Geometry: A Complete Course
(with Trigonometry)

Module D - Student WorkText

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VideoText*Interactive*

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Module D–Student Worktext

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Example 1: In each of the following, write the ratios of the two given numbers in lowest terms.

a) 2 to 3

b) 8 to 24

c) $4x : 5x$

d) $10x^2$ to $4x$

e) 90 inches to 360 inches

f) 1 meter to 10 meters

g) $(x^2 + 4x)$ to $(x + 4)$

h) $3xy : 6y$

Solution:

a) $\frac{2}{3}$

b) $\frac{8}{24} = \frac{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot 1}{\cancel{2}\cdot\cancel{2}\cdot\cancel{2}\cdot 3} = \frac{1}{3}$

c) $\frac{4x}{5x} = \frac{4\cdot\cancel{x}}{5\cdot\cancel{x}} = \frac{4}{5}$

d) $\frac{10x^2}{4x} = \frac{\cancel{2}\cdot 5\cdot\cancel{x}\cdot x}{\cancel{2}\cdot 2\cdot\cancel{x}} = \frac{5x}{2}$

e) $\frac{90 \text{ inches}}{360 \text{ inches}} = \frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{5}}{\cancel{2}\cdot 2\cdot 2\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{5}} = \frac{1 \text{ inch}}{4 \text{ inches}}$

f) $\frac{1 \text{ meter}}{10 \text{ meters}}$

g) $\frac{x^2 + 4x}{x + 4} = \frac{x\cancel{(x+4)}}{\cancel{(x+4)}\cdot 1} = \frac{x}{1}$ or x

h) $\frac{3xy}{6y} = \frac{\cancel{3}\cdot x\cdot\cancel{y}}{2\cdot\cancel{3}\cdot\cancel{y}} = \frac{x}{2}$

Example 2: In the given figure, AB = 28 and BC = 7. Write the following ratios in lowest terms.

a) AC : AB

b) $\frac{BC}{AB}$

c) BC : AC

d) $\frac{AB}{AC}$

Solution:

a) $\frac{AC}{AB} = \frac{21}{28} = \frac{3\cdot\cancel{7}}{4\cdot\cancel{7}\cdot 1} = \frac{3}{4}$

b) $\frac{BC}{AB} = \frac{7}{28} = \frac{\cancel{7}}{\cancel{7}\cdot 2\cdot 2} = \frac{1}{4}$

c) $\frac{BC}{AC} = \frac{7}{21} = \frac{1\cdot\cancel{7}}{3\cdot\cancel{7}} = \frac{1}{3}$

d) $\frac{AB}{AC} = \frac{28}{21} = \frac{2\cdot 2\cdot\cancel{7}}{3\cdot\cancel{7}} = \frac{4}{3}$

31. Suppose you ride your bicycle 90km in 3 hours. Compare your distance traveled, to your riding time, as a fraction. Then, express the fraction as a rate in kilometers per hour.

32. On a recent weekend trip, Rebecca traveled 437 miles using 23 gallons of gasoline. Compare the miles Rebecca traveled, to the fuel she used, as a fraction. Then, express that fraction as a rate, in miles per gallon.

33. Two supplementary angles have a ratio of 2 to 1. What is the measure of each angle?

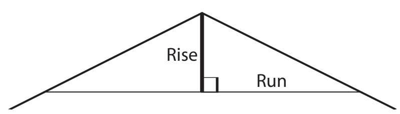
34. The acute angles of a right triangle are in a ratio of 5 to 4. What is the measure of each angle?

35. A 56-inch segment is divided in a ratio of 3 : 5. What is the length of each segment?

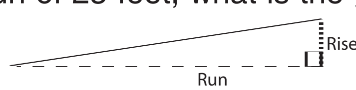
36. Two numbers are in a ratio of 2 : 3. What is the ratio of their squares?

37. Find the measures of the angles of a triangle, if they are in the ratio of 5 : 7 : 8.

38. The pitch of a roof is the ratio of the rise to the run. If a roof has a rise measuring 1.8 feet, and a run measuring 3.6 feet, what is the pitch? Rewrite this ratio as a decimal to the nearest hundredth.



39. The grade of a residential driveway is the ratio of the rise to the run. If a driveway has a rise of 15 inches and a run of 25 feet, what is the grade of the driveway?



40. If $2x = 3y$, find the ratio of x to y.

Unit IV — Triangles

Part C — Similarity – Part 1 (General Geometric Relationship)

Lesson 2 — *Special Properties of Proportions*

Objective: To understand and recognize various special properties of proportions, and to be able to manipulate proportions using those special properties.

Important Terms:

Geometric Mean in a Proportion – In a proportion, when the second and third terms are equal in value, that value is called the geometric mean between the first and fourth terms. Generally:

If $\frac{a}{x} = \frac{x}{d}$, (with $x \neq 0$, and $d \neq 0$), then x is the geometric mean between a and d.

Means-Extremes Product Property of a Proportion – In a proportion, the product of the means, is equal to the product of the extremes. Generally:

If $\frac{a}{b} = \frac{c}{d}$, (with $b \neq 0$, and $d \neq 0$), then $a \bullet d = b \bullet c$

Equivalent Forms of a Proportion – Recognizing that, in a given valid proportion, the means-extremes product property is preserved, there are three additional forms of that proportion which are considered equivalent. Specifically:

If $\frac{a}{b} = \frac{c}{d}$, (with $b \neq 0$, and $d \neq 0$), the following proportions are equivalent forms:

$$\frac{a}{c} = \frac{b}{d}\ ,\ \frac{d}{b} = \frac{c}{a}\ ,\ \frac{b}{a} = \frac{d}{c}$$

Denominator-Addition Property of a Proportion – In a proportion, the value of the denominator of each ratio may be added to the numerator of that ratio without affecting the integrity of the proportion. Specifically:

If $\frac{a}{b} = \frac{c}{d}$, (with $b \neq 0$, and $d \neq 0$), then $\frac{a+b}{b} = \frac{c+d}{d}$.

Lesson 2 — Exercises:

In Exercises 1-9, find the value of x in each proportion.

1. $\frac{13}{24}=\frac{x}{24}$ **2.** $\frac{14}{x}=\frac{7}{8}$ **3.** $\frac{7}{12}=\frac{9.8}{x}$ **4.** $5:x=8:15$ **5.** $\frac{3-x}{x+1}=\frac{2}{1}$

6. $\frac{x+3}{10}=\frac{3x-2}{8}$ **7.** $\frac{2x-3}{3}=\frac{3x-7}{2}$ **8.** $\frac{x-3}{2}=\frac{2}{x}$ **9.** $\frac{x+2}{6}=\frac{6}{x+2}$

In Exercises 10-12, make a list for each, using a-g, and then complete each statement in seven different ways. Give a reason for each answer. (NOTE: Answers will vary, as there are more than seven ways for each.)

10. If $\frac{7}{8}=\frac{14}{16}$, then ____________.

11. If $\frac{x}{y}=\frac{6}{7}$, then ____________.

12. If $\frac{13}{x}=\frac{19}{y}$, then ____________.

In Exercises 13-15, use the proportion $\frac{a}{b}=\frac{x}{y}$ to complete each proportion.

13. $\frac{a}{x}=$ _______. **14.** $\frac{y}{x}=$ _______. **15.** $\frac{y}{b}=$ _______.

In Exercises 16-18, use the proportion $\frac{c+d}{d}=\frac{u+v}{v}$ to complete each proportion.

16. $\frac{c}{d}=$ _______. **17.** $\frac{v}{u}=$ _______. **18.** $\frac{c+2d}{d}=$ _______.

19. A basketball player attempts 156 shots and makes 117 during one basketball season. Set up a proportion to determine the player's shooting percentage, and express that percentage to the nearest whole percent.

20. The mixture for a finish coat of concrete is one part cement to two parts sand. Set up a proportion to determine how much cement should be mixed with 15 pounds of sand, and solve it for the amount of cement needed.

21. A designated hitter for the local minor league baseball team made eight hits in nine games. If he continues hitting at this rate, how many hits will he make in 108 games?

22. An old cake recipe requires 2 parts butter to 3 parts sugar. Set up a proportion to determine the amount of butter to be used with $4\frac{1}{2}$ cups of sugar, and find the amount of butter needed.

In Exercises 23 and 24, find the geometric mean between the given numbers.

23. 9 and 16

24. 2 and 18

In Exercises 25 and 26, find the fourth proportional for the numbers given.

25. 4, 6, 10

26. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$

In Exercises 27-29, find the third proportional for the given pairs of numbers.
(Note: If $\frac{a}{x} = \frac{x}{b}$, then b is often called the third proportional.)

27. 2, 7

28. 1, 9

29. $\frac{1}{5}, \frac{1}{11}$

30. Complete the following generalization: If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \frac{g}{h} = \ldots$, then $\frac{a}{b} = \frac{(a + \_\_\_\_\_)}{(b + \_\_\_\_\_)}$.

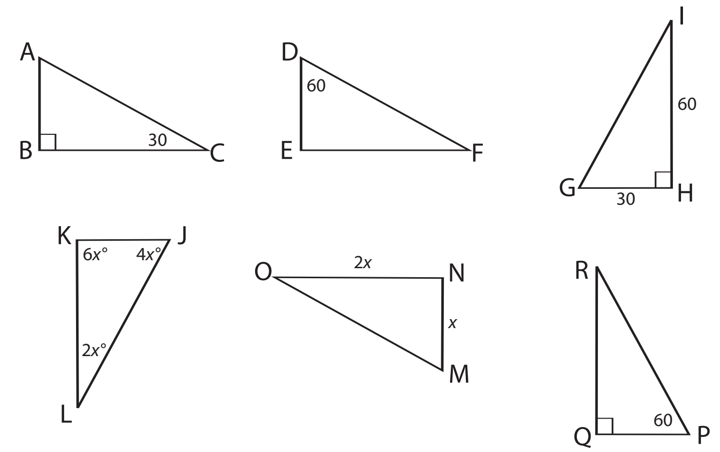
In Exercises 31-33, decide whether each statement is always true, sometimes true, or never true. Give an explanation for your answer.

31. When two terms of the proportion a : b = c : d are interchanged, the two resulting ratios form a proportion.

32. If only one term of the proportion a : b = c : d is added to another term, then the two resulting ratios form a proportion.

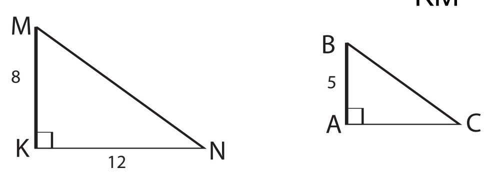
33. If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{b} = \frac{c+b}{d}$.

Example 1: Which of the following triangles are similar to $\triangle ABC$?



Solution: $\triangle JKL$ and $\triangle PQR$ have angles which are congruent to corresponding angles of $\triangle ABC$, and are, therefore, similar to $\triangle ABC$. The other triangles may be similar to $\triangle ABC$, but more information is needed.

Example 2: If $\angle A \cong \angle K$, $\angle B \cong \angle M$, and $\frac{AB}{KM} = \frac{5}{8}$, find the value of AC when KN = 12.



Solution: Since $\triangle ABC \sim \triangle KMN$ (Postulate Corollary 12a or 12b),
we know that $\frac{AB}{KM} = \frac{AC}{KN}$

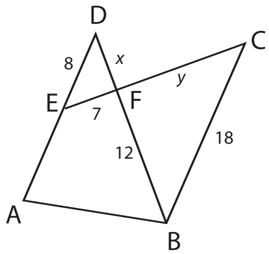
Substituting, we have $\frac{5}{8} = \frac{AC}{12}$

Then $5 \cdot 12 = 8 \cdot AC$ (using the means-extremes property of proportions)

So, $60 = 8 \cdot AC$

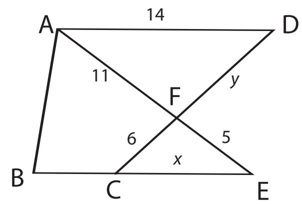
And, $\frac{60}{8} = AC$ or $7\frac{1}{2}$

12.



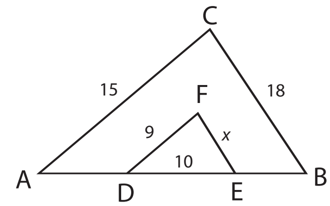
$\overline{AE} \parallel \overline{BC}$

13.



$\overline{AD} \parallel \overline{BE}$

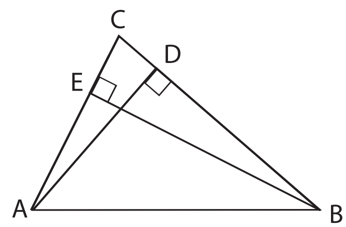
14.



$\overline{DF} \parallel \overline{AC}$; $\overline{EF} \parallel \overline{BC}$;
AB = y

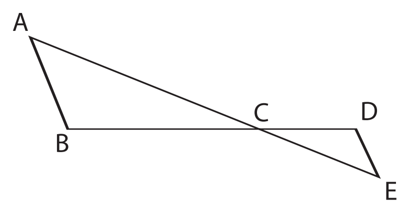
15. Given: $\triangle ABC$ with altitudes $\overline{BE}$ and $\overline{AD}$

Prove: $\dfrac{AD}{AC} = \dfrac{BE}{BC}$



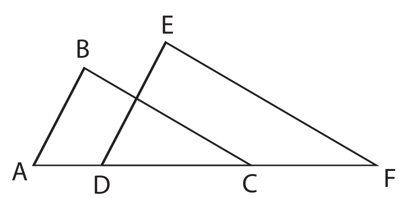
16. Given: $\overline{AB} \parallel \overline{ED}$

Prove: $\dfrac{AB}{ED} = \dfrac{BC}{DC}$



17. Given: $\overline{AB} \parallel \overline{DE}$
$\overline{BC} \parallel \overline{EF}$

Prove: (DE)(AC) = (AB)(DF)



Unit IV — Triangles

Part D — Similarity – Part 2 (Triangles and Their Parts)

Lesson 2 — *Theorem 28: "If a line is parallel to one side of a triangle and intersects the other two sides in different points, then it divides the two sides proportionally."*

Objective: To understand this theorem as an application of previously accepted postulates, definitions and properties, and to demonstrate its proof directly.

Important Terms:

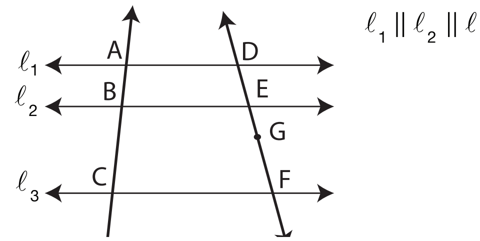
Auxiliary Element – From a Latin word meaning "to help", this is a geometric element which is not specifically referred to in a given diagram, but which does exist, and is needed, in order to logically complete the demonstration of a conditional. This element is usually drawn in a figure, using dashed lines, and must be referenced in the proof with a step justifying its existence. It is important to note that you must not place too many conditions on the element (called "over-determining") thereby denying its existence. Neither must you place too few conditions on the element (called "under-determining"), thereby allowing the existence of more than one such element. In other words, a geometric element is considered to be "determined", if exactly one such element can be drawn to meet the conditions.

"Divides Proportionally" – Noting the illustration below, $\overline{AB}$ and $\overline{CD}$ are said to be "divided proportionally" by points X on $\overline{AB}$, and Y on $\overline{CD}$, if $\frac{AX}{XB} = \frac{CY}{YD}$.

Corollary 28a – "If a line intersects two sides of a triangle in different points so that the sides are divided proportionally, then the line is parallel to the third side." (This is the converse of Theorem 28)

Corollary 28b – "If three or more parallel lines intersect two transversals, then they divide the transversals proportionally."

For Exercises 14-18, complete each statement using the following diagram.



14. AB = 3, BC = 6, DE = 2, EF = _____

15. DE = 9, AB = 12, BC = 16, EF = _____

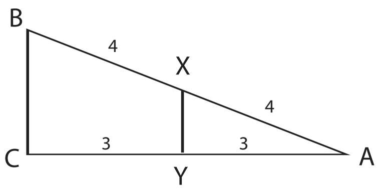
16. DE = 9, EF = 11, AB = x, BC = 40-x

17. AC = 16, AB = 4, DE = 6, DF = _____

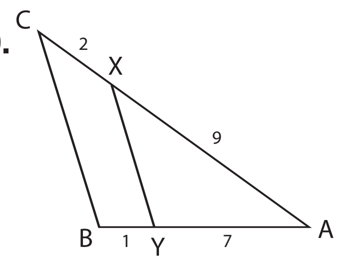
18. AB = 4, BC = 5, FG = 6, ED = 8, EF = _____

For Exercises 19-23, apply Corollary 28a to the given triangles to determine if $\overline{XY} \parallel \overline{BC}$.

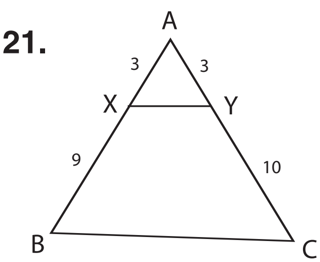
19.



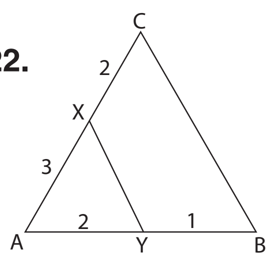
20.



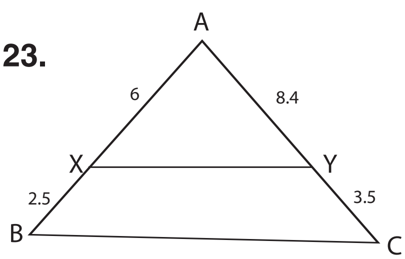
21.



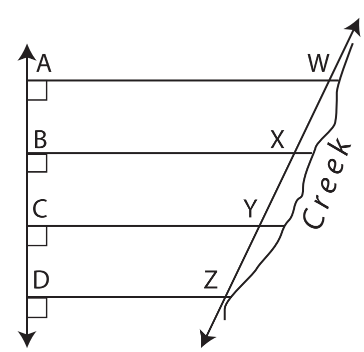
22.



23.



24. In a new subdivision, there is a section of lots of equal width, where the front lot-line runs along a north-south street, the side lot-lines are all perpendicular to the street, and the back lot-line runs along a creek that crosses the lots at a northeast angle as shown in the diagram. Note: WZ = 372 ft. What is the approximate length (to the nearest foot) of each rear lot-line?



Important Terms: (continued)

Geometric Mean in a Proportion – In a proportion, when the second and third terms are equal in value, that value is called the geometric mean between the first and fourth terms.

Corollary 30a – "If you have a right triangle, then either leg is the geometric mean between the hypotenuse of the triangle, and the projection of that leg on that hypotenuse."

Corollary 30b – "If you have a right triangle, then the altitude drawn to the hypotenuse, divides that hypotenuse into two segments, in such a way that, the altitude is the geometric mean between the two segments formed by drawing that altitude."

Corollary 30c – "If you have an altitude drawn to the hypotenuse of a right triangle, then the product of the lengths of that altitude and the hypotenuse, is equal to the product of the lengths of the two legs."

Example 1: Simplify each radical.

a) $\sqrt{32}$ b) $\sqrt{\frac{17}{2}}$ c) $\frac{2}{\sqrt{5}}$

Solution: a) $\sqrt{32} = \sqrt{16 \cdot 2} = \sqrt{16} \cdot \sqrt{2} = 4\sqrt{2}$

or

$$\sqrt{32} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2} = \sqrt{2^2 \cdot 2^2 \cdot 2} = \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{2}$$
$$= 2 \cdot 2 \cdot \sqrt{2}$$
$$= 4\sqrt{2}$$

b) $\sqrt{\frac{17}{2}} = \frac{\sqrt{17} \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{17 \cdot 2}}{\sqrt{2^2}} = \frac{\sqrt{34}}{2}$

c) $\frac{2}{\sqrt{5}} = \frac{2 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{2\sqrt{5}}{\sqrt{25}} = \frac{2\sqrt{5}}{5}$

Unit IV — Triangles

Part D — Similarity – Part 2 (Triangles and Their Parts)

Lesson 5 — *Theorem 31: "If you have a given right triangle, then the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the two legs." (The Pythagorean Theorem)*

Objective: To understand this theorem as an application of previously accepted postulates, corollaries and properties, and to demonstrate its proof directly.

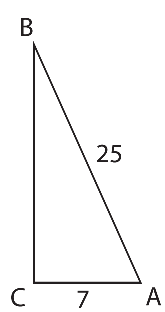
Important Terms:

Corollary 31a – "If you have a right triangle whose acute angles have measures of 30° and 60°, then the measure of the hypotenuse is twice the measure of the shorter leg, and the measure of the longer leg is $\sqrt{3}$ times the shorter leg." (This special right triangle is usually referred to as a "30-60-90 Right Triangle".)

Corollary 31b – "If you have a right triangle whose acute angles each have measures of 45°, then the measure of the hypotenuse is $\sqrt{2}$ times the measure of either leg." (This special right triangle is usually referred to as a "45-45-90 Right Triangle".)

Example 1: If a 25 foot ladder, represented by $\overline{AB}$, is placed 7 feet, represented by $\overline{AC}$, from a vertical wall, represented by $\overline{BC}$, how high up the wall will the ladder reach?

Solution: a)



$a^2 + b^2 = c^2$

$b = 7 \quad c = 25$

$a^2 + 7^2 = 25^2$

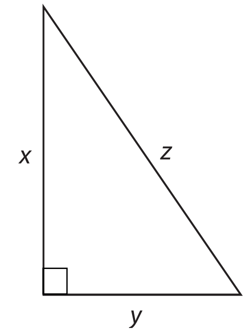
$a^2 + 49 = 625$

$a^2 = 576$

$a = 24$ So, ladder will reach 24 feet up the wall.

4. Write a paragraph argument explaining why the 45-45-90 triangle corollary must be true.

5. Using the figure to the right, find each missing length. Simplify all radical expressions.



a) If $x = 6$ and $y = 8$, then $z =$ _____.

b) If $z = 15$ and $x = 9$, then $y =$ _____.

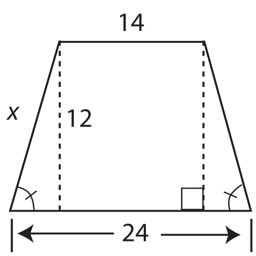
c) If $z = \sqrt{15}$ and $x = \sqrt{10}$, then $y =$ _____.

d) If $y = \sqrt{2}$ and $x = \sqrt{3}$, then $z =$ _____.

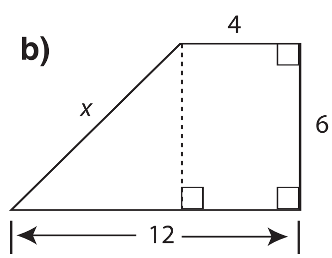
e) If $x = 2\sqrt{3}$ and $z = 6$, then $y =$ _____.

6. In the diagrams below, find x.

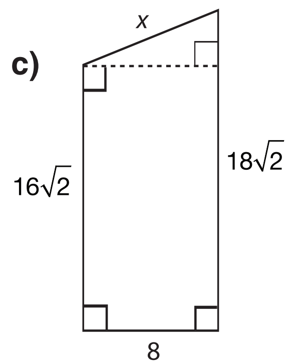
a)



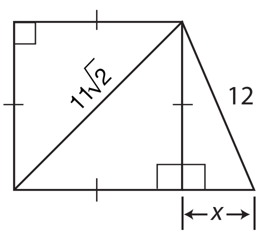
b)



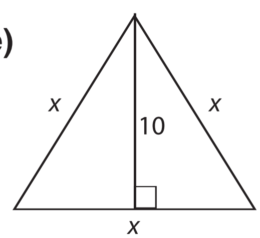
c)



d)



e)

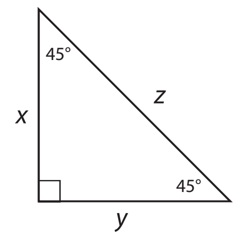


7. Which of these triples could be sides of a right triangle?

a) (2, 3, 4) **b)** $\left(\sqrt{2}, \sqrt{3}, \sqrt{5}\right)$ **c)** (1, 1, 2) **d)** $\left(\frac{1}{3}, \frac{1}{4}, \frac{1}{5}\right)$

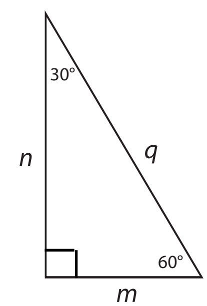
Refer to the figure at the right to find the missing lengths in Exercise 8.

8.	*x*	*y*	*z*
a)	6	______	______
b)	______	10	______
c)	2.5	______	______
d)	______	$\sqrt{2}$	______
e)	______	______	$7\sqrt{2}$
f)	______	______	8



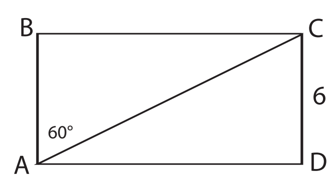
Refer to the figure at the right to find the missing lengths in Exercise 9.

9.	*m*	*n*	*q*
a)	4	______	______
b)	$\frac{2}{3}$	______	______
c)	______	______	6
d)	______	______	$\frac{3}{4}$
e)	______	$5\sqrt{3}$	______
f)	______	9	______



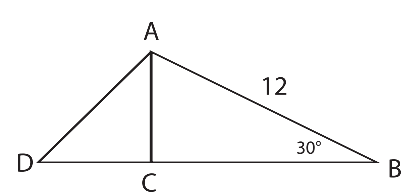
Rectangle ABCD is shown to the right. Use this figure for Exercise 10.

10. a) Find the length of the diagonal $\overline{AC}$.
b) Find the length of side $\overline{BC}$ of the rectangle.
c) Find the perimeter of rectangle ABCD.



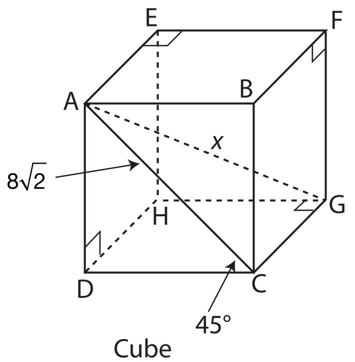
For Exercise 11, find the indicated measures, referring to the figure at the right, where $m\angle BAD = 105°$ and $\overline{AC} \perp \overline{DB}$.

11. a) AC = ______
b) BC = ______
c) DC = ______
d) AD = ______
e) $m\angle BAC$ = ______
f) $m\angle CDA$ = ______

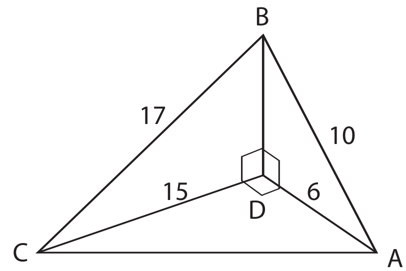


Lesson 6 — Exercises:

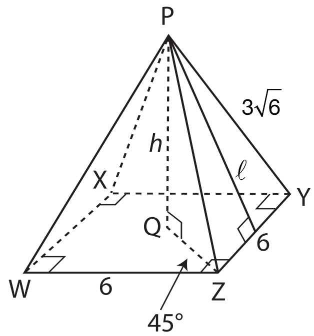
1. Find x.



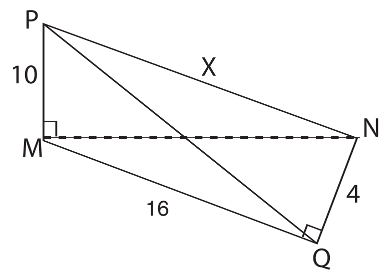
2. Show that △ABC is not a right triangle.



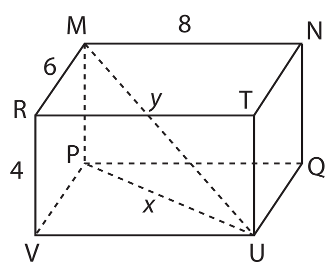
3. Find altitude h and slant height ℓ.



4. Find x.

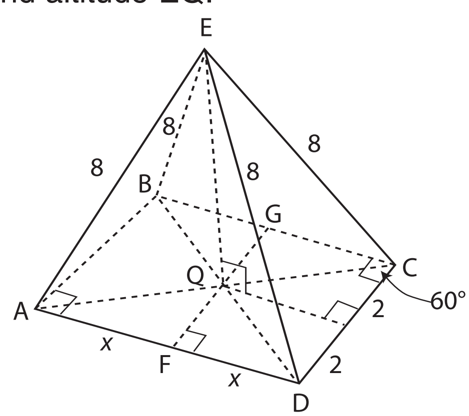


5. Find x and y.

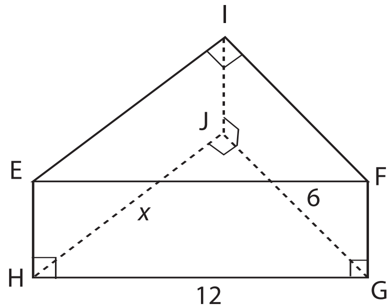


Note: This is a right rectangular prism.

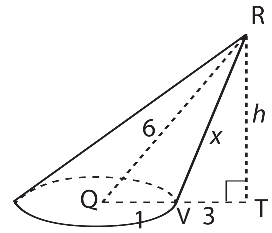
6. Find the perimeter of the base.
Find altitude EQ.



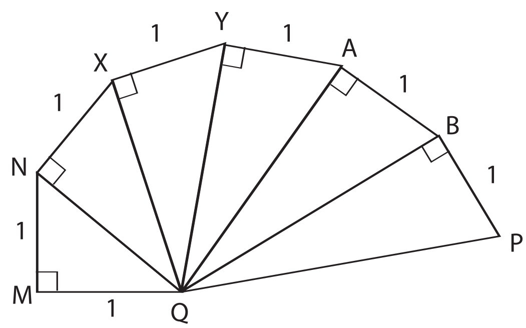
7. Find x. Find m∠JHG.



8. Find h. Find x.

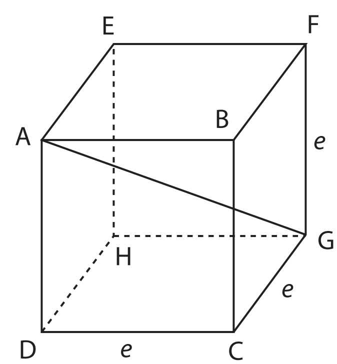


Use the figure below to find each of the lengths asked for in Exercise 9-14.

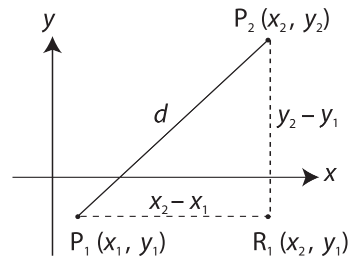


9. NQ **10.** XQ **11.** YQ **12.** AQ **13.** BQ **14.** PQ

15. Suppose each edge of the cube at the right is e. Develop a formula for the measure of the distance from point A to G in terms of e.



Extending Your Mind:



Recall from Algebra (Videotext Algebra: A Complete Course, Unit VII, Part D, Lesson 1), the mathematical relation which is obtained by applying the Pythagorean Theorem to two points in the Cartesian coordinate system.

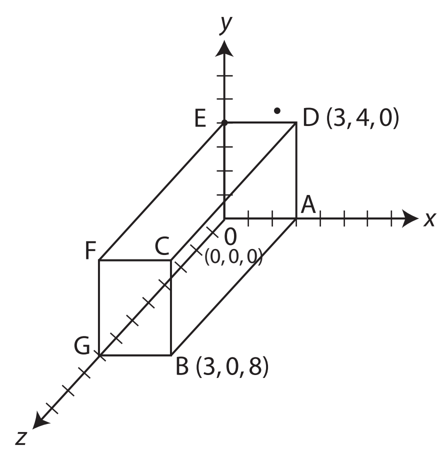
$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

The Distance Formula: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

The distance between two points in three dimensions, $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$, can be found using a formula similar to the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Consider the rectangular prism to the right, sketched in a three-dimensional (*x, y, z*) coordinate system for Exercises 16-19.



16. Write the coordinates of vertices A, C, E, F, and G.

17. Find the length of diagonals $\overline{DF}$, $\overline{DG}$, and $\overline{DB}$.

18. Give the dimensions of the prism.

19. Find the volume of the prism.

20. Five of the eight vertices of a rectangular prism are A (1, -1, 2), B (1, -1, 6), C (1, 5, 2), D (1, 5, 6) and E (9, -1, 2).

Find:

a) The dimensions of the prism.

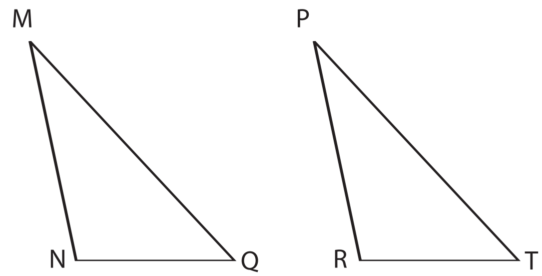
b) The coordinates of the other three vertices.

c) The length of diagonal $\overline{DE}$

Note: It might be helpful to sketch the prism in a three-dimensional coordinate system.

Lesson 1 — Exercises:

1. In the figures to the right, △MNQ ≅ △PRT. Complete each statement below by supplying the missing symbols.



a) The correspondence M __ __ to __ RT is a congruence.

b) The correspondence QM__ to __ __ R is a congruence.

c) ∠M ≅ ∠___

d) ∠Q ≅ ∠___

e) ∠N ≅ ∠___

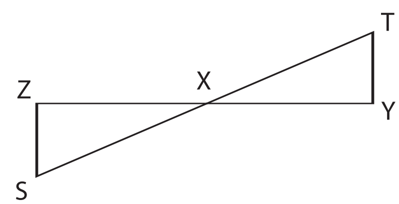
f) $\overline{MQ}$ ≅ ___

g) $\overline{NQ}$ ≅ ___

h) $\overline{NM}$ ≅ ___

i) △TRP ≅ △____

2. In the given figure, △TXY ≅ △SXZ. Use this correspondence for a-f below.



a) ∠YXT corresponds to ∠__ X __.

b) ∠YTX corresponds to ∠__ __ X.

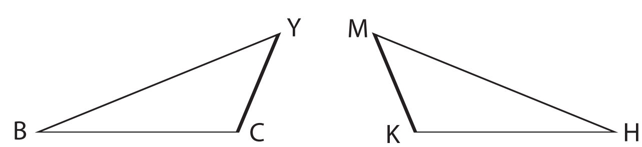
c) ∠TYX corresponds to ∠__ __ __.

d) Is a line segment congruent to itself? Why?

e) Is an angle congruent to itself? Why?

f) Is a triangle congruent to itself? Why?

3. Given △BCY ≅ △HKM. Determine which of the correspondences below are congruences.



a) ∠BYC corresponds to ∠HMK.

b) ∠BYC corresponds to ∠HKM.

c) ∠YCB corresponds to ∠HKM.

d) ∠YCB corresponds to ∠MKH.

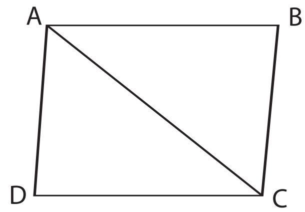
e) ∠CBY corresponds to ∠KMH.

f) ∠YBC corresponds to ∠MHK.

g) ∠CBY corresponds to ∠KHM.

h) ∠BCY corresponds to ∠HKM.

4. In the figure below, △ABC ≅ △CDA. List the congruent parts.



5. In the diagram to the right, △ABC ≅ △XYZ. Use this relationship to complete each of the following statements.

a) XY = ____

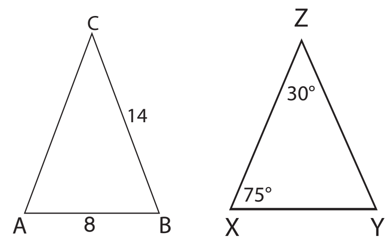
b) m∠Y = ____

c) m∠C = ____

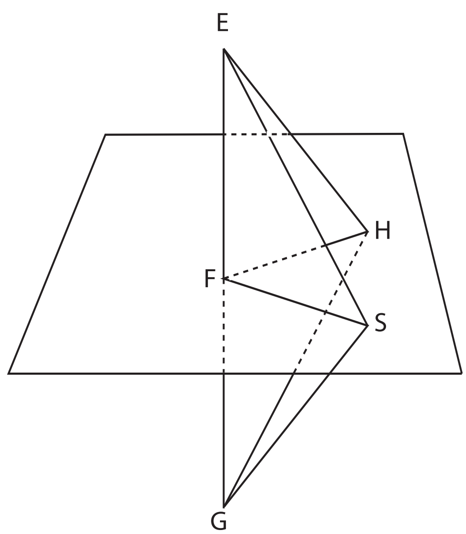
d) ZY = ____

e) m∠A = ____

f) m∠B = ___



6. Use the figure below to name three pairs of triangles that appear to be congruent.



7. Use the figure and your answers for Exercise 6 to complete each of the following statements.

a) ∠FES ≅ ____

b) $\overline{FE}$ ≅ ____

c) ∠HFE ≅ ____

d) $\overline{GS}$ ≅ ____

e) ∠EGS ≅ ____

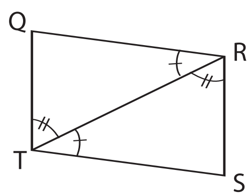
f) $\overline{FS}$ ≅ ____

4. Prove Postulate Corollary 13a –"If two angles and a non-included side of one triangle are congruent to the two corresponding angles and the corresponding non-included side of another triangle, then the two triangles are congruent." (Note: This is the same postulate corollary we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)

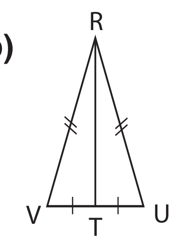
a) State the corollary.
b) Draw and label a diagram to accurately show the conditions of the corollary.
c) List the given information.
d) Write the statement we wish to prove.
e) Demonstrate the direct proof using the two-column format.

5. Name the pairs of triangles that are congruent and name the Postulate 13 Assumption or Corollary which justifies the congruence.

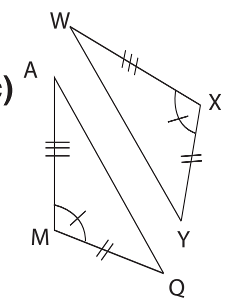
a)



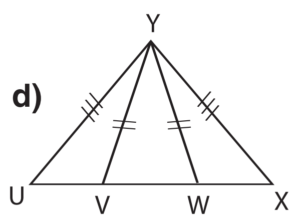
b)



c)

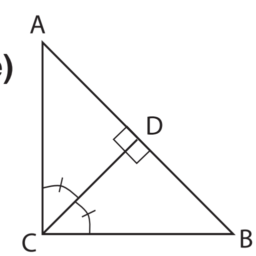


d)

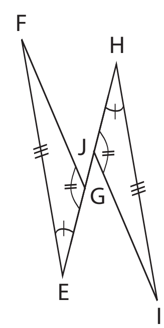


For △UWY and △XVY where $\overline{UV} \cong \overline{XW}$

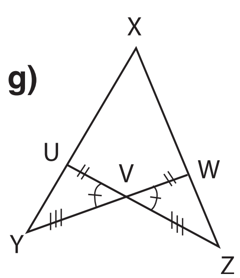
e)



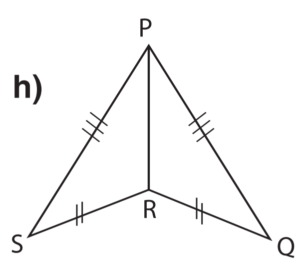
f)



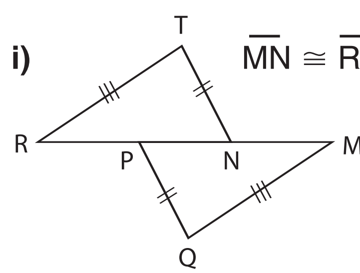
g)



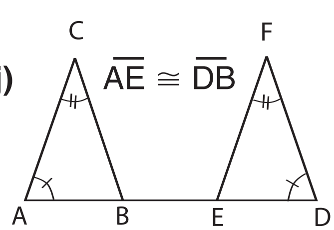
h)



i) $\overline{MN} \cong \overline{RP}$



j) $\overline{AE} \cong \overline{DB}$



Lesson 3 — *Congruence Postulate Corollaries*

Objective: To understand the applications of the Congruence Postulate to the relationships between right triangles, and to prove those applications directly, as additional Congruence Postulate Corollaries.

Important Terms:

Congruence – From the Latin, meaning “to agree”, this is a relationship between geometric figures which have the same shape, and the same size. For practical purposes, two line segments are said to be congruent, if and only if, their measures are equal. Additionally, two angles are said to be congruent, if and only if, their measures are equal. Finally, two polygons are said to be congruent, if and only if, for some pairing of their vertices, the corresponding angles are congruent, and the corresponding sides are congruent.

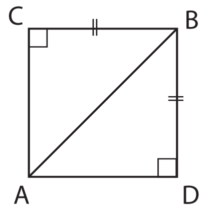
Postulate Corollary 13b – “If the hypotenuse and one acute angle of one right triangle are congruent to the hypotenuse and the corresponding acute angle of another right triangle, then the two right triangles are congruent.” (This is often called the “HA Postulate Corollary”.)

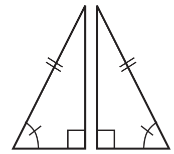
Postulate Corollary 13c – “If a leg and one acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the two right triangles are congruent.” (This is often called the “LA Postulate Corollary”.)

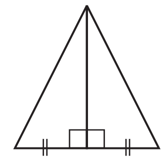
Postulate Corollary 13d – “If the two legs of a right triangle are congruent to the two corresponding legs of another right triangle, then the two right triangles are congruent.” (This is often called the “LL Postulate Corollary”.)

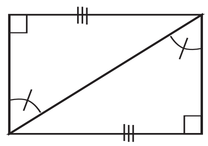
Postulate Corollary 13e – “If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the two right triangles are congruent.” (This is often called the “HL Postulate Corollary”.)

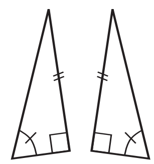
Example 1: Indicate in each of the following whether the given pair of triangles is congruent by the HA, LA, HL, or LL Postulate Corollaries.

a) 

b) 

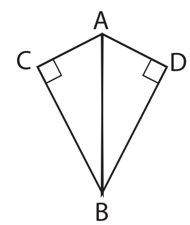
c) 

d) 

e) 

Solution: a) HL b) HA c) LL d) HL, HA or LA e) LA

Example 2: State the additional information you would need to prove $\triangle ABC \cong \triangle ABD$, using each of the following corollaries.



a) HA
b) HL
c) LL
d) LA

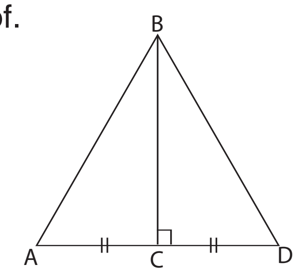
Solution:

a) $\angle CAB \cong \angle DAB$ or $\angle CBA \cong \angle DBA$
b) $\overline{AC} \cong \overline{AD}$ or $\overline{CB} \cong \overline{DB}$
c) $\overline{AC} \cong \overline{AD}$ and $\overline{BC} \cong \overline{DB}$
d) $\overline{AC} \cong \overline{AD}$ or $\overline{CB} \cong \overline{DB}$ and
$\angle BAC \cong \angle BAD$ or $\angle ABC \cong \angle ABD$

Example 3: Supply the missing reasons in the following proof.

Given: $\overline{BC} \perp \overline{AD}$
$\overline{DC} \cong \overline{AC}$

Prove: $\triangle ABC \cong \triangle DBC$



STATEMENT	REASON
1. $\overline{BC} \perp \overline{AD}$	1. Given
2. $\angle DCB$ is a right angle $\angle ACB$ is a right angle	2. ____________________
3. $\triangle DCB$ is a right triangle $\triangle ACB$ is a right triangle	3. ____________________
4. $\overline{DC} \cong \overline{AC}$	4. ____________________
5. $\overline{BC} \cong \overline{BC}$	5. ____________________
6. $\triangle ABC \cong \triangle DBC$	6. ____________________

Solution:
2. Definition of perpendicular lines
3. Definition of right triangle
4. Given
5. Reflexive Property for Congruence
6. LL Postulate Corollary

Lesson 3 — Exercises:

For Exercises 1-3, state the given postulate corollary, draw and label a diagram to accurately show the conditions of the corollary, state the given and the prove, and write the demonstration of the corollary.

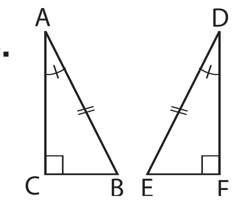
1. Postulate Corollary 13-b: Hypotenuse - Acute Angle Postulate Corollary

2. Postulate Corollary 13-c: Leg - Acute Angle Postulate Corollary

3. Postulate Corollary 13d: Leg - Leg Postulate Corollary

4. Postulate Corollary 13e: Hypotenuse - Leg Postulate Corollary. This corollary is included in this lesson because it is part of this family of postulates and corollaries in Lessons 2 and 3 about triangle congruence. State the corollary and sketch a diagram to accurately illustrate the conditions necessary. A demonstration or proof of this corollary will be asked for in Unit IV, Part F, Lesson 4.

For Exercises 5-12, name the pairs of triangles that are congruent. Then state the postulate corollary (HA, LA, HL, or LL) which justifies each congruence relationship.

5.


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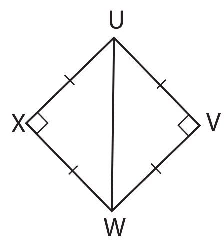

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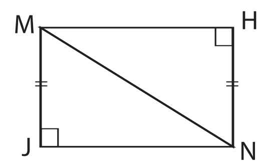

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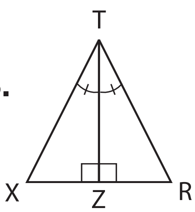

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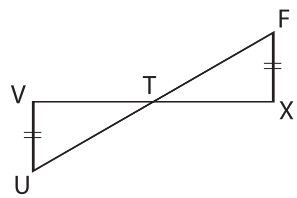

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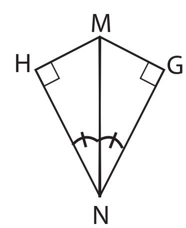

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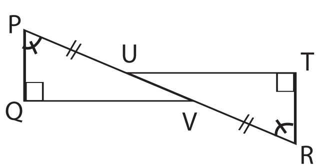

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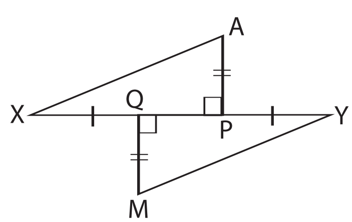
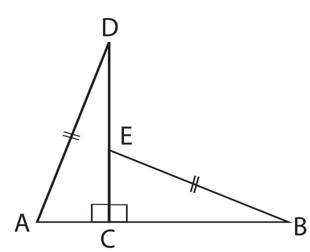


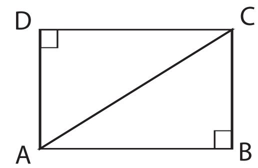
For Exercises 13-16, state the additional information you would need to prove $\triangle ACD \cong \triangle BCE$, using each of the indicated postulate corollaries.

13. HA
14. HL
15. LA
16. LL



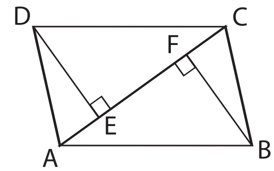
17. Given: $\angle ADC \cong \angle CBA$ are right angles
$\overline{DC} \parallel \overline{BA}$

Prove: $\triangle ADC \cong \triangle CBA$



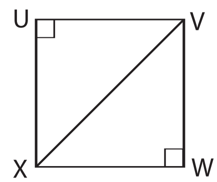
18. Given: ABCD is a parallelogram
$\overline{DE} \perp \overline{AC}$; $\overline{BF} \perp \overline{AC}$; $\overline{AE} \cong \overline{CF}$

Prove: $\triangle AFB \cong \triangle CED$



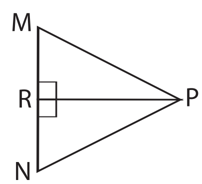
19. Given: UVWX is a square

Prove: $\triangle UVX \cong \triangle WXV$
a) Using the LA Postulate Corollary
b) Using the LL Postulate Corollary



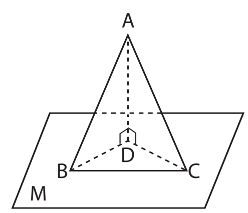
20. Given: $\overline{PR} \perp \overline{MN}$
R is midpoint of $\overline{MN}$

Prove: $\triangle MRP \cong \triangle NRP$



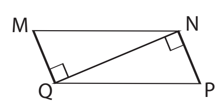
21. Given: $\overline{AD} \perp$ plane of $\triangle BDC$ (Plane M)
$\triangle BDC$ is an equilateral triangle

Prove: $\triangle ADB \cong \triangle ADC$



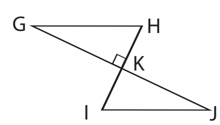
22. Given: $\overline{NQ} \perp \overline{QM}$; $\overline{QN} \perp \overline{NP}$; $\overline{MQ} \cong \overline{PN}$

Prove: $\triangle MQN \cong \triangle PNQ$



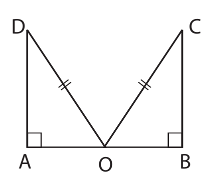
23. Given: $m\angle GKH = 90$
$\overline{GJ}$ and $\overline{HI}$ bisect each other at K

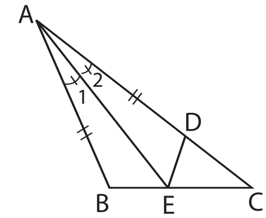
Prove: $\triangle GKH \cong \triangle JKI$



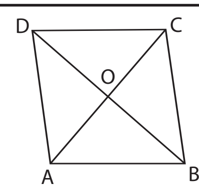
24. Given: $\angle A$ and $\angle B$ are right angles. Point O is the midpoint of $\overline{AB}$. $\overline{OD} \cong \overline{OC}$

Prove: $\triangle DAO \cong \triangle CBO$



Example 1:	Given: $\overline{AB} \cong \overline{AD}$ $\angle 1 \cong \angle 2$ Prove: $\overline{BE} \cong \overline{DE}$ Write an analysis and demonstration of the proof.	
Solution:	Analysis: I can prove $\overline{BE} \cong \overline{DE}$ if I can find a pair of congruent triangles that contain these segments, as corresponding parts. △ABE and △ADE are two such triangles. Finally, since angles 1 and 2 are included angles between two congruent sides, we can use the SAS Congruence Assumption to prove the two triangles congruent, and then use the definition of congruent triangles.	

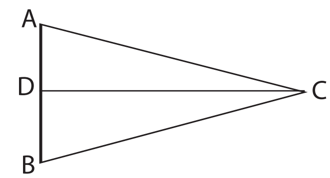
STATEMENT	REASON
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\angle 1 \cong \angle 2$	2. Given
3. $\overline{AE} \cong \overline{AE}$	3. Reflexive Property
4. △ABE ≅ △ADE	4. SAS Congruence Assumption
5. $\overline{BE} \cong \overline{DE}$	5. CPCTC

Example 2:	Given: $\overline{AC}$ and $\overline{BD}$ bisect each other Prove: ∠DAC ≅ ∠BCA Write an analysis and demonstration of the proof.	
Solution:	Analysis: I can prove ∠DAC ≅ ∠BCA if I can find a pair of congruent triangles that contain these angles, as corresponding parts. △DAO and △BCO are two such triangles. And, since $\overline{AC}$ and $\overline{BD}$ bisect each other, I have two pairs of congruent segments which are in △DAO and △BCO. Then, since ∠DOA and ∠BOC are congruent vertical angles, I can prove △DOA ≅ △BOC by the SAS Congruence Assumption, and then use the definition of congruent triangles. (Note: △DAC and △BCA could also be used.)	

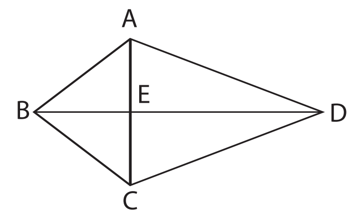
STATEMENT	REASON
1. $\overline{AC}$ and $\overline{BD}$ bisect each other	1. Given
2. $\overline{AO} \cong \overline{CO}$; $\overline{DO} \cong \overline{BO}$	2. A segment bisector divides a segment into two congruent parts
3. ∠DOA ≅ ∠BOC	3. If two lines intersect, then the vertical angles formed are congruent
4. △DOA ≅ △BOC	4. SAS Congruence Assumption
5. ∠DAC ≅ ∠BCA	5. CPCTC

5. Given: $\overline{DC}$ bisects $\angle ACB$
$\overline{AC} \cong \overline{BC}$

Prove: $\overline{AD} \cong \overline{BD}$



Use the diagram to the right for Exercises 6-8.



6. Given: $\overline{DB} \perp \overline{AC}$
$\overline{DB}$ bisects $\overline{AC}$

Prove: $\angle ABD \cong \angle CBD$

7. Given: $\overline{BD}$ bisects $\angle ABC$
$\overline{BD}$ bisects $\angle CDA$

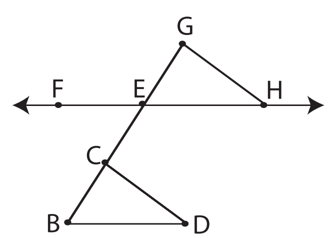
Prove: $\angle BAD \cong \angle BCD$

8. Given: $\overline{AC} \perp \overline{BD}$
$\overline{DB}$ bisects $\overline{AC}$

Prove: $\overline{AD} \cong \overline{CD}$

9. Given: $\overline{BC} \cong \overline{EG}$; $\overline{DB} \cong \overline{HE}$
$\angle FEG$ is supplementary to $\angle DBC$

Prove: $\angle BDC \cong \angle EHG$



Unit IV — Triangles

Part F — Congruence – Part 2 (Applications)

Lesson 3 — *Theorem 32: "If two given triangles are both congruent to a third triangle, then the two given triangles are congruent to each other."*

Objective: To understand this theorem as an application of the definition of congruence, and to demonstrate its proof directly.

Important Terms:

Congruence – From the Latin, meaning "to agree", this is a relationship between geometric figures which have the same shape, and the same size. For practical purposes, two line segments are said to be congruent, if and only if, their measures are equal. Additionally, two angles are said to be congruent, if and only if, their measures are equal. Finally, two polygons are said to be congruent, if and only if, for some pairing of their vertices, the corresponding angles are congruent, and the corresponding sides are congruent.

Lesson 3 — Exercises:

1. Prove Theorem 32 – "If two given triangles are both congruent to a third triangle, then the two given triangles are congruent to each other." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)

a) State the theorem.

b) Draw and label a diagram to accurately show the conditions of the theorem.

c) List the given information.

d) Write the statement we wish to prove.

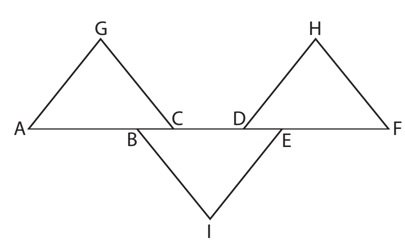
e) Demonstrate the direct proof using the two-column format.

2. Given: $\triangle AGC \cong \triangle FHD$

$\overline{AB} \cong \overline{EC}$, $\overline{GC} \parallel \overline{BI}$

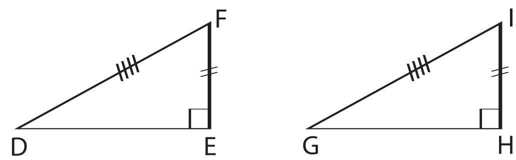
$\overline{AG} \parallel \overline{EI}$

Prove: $\triangle BIE \cong \triangle DHF$



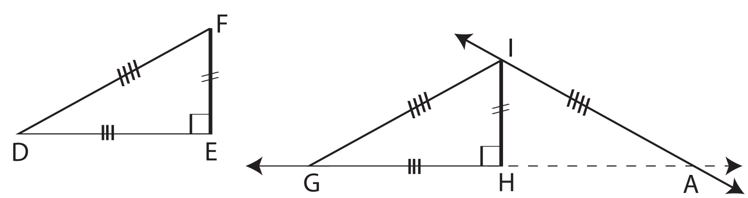
4. Write a demonstration (proof) for Postulate-Corollary 13e, the HL Congruence Postulate (Unit IV, Part E, Lesson 3) by supplying the missing reasons in the following outline.

Postulate Corollary 13e - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the two right triangles are congruent. (HL Congruence Postulate)



Given: $\triangle DEF$ is a right triangle with right angle at E
$\triangle GHI$ is a right triangle with right angle at H
$\overline{DF} \cong \overline{GI}$; $\overline{EF} \cong \overline{HI}$

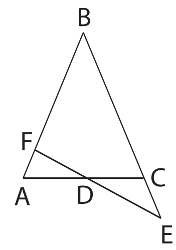
Prove: $\triangle DEF \cong \triangle GHI$



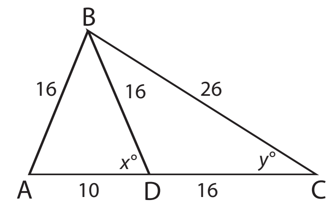
STATEMENT	REASON
1. $\triangle DEF$ is a right triangle with right angle at E	1. Given
2. $\triangle GHI$ is a right triangle with right angle at H	2. Given
3. On the ray opposite $\overrightarrow{HG}$, choose a point A such that HA = ED.	3. Postulate 6-Ruler-Second Assumption: To every point on a line, there corresponds exactly one real number, called the unique distance between the points.
4. $\overline{HA} \cong \overline{ED}$	4. ___________
5. Draw $\overleftrightarrow{AI}$	5. ___________
6. $\angle GHI$ and $\angle AHI$ are supplementary angles	6. ___________

7. $m\angle GHI + m\angle AHI = 180$	7. ___________
8. $m\angle GHI = 90$	8. ___________
9. $90 + m\angle AHI = 180$	9. ___________
10. $m\angle AHI = 90$	10. Properties of Algebra
11. $\angle AHI$ is a right angle	11. Definition of a right angle
12. $\angle AHI \cong \angle DEF$	12. ___________
13. $\overline{EF} \cong \overline{HI}$	13. ___________
14. $\triangle AHI \cong \triangle DEF$	14. ___________
15. $\overline{AI} \cong \overline{DF}$	15. ___________
16. $\overline{DF} \cong \overline{GI}$	16. ___________
17. $\overline{AI} \cong \overline{GI}$	17. Transitive Property of Congruent Line Segments (or substitution)
18. In $\triangle IGA$, $\angle G \cong \angle A$	18. Theorem 33: If two sides of a triangle are congruent, then the angles opposite them are congruent.
19. $\angle A \cong \angle D$	19. CPCTC
20. $\angle G \cong \angle D$	20. Transitive Property of Congruent Angles (or substitution)
21. $\triangle DEF \cong \triangle GHI$	21. ___________

5. In the figure to the right, BA = BC, CE = CD, ∠EDF is a straight angle, and m∠E = 15°. Find m∠ACB, m∠A, m∠B, and m∠BFD.



6. Using the diagram to the right, find:
a) m∠A
b) m∠CBD
c) m∠ABC

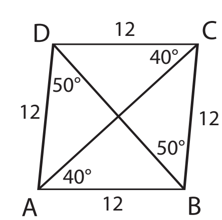


7. △ABC is isosceles with AB = BC. If AB = 4x and BC = 6x – 15, find AB and BC.

8. △DEF is isosceles with base $\overline{DF}$. If DE = 4x + 15, EF = 2x + 45, and DF = 3x + 15, find the lengths of the sides of the triangle.

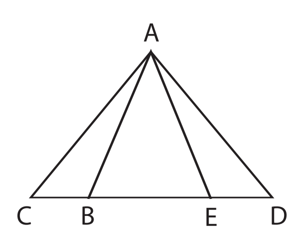
9. In △XYZ, XY = YZ. If m∠X = 4x + 60, m∠Y = 2x + 30, and m∠Z = 14x + 30, find m∠X, m∠Y, and m∠Z.

10. Using the diagram to the right, find:
a) m∠BDC
b) m∠DAC
c) m∠DBA
d) m∠ACB



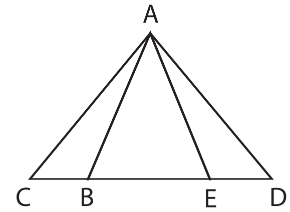
11. Given: $\overline{AC} \cong \overline{AD}$
$\overline{CB} \cong \overline{DE}$

Prove: △ACE ≅ △ADB



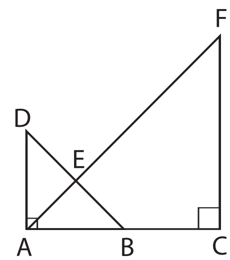
12. Given: $\overline{AB} \cong \overline{AE}$
$\overline{AC} \cong \overline{AD}$

Prove: $\triangle ACE \cong \triangle ADB$



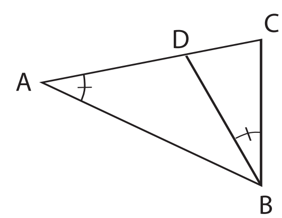
13. Given: $\triangle AEB$ is isosceles
$\overline{DA} \perp \overline{AC}$
$\overline{FC} \perp \overline{AC}$

Prove: $AD \cdot AF = CF \cdot BD$



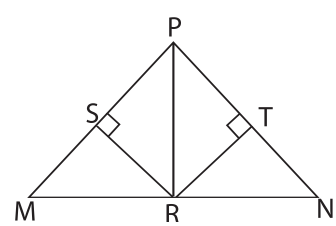
14. Given: $\overline{AB} \cong \overline{AC}$
$\angle A \cong \angle CBD$

Prove: $\frac{AB}{BD} = \frac{BC}{CD}$



15. Given: $\triangle MNP$ with $\overline{MP} \cong \overline{NP}$
$\overline{RT} \perp \overline{PN}$
$\overline{RS} \perp \overline{PM}$

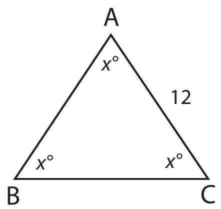
Prove: $RT \cdot RM = RS \cdot RN$



16. Prove: A triangle is isosceles if and only if a median and an altitude of the triangle are the same segment.

For Exercises 7-9, use the information indicated on each figure to state any valid conclusions you can, about the unmarked sides and angles in the figure.

7.



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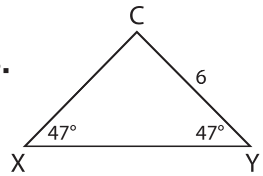

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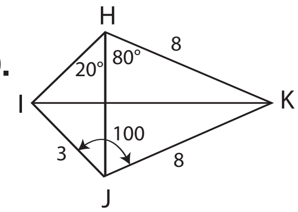
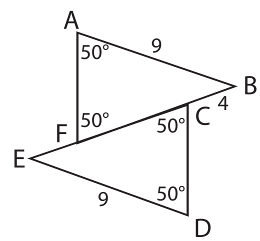


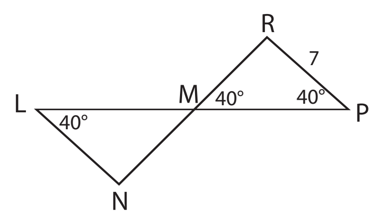
Use △ABF and △CED in the figure below to complete Exercises 10-14.

10. Find BF
11. Find EC
12. Find FE
13. Find CF
14. Find m∠B



In the figure below, M is the midpoint of $\overline{LP}$. Use this illustration to complete Exercises 15-17.

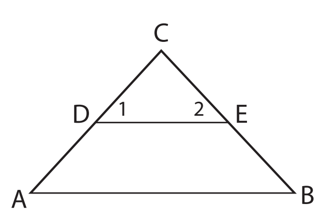
15. Find MR
16. Find m∠NML
17. Find LN



18. In a given triangle △ABC, ∠A ≅ ∠C. If AB = 4x + 25, BC = 2x + 45, and AC = 3x – 15, find the lengths of all three sides.

19. Given: ∠ADE ≅ ∠BED

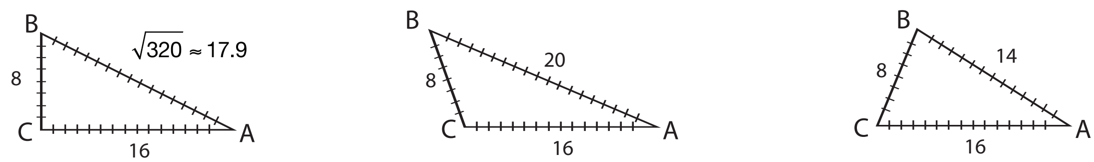
Prove: $\overline{DC} \cong \overline{EC}$



Statements	Reasons
4. $\angle PRQ$ is a right angle	4. __________
5. $\triangle PQR$ is a right triangle	5. __________
6. Draw $\triangle PQR$ with $PR = b$ and $RQ = a$	6. Postulate 6 - Ruler - Second Assumption – To ever pair of points on a line, there corresponds exactly one real number, called the unique distance between the points.
7. $(PQ)^2 = a^2 + b^2$	7. __________
8. $(PQ)^2 = c^2$	8. __________
9. $PQ = c$	9. Properties of Algebra
10. $AB = PQ$; $AC = PR$; $CB = RQ$	10. __________
11. $\overline{AB} \cong \overline{PQ}$; $\overline{AC} \cong \overline{PR}$; $\overline{CB} \cong \overline{RQ}$	11. __________
12. $\triangle ABC \cong \triangle PQR$	12. __________
13. $\angle C \cong \angle R$	13. __________
14. $m\angle C = m\angle R$	14. __________
15. $m\angle C = 90$	15. __________
16. $\angle C$ is a right angle	16. __________
17. $\triangle ABC$ is a right triangle	17. __________

Extending Your Mind:

Consider the diagrams below, showing the result of increasing or decreasing the length of the hypotenuse *c*, of right △ABC.



Original	Increase c	Decrease c
$a^2 + b^2 = c^2$	$a^2 + b^2 < c^2$	$a^2 + b^2 > c^2$
$8^2 + 16^2 = \sqrt{320}^2$	$8^2 + 16^2 < 20^2$	$8^2 + 16^2 > 14^2$
$64 + 256 = 320$	$64 + 256 < 400$	$64 + 256 > 196$
$320 = 320$	$320 < 400$	$320 > 196$

These diagrams suggest the following: If *c* is decreased, while keeping *a* and *b* the same, $a^2 + b^2$ will be greater than c^2, and m∠C will be less than 90° (acute). If *c* is increased, while keeping *a* and *b* the same, $a^2 + b^2$ will be less than c^2, and m∠C will be greater than 90° (obtuse). Summarizing these ideas, we can state:

If $a^2 + b^2 = c^2$ then, the triangle is a right triangle
If $a^2 + b^2 > c^2$ then, the triangle is an acute triangle
If $a^2 + b^2 < c^2$ then, the triangle is an obtuse triangle

Use these ideas in Exercises 11-19 to classify each triangle with the given side lengths as acute, right, or obtuse.

11. 4, 5, 7

12. $\sqrt{3}, \sqrt{2}, \sqrt{5}$

13. 6, 8, 10

14. $\sqrt{2}, \sqrt{3}, \sqrt{4}$

15. $\sqrt{3}, \sqrt{4}, \sqrt{5}$

16. 0.4, 0.5, 0.6

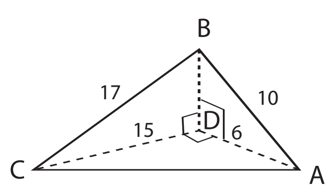
17. 9, 10, 12

18. 0.9, 4.0, 4.1

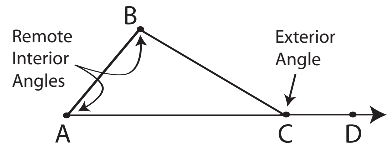
19. $\frac{1}{5}, \frac{4}{5}, 1$

For Exercise 20, decide if △ABC is right, acute, or obtuse. Explain your answer.

20.



Exterior Angle of a Polygon – The angle formed when one side of a polygon is extended. For example, in the diagram below, $\angle BCD$ is an exterior angle of $\triangle ACB$.



Note: An exterior angle of a triangle forms a **linear pair** with the adjacent interior angle of the triangle. Further, the two angles of the triangle that are not adjacent to the exterior angle are called the **remote interior angles** of that exterior angle.

Lesson 1 — Exercises:

1. Prove Theorem 37 – "If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle." (Exterior Angle Inequality Theorem) (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)
a) State the theorem.
b) Draw and label a diagram to accurately show the conditions of the theorem.
c) List the given information.
d) Write the statement we wish to prove.
e) Demonstrate the direct proof using the two-column format.

2. Complete the following demonstration of Theorem 37 which does not use Theorem 26. (Note: It will be helpful for you to draw an appropriate diagram to see the logic of the proof.)

STATEMENT	REASON
1. $\triangle ABC$ with exterior $\angle BCD$	1. Given
2. Choose point M on $\overline{BC}$ so that point M is the midpoint of $\overline{BC}$	2. Theorem 4: __________
3. Draw $\overrightarrow{AE}$ through point M	3. Postulate 2 – Uniqueness of Lines, Planes, and Spaces

Statements	Reasons
4. Choose point E on $\overrightarrow{AE}$ so that AM = EM	4. Postulate 6 – Ruler – Second Assumption
5. Draw $\overline{CE}$	5. Postulate 2 – Uniqueness of Lines, Planes, and Spaces
6. BM = MC	6. ___________
7. $\overline{BM} \cong \overline{MC}$	7. ___________
8. $\angle AMB \cong \angle EMC$	8. ___________
9. $\overline{AM} \cong \overline{EM}$	9. ___________
10. $\triangle AMB \cong \triangle EMC$	10. ___________
11. $\angle ABM \cong \angle ECM$	11. ___________
12. $m\angle BCD = m\angle DCE + m\angle ECM$	12. ___________
13. $m\angle BCD = m\angle DCE + m\angle ABM$	13. ___________
14. $m\angle BCD > m\angle B$	14. Definition of "is greater than" – For any numbers *a* and *b*, *a* > *b*, if and only if, there is a positive number *c* such that *a* = *b* + *c*
15. $\overline{AB} \parallel \overline{EC}$	15. ___________
16. $\angle BAC \cong \angle DCE$	16. ___________
17. $m\angle BAC = m\angle DCE$	17. ___________
18. $m\angle BCD = m\angle DCE + m\angle ECM$	18. ___________
19. $m\angle BCD = m\angle BAC + m\angle ECM$	19. ___________
20. $m\angle BCD > m\angle BAC$	20. ___________

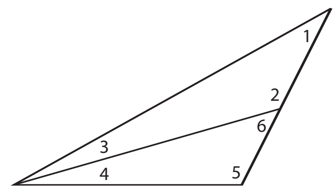
3. Referring to the figure to the right, use Theorem 37 to determine which angle is larger, or state that this cannot be determined from the information given.

a. $\angle 1$ and $\angle 6$

b. $\angle 1$ and $\angle 2$

c. $\angle 2$ and $\angle 4$

d. $\angle 5$ and $\angle 2$



For Exercises 4-7, write an inequality that shows a relationship between only x and y.

4. $x + 30 = y$

5. $75 - y = 180 - x$

6. $60 + 45 + x = 180 + y$

7. $46 + x + 89 = 91 + y$

For Exercises 8-11, write an inequality that shows a relationship between only $m\angle 1$ and $m\angle 2$.

8. $m\angle 1 = 46 + m\angle 2$

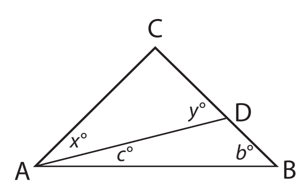
9. $25 + 30 + m\angle 1 = m\angle 2$

10. $175 + m\angle 1 - 89 = m\angle 2 - 46$

11. $m\angle 1 + m\angle 3 = m\angle 2 - m\angle 4$

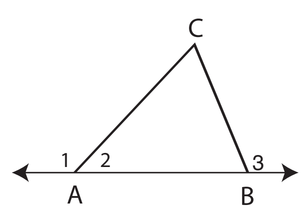
12. Given: $b = x + c$

Prove: $y > x$



13. Given: $m\angle 1 > m\angle 3$

Prove: $m\angle 1 > m\angle 2$



The "Hinge" Theorem (Side-to-Angle version) – "If two sides of one triangle are congruent to two sides of a second triangle, and the length of the third side of the first triangle is greater than the length of the third side of the second triangle, then the measure of the angle opposite the third side of the first triangle, is greater than the measure of the angle opposite the third side of the second triangle."

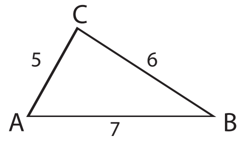
Lesson 2 — Exercises:

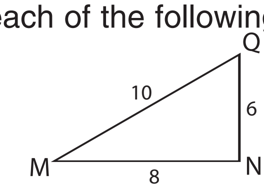
1. Theorem 38: "In a given triangle, if two sides are not congruent, then the angles opposite those sides are not congruent." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)
a) State the theorem.
b) Draw and label a diagram to accurately show the conditions of the theorem.
c) List the given information.
d) Write the statement we wish to prove.
e) Demonstrate the direct proof using the two-column format.

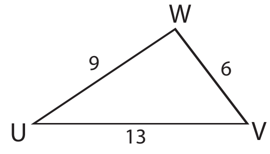
2. Prove: If the measure of one side of a triangle is greater than the measure of a second side of the triangle, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side. Use the same outline as used in Exercise 1.

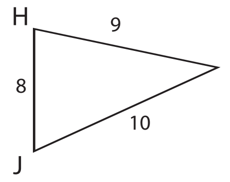
3. In each of the following, the measure of the sides of △ABC are given. List the angles of △ABC from the smallest to the largest.
a) AB = 17 BC = 21 AC = 18
b) AB = 15 BC = 17 AC = 16
c) AB = 9 BC = 40 AC = 41
d) AB = 13 BC = 12 AC = 5

4. Name the largest and the smallest angle in each of the following figures.

a)


b)


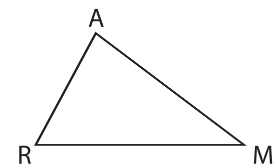
c)


d)


In Exercises 5 and 6, state the conclusion(s) that can be drawn from the given information. Give a reason for each conclusion.

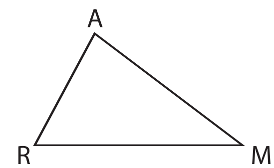
5. Given: $\triangle AMR$ with $\overline{RA} \ncong \overline{AM}$

Conclusion: $\angle M \ncong$ _____



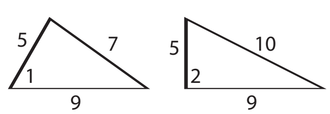
6. Given: AR < RM
RM > MA

Conclusion: $m\angle M <$ _____
$m\angle A$ __ _____

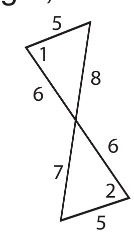


In Exercises 7-11, tell which angle is larger, $\angle 1$ or $\angle 2$.

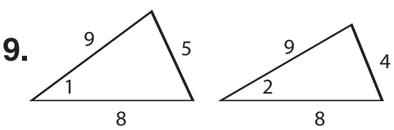
7.



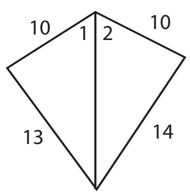
8.



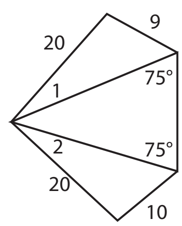
9.



10.

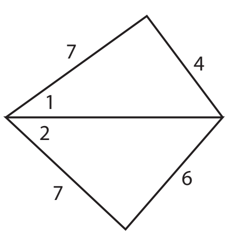


11.



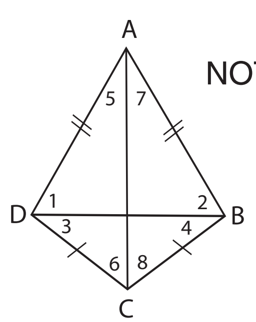
For Exercises 12 and 13, fill in the blank with >, <, or =.

12.



$m\angle 1$ ____ $m\angle 2$

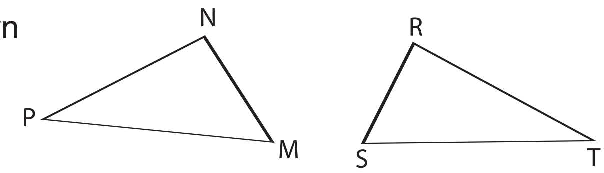
13.



NOTE: DC < DA

$m\angle 1$____$m\angle 2$
$m\angle 3$____$m\angle 4$
$m\angle 5$____$m\angle 6$

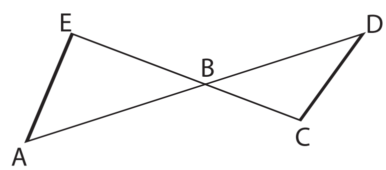
For Exercises 14-18, use the two triangles shown to the right, and determine whether each statement is always true, sometimes true, or never true.



14. If NP = RT, MP = TS, and NM > RS, then m∠P < m∠T.
15. If NP = RT, NM = RS, and PM > ST, then m∠N > m∠R.
16. If NP = RT, NM = RS, then m∠N = m∠R.
17. If NP = RT, NM = RS, and PM = ST, then m∠P = m∠S.
18. If NP > RT, NM = RS, and PM = ST, then m∠P < m∠T.

19. Given: EB > AE
CD > BC

Prove: m∠A > m∠D

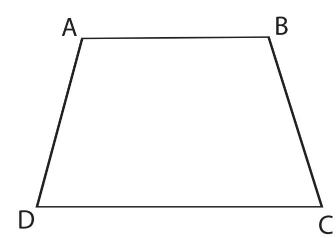


20. Prove, or find a counterexample: "The sum of the measures of any two angles of a triangle is greater than the measure of the third angle."

21. Write an indirect proof for the following:

Given: ABCD is a trapezoid with $\overline{AB} \parallel \overline{CD}$

Prove: ∠C and ∠D are not both right angles



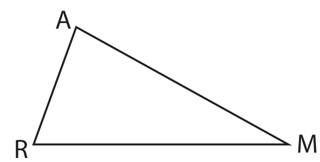
4. In each of the following, the measures of the angles of $\triangle ABC$ are given. List the sides of $\triangle ABC$ from the longest to the shortest.

a. $m\angle A = 46$ $m\angle B = 30$ $m\angle C = 104$
b. $m\angle A = 9$ $m\angle B = 70$ $m\angle C = 101$
c. $m\angle A = 60$ $m\angle B = 59$ $m\angle C = 61$
d. $m\angle A = 47$ $m\angle B = 48$ $m\angle C = 85$

In Exercises 5 and 6, state the conclusions that can be drawn from the given information. Give a reason for each conclusion.

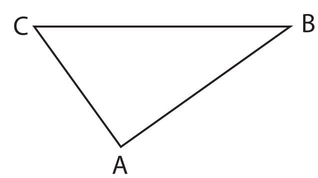
5. Given: $\angle R \ncong \angle A$

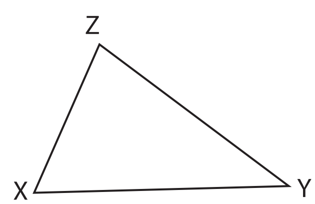
Conclusion: $\overline{AM} \ncong$ _____



6. Given: $m\angle A > m\angle C$
$m\angle C < m\angle B$

Conclusion: BC > _____
AB __ _____

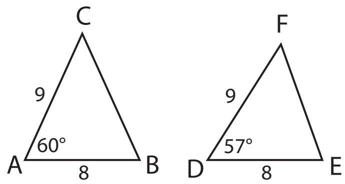




7. Using a ruler, trace the triangle at the right.
a. Measure $\overline{XY}$ and $\overline{XZ}$.
b. Choose a point K on your paper away from the triangle you traced, so that it corresponds to point X. Now, draw a new triangle KMN where XY = KM, XZ = KN, and $m\angle K < m\angle X$.
c. Measure ZY and NM. Which is longer?
d. How do the measures of ZY and NM relate to the $m\angle X$ and $m\angle K$?
e. Complete this statement of the Angle-to-Side version of The Hinge Theorem: "If two sides of one triangle are congruent to two sides of a second triangle and the measure of the included angle of the first triangle is greater than the measure of the included angle of the second triangle, then ______________."
f. Why is this theorem referred to as the "Hinge" theorem?

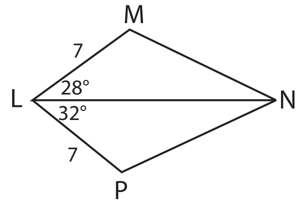
In Exercises 8 - 14, fill in the blank with >, <. or =.

8.



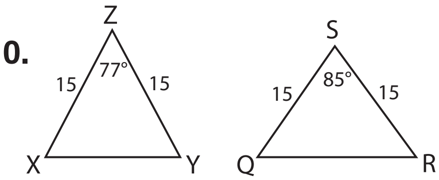
BC _____ EF

9.



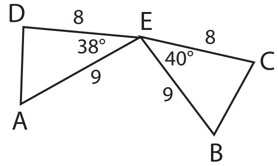
MN _____ NP

10.



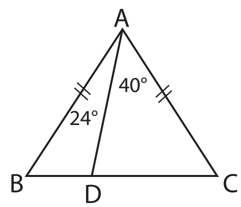
XY _____ QR

11.



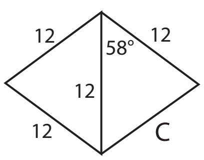
BC _____ AD

12.

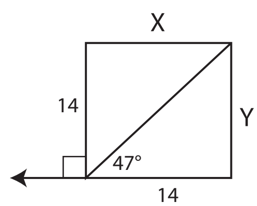


BD _____ CD

13. C __?__ 12

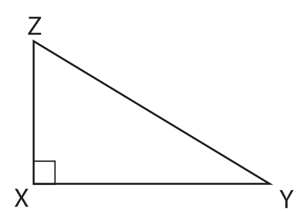


14. X __?__ Y



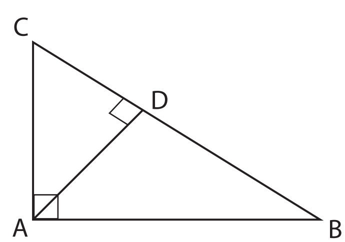
15. Given: ∠YXZ is a right angle

Prove: YZ > XZ
YZ > XY
{In other words, prove that the hypotenuse of a right triangle is the longest side of the triangle.}



16. Given: $\overline{AD} \perp \overline{CB}$
$\overline{AC} \perp \overline{AB}$

Prove: BC > AD



Example 1: The lengths of two sides of a triangle are 6 and 8. Find the possible length, *x,* for the third side.

Solution: $x > |8 - 6|$ and $x < |8 + 6|$

$x > 2$ $x < 14$

or $2 < x$ and $x < 14$

$2 < x < 14$

So the length of the third side is between 2 and 14.

Lesson 4 — Exercises:

1. Theorem 40: "In a given triangle, the sum of the lengths of any two sides, is greater than the length of the third side." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)

a) State the theorem.
b) Draw and label a diagram to accurately show the conditions of the theorem.
c) List the given information.
d) Write the statement we wish to prove.
e) Demonstrate the direct proof using the two-column format.

2. Can the sum of the lengths of two side of a triangle equal the length of the third side? Explain why or why not.

3. In △ABC, AB + BC > AC. Draw triangle ABC and write two other inequalities.

In Exercises 4-9, tell whether it is possible for a triangle to have sides of the given lengths. Assume that all given lengths are positive. Justify your answers.

4. 9, 7, 14

5. 4, 3, 7

6. 3, 4, 5

7. 18, 10, 7

8. x, 2, $x - 2$ (where x is a natural number)

9. $2x$, x, $x + 1$ (where x is a natural number)

Unit IV — Triangles

Appendix A – Properties of Real Numbers

Properties of the Real Numbers — This term refers to the postulates, or axioms, which we accepted without proof, in the study of Arithmetic. Following is a comprehensive list for your reference.

1. Properties of Relations:

- **Trichotomy** - For any real numbers a and b, only one of the following can be true: $a = b, a > b, a < b.$
- **Reflexivity for Equality** - For any real number a, $a = a.$
- **Symmetry for Equality** - For any real numbers a and b, if $a = b$, then $b = a.$
- **Transitivity for Equality** - For any real numbers $a, b,$ and $c,$ if $a = b,$ and $b = c,$ then $a = c.$
- **Substitution** - For any real numbers $a,$ and $b,$ if $a = b,$ then a can be substituted for b in any expression (or b can be substituted for a).
- **Transitivity for Inequality** - For any real numbers $a, b,$ and c, if $a > b,$ and $b > c,$ then $a > c.$ Likewise, if $a < b,$ and $b < c,$ then $a < c.$

2. Properties of Well-Defined Operations:

- **Existence** - For any real numbers a and $b, a + b, a - b,$ and $a \cdot b$ exist. Further, $a \div b$ exists, as long as $b = 0$.
- **Uniqueness** - For any real numbers a and b, $a + b, a - b,$ and $a \cdot b$ are unique. Further, $a \div b$ is unique, as long as $b = 0$.
- **Closure** - For any real numbers a and b, $a + b, a - b,$ and $a \cdot b$ are real numbers. Further, $a \div b$ is a real number, as long as $b = 0$.

3. Properties of Operations in General:

- **Commutativity of Addition** - For any real numbers a and $b, a + b = b + a.$
- **Commutativity of Multiplication** - For any real numbers a and $b, a \cdot b = b \cdot a.$
- **Associativity of Addition** - For any real numbers $a, b,$ and $c,$ $(a + b) + c = a + (b + c).$
- **Associativity of Multiplication** - For any real numbers $a, b,$ and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c).$
- **Distributivity of Multiplication over Addition** - For any real numbers $a, b,$ and $c,$ $a \cdot (b + c) = a \cdot b + a \cdot c.$
- **Distributivity of Multiplication over Subtraction** - For any real numbers $a, b,$ and $c, a \cdot (b - c) = a \cdot b - a \cdot c.$

Arc of a Circle (p. 158) – Any set of continuous points on a circle.

Area (p. 43, p. 48, p. 52, p. 55, p. 59, p. 62) – Intuitively, the number of non-overlapping unit squares and parts of unit squares, which will exactly cover the interior of a simple closed plane curve

Area of a Circle (p. 62, p. 76) – The product of the square of the radius of the circle, and the irrational number π.

Area of a Parallelogram (p. 48) – The product of the base and height of a parallelogram.

Area of a Rectangle (p. 43) – The product of the base and height of a rectangle.

Area of a Regular Polygon (p. 59) – One-half the product of the measure of the apothem, and the perimeter of the polygon.

Area of a Rhombus (p. 48) – The product of the base and height of a rhombus.

Area of a Square (p. 43) – The square of the measure of one side of a square.

Area of a Triangle (p. 52) – One-half of the product of the base and height of a triangle.

Area of a Trapezoid (p. 55) – One-half of the product of the height and the sum of the bases of a trapezoid.

Auxiliary Element (p. 291, p. 351, p. 365, p. 404, p. 426, p. 430, p. 442, p. 446, p. 451) – From a Latin word meaning “to help”, this is a geometric element which is not specifically referred to in a given diagram, but which does exist, and is needed, in order to logically complete the demonstration of a conditional. This element is usually drawn in a figure, using dashed lines, and must be referenced in the proof with a step justifying its existence. It is important to note that you must not place too many conditions on the element (called “over-determining”) thereby denying its existence. Neither must you place too few conditions on the element (called “under-determining”), thereby allowing the existence of more than one such element. In other words, a geometric element is considered to be “determined”, if exactly one such element can be drawn to meet the conditions.

Base Angles of an Isosceles Triangle (p. 320) – The angles opposite the legs in an isosceles triangle.

Base of an Isosceles Triangle (p. 320) – The third side of an isosceles triangle, when the triangle has exactly two congruent sides.

Betweenness of Points (p. 124) – For three collinear points, A, B, and C, with coordinates a, b, and c, respectively, point B is said to lie between points A and C, if and only if, either $a < b < c$, or, $a > b > c$.

Disjunction (p. 92) – An operation in logic which joins two simple statements, using the word “or”.

“Divides Proportionally” (p. 351) – A term referring to points on line segments which “divide” the segments into smaller segments which are in the same ratio.

Diagonal of a Polygon (p. 377) A line segment joining any two non-consecutive vertices of the polygon.

Diagonal of a Rectangular Solid (p. 377) – A line segment joining any two vertices in a rectangular solid which are not vertices of the same face. Vertices of this type are called “opposite” vertices.

Diameter of a Circle (p. 62, p. 158) – A line segment which is a chord of a circle, and passes through the center of that circle.

Dihedral Angle (p. 142, p. 307) – The union of two half-planes (called faces), with the same edge.

Dilation (p. 8) – A transformation in which the distance between any point of the pre-image, and a specified point (called the center of dilation), is multiplied by some constant factor, to produce the image.

Direct Proof (p. 215) – The process of reaching a desired conclusion, logically and deductively, from given statements, from already accepted definitions and postulates, and from any previously proved theorems.

Discrete Geometry (p. 25) – A Geometry in which every point is a “dot”, and every line is made up of separate points, with a space between them.

Disjoint Sets (p. 18) – Two or more sets which have no members, or elements, in common.

Dodecagon (p. 34) – A polygon made with twelve line segments.

Edge (p. 141) – Another name for a separation line in a plane.

Element of a Set (p. 18) – Also referred to as a “member” of a set, this is one of the objects in a set.

Empty Set (p. 18) – A set which contains no members.

Equal Angles (p. 151) – Two angles whose measures are equal.

Equal Line Segments (p. 134) – Line segments whose lengths are equal.

Interior Angles on the Same Side of a Transversal (p. 277) – Pairs of angles which are between parallel lines and are on the same side of a transversal.

Interior of an Angle (p. 141) – The set of points between two rays when one ray lies in the edge of a half-plane.

Intersecting Lines (p. 128) – Two lines which have a point in common.

Intersection (p. 18) – An operation on two or more sets, which selects only those elements common to (or belonging to) all of the original sets.

Intuition (p. 79, p. 81) – A type of mental activity which gives information or beliefs, based on hunches or insight.

Inverse (p. 107) – A conditional which results from negating both the hypothesis and the conclusion in a given conditional.

"Is Greater Than" (p. 442, p. 446, p .451) – By definition, for any real numbers *a* and *b*, *a* "is greater than" *b*, if and only if, there is a positive real number *c*, such that, $a = b + c$.

"Is Less Than" (p. 442, p. 446, p. 451) – By definition, for any real numbers a and b, a "is less than" *b*, if and only if, there is a positive real number *c*, such that, $b = a + c$.

Isosceles Trapezoid (p. 33) – A trapezoid in which the two non-parallel sides are congruent.

Isosceles Triangle (p. 33, p. 320, p. 415, p. 422) – A triangle is an isosceles triangle, if and only if, it has at least two congruent sides.

Isometry Transformation (p. 8, p. 382) – A transformation or combination of transformations which results in the image being exactly the same shape and size as the pre-image.

Kite (p. 33) – A quadrilateral in which there are two distinct pairs of consecutive sides which are of equal measure.

Law of Syllogism (p. 101) – An application of syllogistic reasoning involving two related conditionals, which, if considered together, using the "law of detachment", will result in a third valid conditional.

Legs of an Isosceles Triangle (p.320) – The two congruent sides of an isosceles triangle, when that triangle has exactly two congruent sides.

Length of a Line Segment (p.134) – A real number which represents the distance between the endpoints of a line segment.

Line (p. 3) – A basic element of Geometry, which has infinite length, but no thickness. In a drawing, it is represented by a line segment with arrowheads on each end, to show that it goes on forever. We name a line by choosing any two points on the line, and labeling each with a capital letter.

Line Segment (p. 134) – The union of two points on a line, and the set of all the points between them

Linear Pair (p. 151, p. 316, p. 327) – Two angles which have a common side (they are adjacent), and whose exterior sides are opposite rays.

Logic (p. 92, p. 100, p.107) – A system of reasoning, in an orderly fashion, which draws conclusions from specific premises.

Major Arc of a Circle (p. 159) – An arc which is the union of two points on a circle, not the endpoints of a diameter, and the set of points on the circle which lie in the exterior of the angle formed by the radii containing the two points.

Mapping (p. 7) – Another name for a transformation in Geometry.

Means-Extremes Product Property of a Proportion (p. 336) – A property of a valid standard proportion, which states that the product of the means is equal to the product of the extremes.

Means of a Proportion (p. 331) – In a standard proportion, the second and third terms.

Measure of a Dihedral Angle (p. 307) – A real number which is defined to be the measure of any of its plane angles.

Measure of a Major Arc of a Circle (p. 159) – A real number which is equal to 360 minus the measure of its related minor arc.

Measure of a Minor Arc of a Circle (p.159) – A real number which is equal to the measure of its related central angle.

Measure of the Arc making up a Complete Circle (p. 159) – Related to a central angle of 360° (a complete rotation about the center of a circle), this defined to be 360°.

Median in a Triangle (p.316, p.416, p.423) – A segment drawn from a vertex of a triangle, to the midpoint of its opposite side.

Midpoint of a Line Segment (p. 135, p. 239) – A point on a line segment which is between the endpoints, and divides the given segment into two congruent segments.

Unit IV — Triangles

Appendix C - Postulates and Postulate Corollaries

Postulate 1 – Existence of Points (WT- p. 170)

- "Every line contains at least 2 different points."
- "Every plane contains at least 3 different, non-collinear points."
- "Space contains at least 4 different, non-coplanar points, no three of which are collinear."

Postulate 2 – Uniqueness of Lines, Planes, and Spaces (WT- p. 175)

- "For any 2 different points, there is exactly 1 line containing them."
- "For any 3 different, non-collinear points, there is exactly 1 plane containing them."
- "For any 4 different, non-coplanar points, no 3 of which are collinear, there is exactly 1 space containing them."

Postulate 3 – One, Two, and Three Dimensions (WT- p. 178)

- "For any 2 different points in a plane, the line containing them is in the plane."
- "For any line in a plane, there is at least 1 point in the plane that is not on the line."
- "For any plane in space, there is at least 1 point in space that is not on the plane."

Postulate 4 – Separation of Lines, Planes, and Spaces (WT- p. 180)

- "A point separates a line into two non-empty sets called half-lines. If two points are in the same half-line, then the segment joining them does not contain the given point. If two points are in different half-lines, then the segment joining them does contain the given point."
- "A line separates a plane into two non-empty sets called half-planes. If two points are in the same half-plane, then the segment joining them does not intersect the given line. If two points are in different half-planes, then the segment joining them does intersect the given line."
- "A plane separates space into two non-empty sets called half-spaces. If two points are in the same half-space, then the segment joining them does not intersect the given plane. If two points are in different half-spaces, then the segment joining them does intersect the given plane."

Postulate 5 – Intersection of Lines or Planes (WT- p. 184)

- "If 2 different lines intersect, the intersection is a unique point."
- "If 2 different planes intersect, the intersection is a unique line."

Postulate 6 – Ruler (WT- p. 187)

- "The set of all points on a line can be put into a one-to-one correspondence with the real numbers, so that any point may correspond to 0, and any other point may correspond to 1."
- "To every pair of points on a line, there corresponds exactly one real number, called the unique distance between the points."
- "The distance between any 2 points on a line is the absolute value of the difference between their coordinates."
- "If, on a line, point B lies between points A and C, then:
  mAB + mBC = mAC (Segment-Addition Assumption)

Postulate 7 – Protractor (WT- p. 194)

- "In a half-plane, the set of all rays with a common endpoint in the edge of the half-plane, can be put into a one-to-one correspondence with the real numbers from 0 to 180, inclusive, pairing either ray in the edge of the half-plane with 0."
- "To every pair of rays with a common endpoint in the edge of a half-plane, there corresponds exactly one real number from 0 to 180, inclusive, called the unique measure of the angle formed by the rays."
- "The measure of an angle is the absolute value of the difference between the coordinates of its rays."
- "If, in a half-plane, a ray OB lies between rays OA and OC, then:
  $m\angle AOB + m\angle BOC = m\angle AOC$ (Angle-Addition Assumption)

Postulate 8 – Circle (WT- p. 198)

- "The set of all points on a circle can be put into a one-to-one correspondence with the real numbers from 0 to 360, inclusive, with the exception of any one point which may be paired with 0 and 360."
- "To every pair of points on a circle, there correspond exactly 2 real numbers whose sum is 360, each of which may be called the distance between the 2 points."
- "The distance between any 2 points on a circle is the absolute value of the difference between their coordinates."
- "If, on a circle, a point B lies between points A and C, then:
  mAB + mBC = mAC (Arc-Addition Assumption)

Postulate 9 – Uniqueness of Parallel Lines (WT- p. 202)

- "In a plane, through a point not on a given line, there is exactly one line parallel to the given line."

Postulate 10 – Uniqueness of Perpendicular Lines (WT- p. 207)

- "In a plane, through a point not on a given line, there is exactly one line perpendicular to the given line."
- "Through a point not in a given plane, there is exactly one line perpendicular to the given plane."

Postulate 11 – Corresponding Angles of Parallel Lines (WT- p. 277)

- "If two parallel lines are cut by a transversal, then corresponding angles are congruent."

Postulate Corollary 13b (WT- p. 395)

- "If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and the corresponding acute angle of another right triangle, then the two right triangles are congruent." (HA Congruence Postulate Corollary)

Postulate Corollary 13c (WT- p. 395)

- "If a leg and an acute angle and of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the two right triangles are congruent." (LA Congruence Postulate Corollary)

Postulate Corollary 13d (WT- p. 395)

- "If the two legs of a right triangle are congruent to the two corresponding legs of another right triangle, then the two right triangles are congruent." (LL Congruence Postulate Corollary)

Postulate Corollary 13e (WT- p. 395)

- "If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the two right triangles are congruent." (HL Congruence Postulate Corollary)

Theorem 18 (WT- p. 287)
"If a given line is perpendicular to one of two parallel lines, then it is perpendicular to the other."

Theorem 19 (WT- p. 291)
"If two lines are cut by a transversal so that corresponding angles are congruent, then the two lines are parallel."

Theorem 20 (WT- p. 294)
"If two lines are cut by a transversal so that alternate interior angles are congruent, then the two lines are parallel."

Corollary 20a (WT- p. 294)
"If two lines are cut by a transversal so that alternate exterior angles are congruent, then the two lines are parallel."

Theorem 21 (WT- p. 297)
"If two lines are cut by a transversal so that interior angles on the same side of the transversal are supplementary, then the two lines are parallel."

Corollary 21a (WT- p. 297)
"If two lines are cut by a transversal so that exterior angles on the same side of the transversal are supplementary, then the two lines are parallel."

Theorem 22 (WT- p. 301)
"If two lines are perpendicular to a third line, then the two lines are parallel."

Theorem 23 (WT- p. 304)
"If two lines are parallel to a third line, then the two lines are parallel to each other."

Theorem 24 (WT- p. 307)
"If two parallel planes are cut by a third plane, then the two lines of intersection are parallel."

Theorem 25 (WT- p. 324)
"If you have any given triangle, then the sum of the measures of its angles is 180."

Corollary 25a (WT- p. 324)
"If two angles of one triangle are congruent to two angles of another triangle, then the third pair of angles are congruent."

Corollary 25b (WT- p. 324)
"If all of the angles of a triangle are congruent, then the measure of each angle is 60."

Corollary 30a (WT- p. 364)
"If you have a right triangle, then either leg is the geometric mean between the hypotenuse of the triangle, and the projection of that leg on that hypotenuse."

Corollary 30b (WT- p. 364)
"If you have a right triangle, then the altitude drawn to the hypotenuse of that triangle is the geometric mean between the segments of that hypotenuse, formed by drawing that altitude."

Corollary 30c (WT- p. 364)
"If you have an altitude drawn to the hypotenuse of a right triangle, then the product of the lengths of that altitude and the hypotenuse, is equal to the product of the lengths of the two legs."

Theorem 31 (WT- p. 369)
"If you have a right triangle, then the square of the measure of the hypotenuse, is equal to the sum of the squares of the measures of the two legs."
(The Pythagorean Theorem)

Corollary 31a (WT- p. 369)
"If you have a right triangle whose acute angles have measures of 30° and 60°, then the measure of the hypotenuse is twice the measure of the shorter leg, and the measure of the longer leg is $\sqrt{3}$ times the measure of the shorter leg."

Corollary 31b (WT- p. 369)
"If you have a right triangle whose acute angles each have measures of 45°, then the measure of the hypotenuse is $\sqrt{2}$ times the measure of either leg."

Theorem 32 (WT- p. 414)
"If two given triangles are both congruent to a third triangle, then the two given triangles are congruent to each other."

Theorem 33 (WT- p. 415)
"If two sides of a triangle are congruent, then the angles opposite them are congruent."

Corollary 33a (WT- p. 416)
"If a triangle is equilateral, then it is equiangular."

Corollary 33b (WT- p. 416)
"If a triangle is equilateral, then the measure of each of its angles is 60°."

Theorem 34 (WT- p. 422)
"If two angles of a triangle are congruent, then the sides opposite them are congruent."