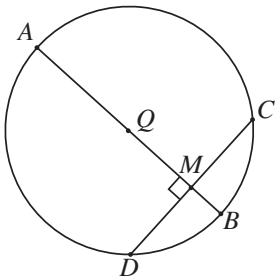


16. Find the measure of $\overline{XY}$.



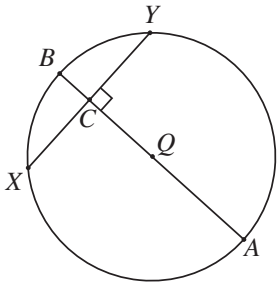
Given:
BX = 7
BC = 4
$\overline{AD} \perp \overline{XY}$

17. Find the measure of $\overline{QM}$.



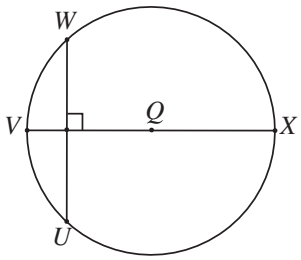
Given:
DC = 24
QA = 13
$\overline{AB} \perp \overline{DC}$

18. Find the measure of $\overline{QY}$.



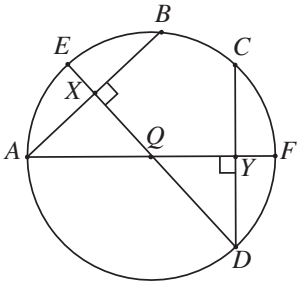
Given:
QC = 9
XY = 18
$\overline{AB} \perp \overline{XY}$

19. Find the measure of ∠VQW.



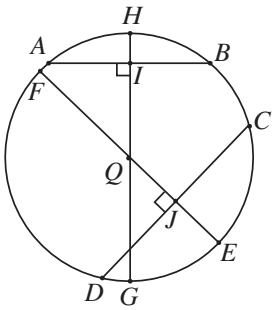
Given:
$m\widehat{UXW} = 288$
$\overline{XV} \perp \overline{UW}$

20. Find the measure of $\overline{BX}$, $\overline{DY}$, and $\widehat{BF}$.



Given:
$\overline{DE} \perp \overline{AB}$
$\overline{AF} \perp \overline{CD}$
CD = 16
AX = 7
$m\widehat{BE} = 58$

21. Find the measure of $\overline{CD}$ and $\widehat{AB}$.

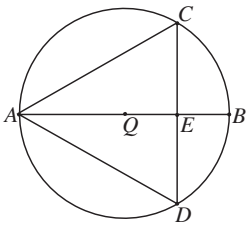


Given:
AB = 18
QI = 12
QJ = 10
$m\widehat{GB} = 140$

8. Prove: If a diameter of a circle bisects one of two parallel chords (which are not diameters), it bisects the other chord.

9. Given: $\overline{AB}$ is a diameter of $\odot Q$.
$\overline{CE} \cong \overline{DE}$.

Prove: $\triangle AEC \cong \triangle AED$



10. How many chords may be drawn from a point on a circle? How many diameters?

11. What is the greatest chord in a circle? An arc has how many chords? A chord has how many arcs?

12. If a chord were extended at either or both ends, what would it become?

13. How long is the chord which is perpendicular to a tangent to a circle having a radius with a measure of 9 inches? Justify your answer.

14. Through what point does the perpendicular bisector of a chord pass?

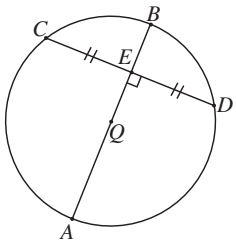
15. Prove: If a diameter of a circle bisects each of two chords which are not diameters, the chords are parallel to each other.

16. Write exercises 8 and 15 as a biconditional.

17. Prove Theorem 75 using the two column format.

Given: Chord $\overline{AB}$ bisects chord $\overline{CD}$ at point E.
$\overline{AB} \perp \overline{CD}$ at point E

Prove: $\overline{AB}$ is a diameter of $\odot Q$.



18. In exercise 1, exercise 8, and exercise 15, all contain the phrase “which are not diameters”. Explain why it is necessary to include this phrase in the statement of these three conditionals. Draw a diagram.

Unit VI — Circles

Part C — Line and Segment Relationships

Lesson 3 — Theorem 76 - "If two chords intersect within a circle, then the product of the lengths of the segments of one chord, is equal to the product of the lengths of the segments of the other chord."

Objective: To investigate the relationship between ordinary chords as they intersect within a circle, and to prove this theorem directly, using previously accepted definitions, postulates and theorems.

Important Terms:

Chord of a Circle – A line segment whose endpoints are two points on a circle.

Means-Extremes Property of a Proportion – A property of a valid, standard proportion, which states that the product of the means is equal to the product of the extremes.

Example 1: The segments of one of two intersecting chords of a circle are 4 inches and 8 inches respectively. One segment of the other chord is 6 inches. Find the measure of the second chord.

Solution:

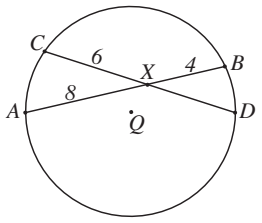
$$AX \cdot XB = CX \cdot XD$$

$$8 \cdot 4 = 6 \cdot XD$$

$$32 = 6 \cdot XD$$

$$\frac{32}{6} = XD$$

$$\frac{16 \cdot \cancel{2}}{3 \cdot \cancel{2}} = \frac{16}{3} = XD$$



The measure of the second chord is given by the expression CX + XD.

$$CD = CX + XD$$

$$CD = 6 + \frac{16}{3}$$

$$CD = \frac{18}{3} + \frac{16}{3}$$

$$CD = \frac{34}{3} \text{ or } 11\frac{1}{3} \text{ inches}$$

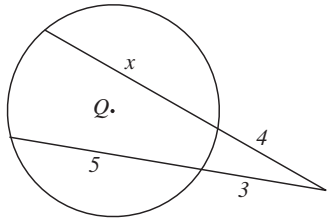
Lesson 4 — Exercises:

1. Prove Theorem 77 - "If two secant segments are drawn to a circle from a single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your Course Notes to check.)
   a) State the theorem.
   b) Draw and label a diagram to accurately show the conditions of the theorem.
   c) List the given information.
   d) Write the statement we wish to prove.
   e) Demonstrate the direct proof using the two column format.

2. Prove Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle, from a single point outside the circle, then the length of that tangent segment is the mean proportional between the length of the secant segment, and the length of its external segment." Use the outline given in Exercise 1.

In exercises 3 through 14, find the value of *x*.

3.



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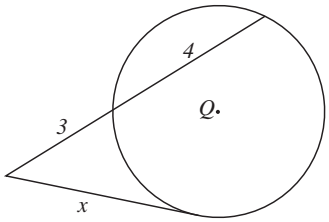

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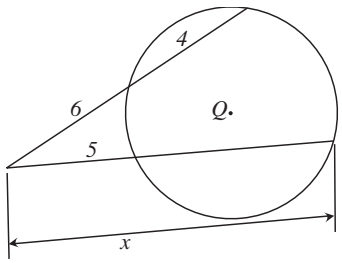

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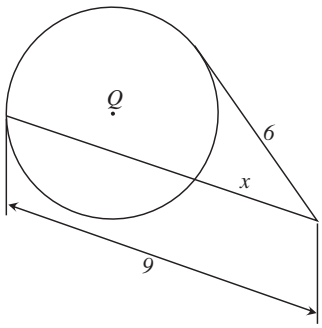

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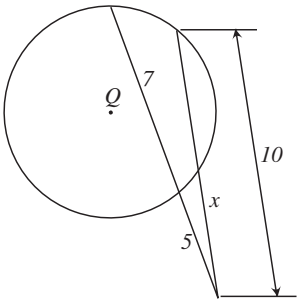

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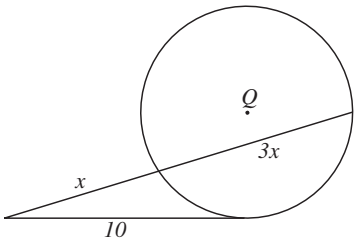
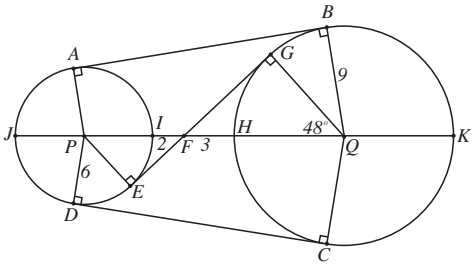


Lesson 5 — Exercises:

1. Prove Theorem 79 using the two column format.

2. Prove Theorem 79 using the indirect proof method.

Use the figure to answer questions 3 through 15.



$\overline{PA} \perp \overline{AB}$; $\overline{PD} \perp \overline{DC}$; $\overline{PE} \perp \overline{EG}$;
$\overline{QB} \perp \overline{BA}$; $\overline{QC} \perp \overline{CD}$; $\overline{QG} \perp \overline{GE}$
QB = 9; PD = 6; FI = 2; FH = 3

3. m∠ABQ = _____.

4. $\overline{FG}$ is a _____ segment. State the Theorem which justifies this statement.

5. PA = _____.

6. m∠GFQ = _____.

7. FG = _____.

8. JK = _____.

9. m∠FPE = _____.

10. $\overline{FE}$ is a _____ segment. State the Theorem which justifies this statement.

11. FE = _____.

12. DC = _____.

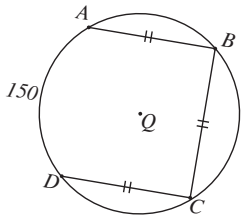
13. m∠PFE = _____.

14. PF = _____.

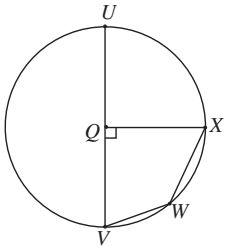
15. $\overline{DC}$ is a tangent segment to circle _____ and circle _____.
State the Theorem which justifies this statement.

16. Theorem 79 is the converse of Corollary 68a. State the relationship as a biconditional.

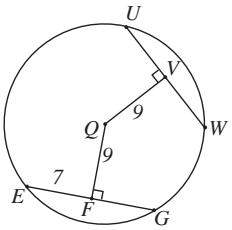
12. $m\overset{\frown}{AD} = 150$
Find $m\overset{\frown}{BC}$.



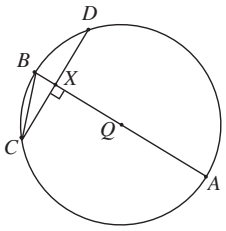
13. $\overline{XQ} \perp \overline{UV}$
$\overline{WX} \cong \overline{VW}$
Find $m\overset{\frown}{VW}$.

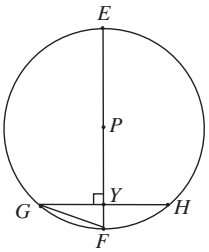


14. QV = 9
QF = 9
EF = 7
$\overline{QV} \perp \overline{UW}$
$\overline{QF} \perp \overline{EG}$
Find UW



Use the following information in exercises 15 through 17.





Given: $\overline{AB}$ is a diameter of $\odot Q$
$\overline{EF}$ is a diameter of $\odot P$.
$\overline{AB} \perp \overline{CD}$
$\overline{EF} \perp \overline{GH}$

15. Given: $\odot Q \cong \odot P$
$\overline{CD} \cong \overline{GH}$

Prove: $\overset{\frown}{CD} \cong \overset{\frown}{GH}$

16. Given: $\odot Q \cong \odot P$
$\overline{QX} \cong \overline{PY}$

Prove: $\overline{CD} \cong \overline{GH}$

17. Given: $\overline{QX} \cong \overline{PY}$
$\odot Q \cong \odot P$

Prove: $\overset{\frown}{CD} \cong \overset{\frown}{GH}$

Unit VI — Circles

Part D — Circles and Concurrency

Lesson 1 — Theorem 83 - "If you have a triangle, then that triangle is cyclic."

Objective: To investigate a relationship between triangles and circles, and to prove this theorem directly, using previously accepted definitions, postulates and theorems.

Important Terms:

Concurrent Lines – From two Latin words meaning "to run together", this term deals primarily with intersecting lines, or line segments. Formally, if two or more lines contain the same point, they are said to be concurrent. (Note: In Geometry, because it is more significant when three or more lines contain the same point, the more frequently used geometric definition of concurrent lines relates to three or more lines.)

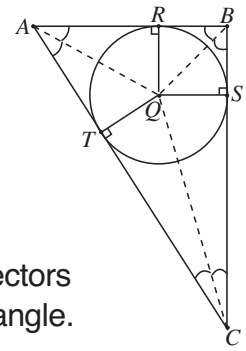
Concyclic Points – From two Latin words meaning "together on a circle", this term deals primarily with points on a circle. Formally, if two or more points lie on the same circle, they are said to be concyclic. (Note: In Geometry, because it is more significant when three of more points lie on the same circle, the more frequently used geometric definition of concyclic points relates to three or more points.)

Cyclic Polygons – A specific term which refers to a relationship between polygons and circles. Formally, when there is a circle which contains all of the vertices of a polygon, then that polygon is cyclic.

Corollary 83a – "If you have a triangle, then the perpendicular bisectors of the sides are concurrent."

Corollary 83b – "If you have a triangle, then the bisectors of the angles are concurrent."

Example 1: The angle bisectors of $\triangle ABC$ meet at point Q from Corollary 83b. Given that AT = 15 and AQ = 17.



a) Which segments are congruent?
b) Find QT and QS.

Solution:

a) $\overline{QT}$, $\overline{QS}$ and $\overline{QR}$ are congruent since the angle bisectors meet at a point equidistant from the sides of the triangle. Using Theorem 80, we can conclude that: $\overline{AR} \cong \overline{AT}$, $\overline{BR} \cong \overline{BS}$, and $\overline{CS} \cong \overline{CT}$.

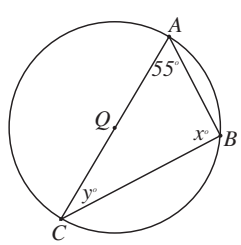
b) $\overline{QT} \perp \overline{AT}$ by corollary 68a.

$$(AT)^2 + (QT)^2 = (AQ)^2$$
$$(15)^2 + (QT)^2 = (17)^2$$
$$225 + (QT)^2 = 289$$
$$(QT)^2 = 64$$
$$QT = \pm\sqrt{64} \text{ (QT cannot be negative)}$$
$$QT = 8$$
$$QT = QS$$
$$QS = 8$$

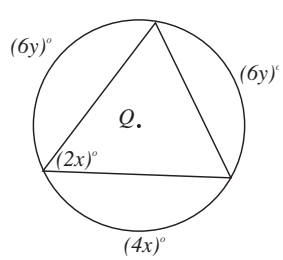
Lesson 1 — Exercises:

1. Prove Theorem 83 - "If you have a triangle, then that triangle is cyclic."

2. Find the values of *x* and *y*.



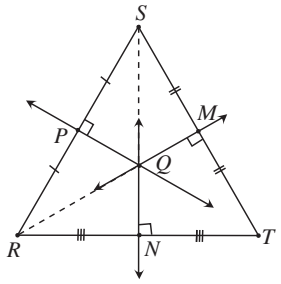
3. Find the values of *x* and *y*.



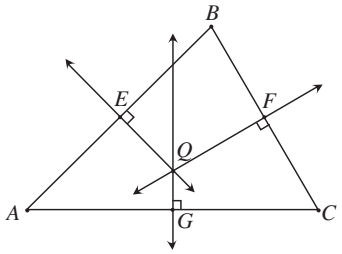
4. Write a two column proof for Corollary 83a - "If you have a triangle, then the perpendicular bisectors of the sides are concurrent."

5. Write a two column proof for Corollary 83b - "If you have a triangle, then the bisectors of the angles are concurrent."

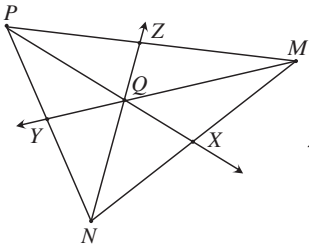
11. The perpendicular bisectors of the sides of △RST meet at point Q. SQ = 11 and QM = 4. Find RQ. Give a reason for your answer.



12. △ABC is given. Perpendicular bisectors of the sides, QE, QF and QG are shown. Can you conclude that QE = QG? If not, explain, and state a correct conclusion that can be deduced from the diagram.

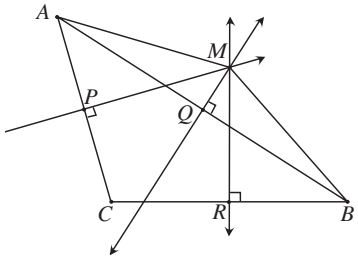


13. Traingle PMN is given. Angle bisectors PX, MY, and NZ are shown. Can you conclude that QZ = QX? If not, explain, and state a correct conclusion that can be deduces from the diagram.



14. The three perpendicular bisectors of the sides of a triangle are concurrent in a point which can be inside the triangle, on the triangle, or outside the triangle. Sketch an obtuse triangle, an acute triangle, and a right triangle showing the perpendicular bisectors of the sides in each to verify each relationship.

15. Triangle ABC is an obtuse triangle. Perpendicular bisectors of the sides meet at point M. MP = 12 and MB = 13. Find AC.



In exercises 16 through 20, complete the statement using always, sometimes, or never.

16. A perpendicular bisector of a side of a triangle _____ passes through the midpoint of a side of the triangle.

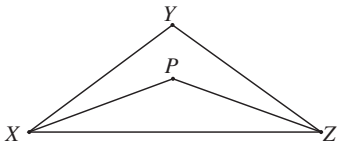
17. The angle bisectors of the angle of a triangle _____ intersect at a single point.

18. The angle bisectors of the angle of a triangle _____ meet at a point outside the triangle.

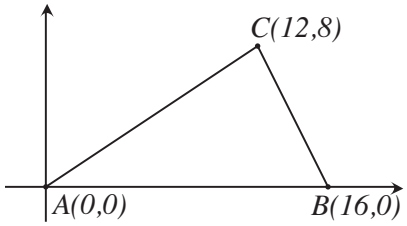
19. The perpendicular bisectors of the sides of a triangle meet at a point which _____ lies outside the triangle..

20. The midpoint of the hypotenuse of a right triangle is _____ equidistant from all vertices of the triangle.

21. $\overline{XP}$ and $\overline{ZP}$ are angle bisectors of $\angle X$ and $\angle Z$ in $\triangle XYZ$. $m\angle XYZ = 112$. Find $m\angle XPZ$.



Use the graph of $\triangle ABC$ and exercises 22 through 24 to illustrate Corollary 83a, about the concurrency of perpendicular bisectors of the sides of a triangle.



22. Find the midpoint of each side of $\triangle ABC$. Use the midpoints to find the equations of the perpendicular bisectors of the sides of $\triangle ABC$.

23. Using your equations from exercise 22, find the intersection of two of the lines.

24. Show that the point in exercise 23 is equidistant from the vertices of $\triangle ABC$.

Unit VI — Circles

Part D — Circles and Concurrency

Lesson 2 — Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic."

Objective: To investigate a relationship between quadrilaterals and circles, and to prove this theorem directly, using previously accepted definitions, postulates and theorems.

Important Terms:

Concurrent Lines – From two Latin words meaning "to run together", this term deals primarily with intersecting lines, or line segments. Formally, if two or more lines contain the same point, they are said to be concurrent. (Note: In Geometry, because it is more significant when three or more lines contain the same point, the more frequently used geometric definition of concurrent lines relates to three or more lines.)

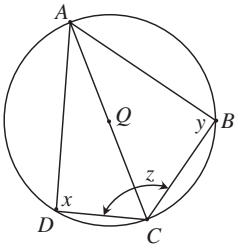
Concyclic Points – From two Latin words meaning "together on a circle", this term deals primarily with points on a circle. Formally, if two or more points lie on the same circle, they are said to be concyclic. (Note: In Geometry, because it is more significant when three of more points lie on the same circle, the more frequently used geometric definition of concyclic points relates to three or more points.)

Cyclic Polygons – A specific term which refers to a relationship between polygons and circles. Formally, when there is a circle which contains all of the vertices of a polygon, then that polygon is cyclic.

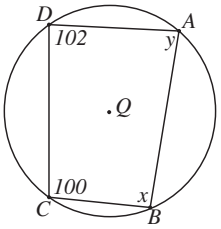
Lesson 2 — Exercises:

1. Prove Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic." (Converse of Corollary 67b)

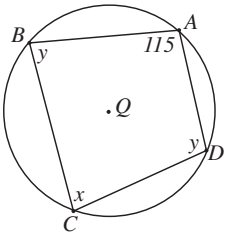
2. Find the values of *x, y,* and *z*.
$m\overset{\frown}{BCD} = 136^{\circ}$



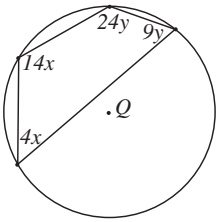
3. Find the values of *x* and *y*.
$m\overset{\frown}{BCD} = z$



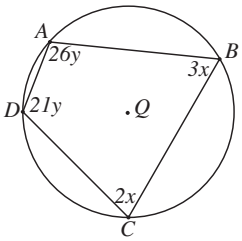
4. Find the values of *x* and *y*.
$m\overset{\frown}{ABC} = z$



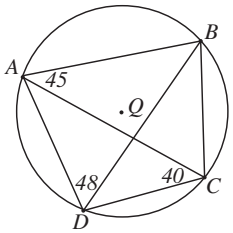
5. Find the values of *x* and *y*.



6. Find the values of *x* and *y*.



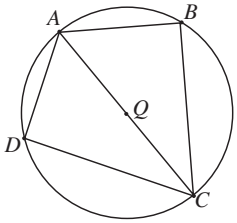
7. Find the measure of the angles of quadrilateral ABCD.



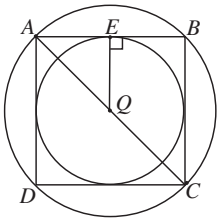
8. Prove that trapezoid inscribed in a circle, is an isosceles trapezoid.

9. Suppose that ABCD is a quadrilateral inscribed in a circle, and that AC is a diameter of the circle.

If $m\angle A$ is three times $m\angle C$, what are the measure of all four angles?



10. Show that the ratio of the radius of the inscribed circle to the radius of the circle circumscribed about a square is $\sqrt{2}$ to 2.

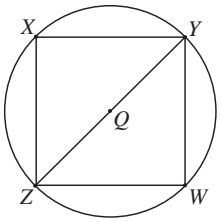


11. All regular simple polygons are cyclic. A circle contains 360 degrees. How many degrees are in each arc of a circle circumscribed about:

a) An equilateral triangle?
b) A square?
c) A regular hexagon?
d) A regular octagon?

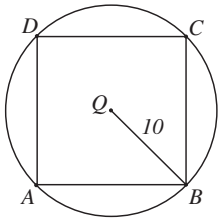
12. Given: Quadrilaterl XYWZ is cyclic.
$\overline{ZY}$ is a diameter of circle Q.
$\overline{XY} \parallel \overline{ZW}$

Prove: $\widehat{XY} \cong \widehat{ZW}$



13. Square ABCD is cyclic

a) Find m$\angle$AQB
b) Find AB
c) Find the distance from point Q to $\overline{AB}$.



In exercises 14 through 19, tell if the given quadrilateral can always be inscribed in a circle. Explain each answer.

14. Square
15. Rectangle
16. Parallelogram
17. Kite
18. Rhombus
19. Isosceles Trapezoid

20. The line segment in a quadrilateral drawn from a midpoint of a side perpendicular to the opposite side is called a maltitude. In a cyclic quadrilateral, maltitudes are concurrent. Using a straightedge, sketch a cyclic quadrilateral and its maltitudes and verify this relationship.