Geometry: A Complete Course

(with Trigonometry)

Module E – Solutions Manual

Written by: Larry E. Collins

ERRATA 3/2015

VideoText Interactive

Geometry: A Complete Course (with Trigonometry)
Module E–Solutions Manual

Send all inquiries to:
Videotext*Interactive*
P.O. Box 19761
Indianapolis, IN 46219

ISBN 1-59676-111-3
3 4 5 6 7 8 9 10 - RPInc -18 17 16 15 14

Table of Contents

Unit V - Other Polygons

Part A - Properties of Polygons

LESSON 1 - Basic Terms 1
LESSON 2 - Parallelograms 3
LESSON 3 - Special Parallelograms (Rectangle, Rhombus, Square) 10
LESSON 4 - Trapezoids 16
LESSON 5 - Kites 22
LESSON 6 - Midsegments 25
LESSON 7 - General Polygons 33

Part B - Areas of Polygons

LESSON 1 - Postulate 14 - Area 37
LESSON 2 - Triangles 40
LESSON 3 - Parallelograms 43
LESSON 4 - Trapezoids 45
LESSON 5 - Regular Polygons 48

Part C - Applications

LESSON 1 - Using Area in Proofs 51
LESSON 2 - Schedules 54

Unit VI - Circles

Part A - Fundamental Terms

LESSON 1 - Lines and Segments 57
LESSON 2 - Arcs and Angles 58
LESSON 3 - Circle Relationships 59

Part B - Angle and Arc Relationships

LESSON 1 - Theorem 65 - "If, in the same circle, or in congruent circles, two 60
central angles are congruent, then their intercepted minor arcs are congruent."
Theorem 66 -"If, in the same circle, or in congruent circles,two minor arcs
are congruent, then the central angles which intercept those minor arcs
are congruent."

LESSON 2 - Theorem 67 - "If you have an inscribed angle of a circle, then the 63
measure of that angle, is one-half the measure of its intercepted arc."

LESSON 3 - Theorem 68 - "If, in a circle, you have an angle formed by a secant 69
ray, and a tangent ray, both drawn from a point on the circle,then the
measure of that angle, is one-half the measure of the intercepted arc."

LESSON 4 - Theorem 69 -"If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines,(or its vertical angle), is equal to one-half the sum of the measures of the arcs intercepted by the angle, and its vertical angle." 74
Theorem 70 - "If, for a circle, two secant lines intersect outside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the difference of the measures of the arcs intercepted by the angle."
LESSON 5 - Theorem 71 - "If, for a circle, a secant line and a tangent line intersect outside a circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle." . . .77
Theorem 72 - "If, for a circle, two tangent lines intersect outside the circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle."

Part C - Line and Segment Relationships

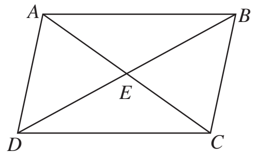
LESSON 1 - Theorem 73 - "If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord." 80
LESSON 2 - Theorem 74 - "If a diameter of a circle bisects a chord of the circle which is not a diameter of the circle, then that diameter is perpendicular to that chord." 83
Theorem 75 - "If a chord of a circle is a perpendicular bisector of another chord of that circle, then the original chord must be a diameter of the circle."
LESSON 3 - Theorem 76 - "If two chords intersect within a circle, then the product of the lengths of the segments of one chord, is equal to the product of the lengths of the segments of the other chord." 86
LESSON 4 - Theorem 77 - "If two secant segments are drawn to a circle from a single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment." 88
Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle, from a single point outside the circle, then the length of that tangent segment is the mean proportional between the length of the secant segment, and the length of its external segment."
LESSON 5 - Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line must be tangent to the circle,at that endpoint," 92
LESSON 6 - Theorem 80 - "If two tangent segments are drawn to a circle from the same point outside the circle, then those tangent segments are congruent." 94
LESSON 7 - Theorem 81 - "If two chords of a circle are congruent, then their intercepted minor arcs are congruent." 98
Theorem 82 - "If two minor arcs of a circle are congruent, then the chords which intercept them are congruent."

Part D - Circles and Concurrency

LESSON 1 - Theorem 83 - "If you have a triangle, then that triangle is cyclic." 104
LESSON 2 - Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic." 113

4. a) Theorem 43 - "If quadrilateral is a parallelogram, then its diagonals bisect each other."

b)



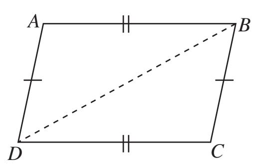
c) Given: Parallelogram ABCD with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at point E.

d) Prove: $\overline{AC}$ bisects $\overline{DB}$; $\overline{DB}$ bisects $\overline{AC}$

STATEMENT	REASONS
1. Parallelogram ABCD with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at point E	1. Given
2. $\overline{AB} \cong \overline{CD}$	2. Theorem 41 - If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent.
3. $\overline{AB} \parallel \overline{CD}$	3. Definition of parallelogram - a quadrilateral with both pairs of opposite sides parallel.
4. $\angle BAC \cong \angle DCA$; $\angle ABD \cong \angle CDB$	4. Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
5. $\triangle ABE \cong \triangle CDE$	5. Postulate 13 - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the two triangles are congruent. (ASA Congruence Assumption)
6. $\overline{AE} \cong \overline{CE}$	6. CPCTC
7. Point E is the midpoint of $\overline{AC}$	7. Definition of midpoint - a point on a line segment which is between the endpoints, and divides the given segment into two congruent parts.
8. $\overline{DB}$ bisects $\overline{AC}$	8. Definition of bisector of a line segment - any point, line segment, ray, or line which intersects a line segment in the midpoint of the line segment, creating two congruent segments.
9. $\overline{BE} \cong \overline{DE}$	9. CPCTC
10. Point E is the midpoint of $\overline{DB}$	10. Definition of midpoint
11. $\overline{AC}$ bisects $\overline{DB}$	11. Definition of bisector of a line segment.

5. a) Corollary 44 - "If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram." (Converse of Theorem 41)

b)



c) Given: Quadrilateral ABCD with $\overline{AB} \cong \overline{CD}$ and $\overline{BC} \cong \overline{DA}$

d) Prove: ABCD is a parallelogram

STATEMENT	REASONS
1. ABCD is a quadrilateral	1. Given
2. $\overline{AB} \cong \overline{CD}$; $\overline{BC} \cong \overline{DA}$	2. Given
3. Draw diagonal $\overline{DB}$	3. Postulate 2 - For any two different points, there is exactly one line containing them.
4. $\overline{DB} \cong \overline{BD}$	4. Reflexive property for congruent line segments.
5. $\triangle ADB = \triangle CBD$	5. Postulate 13 - If three sides of one triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent. (SSS Congruence Postulate)
6. $\angle ABD \cong \angle CDB$; $\angle ADB \cong \angle CBD$	6. CPCTC
7. $\overline{AB} \parallel \overline{CD}$; $\overline{AD} \parallel \overline{CB}$	7. Theorem 20 - If two lines are cut by a transversal so that alternate interior angles are congruent, then the two lines are parallel.
8. ABCD is a parallelogram	8. Definition of parallelogram - a quadrilateral with both pairs of opposite sides parallel

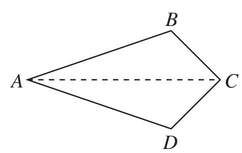
10. **a)** Both pairs of opposite sides parallel
b) Both pairs of opposite sides congruent
c) Any pair of consecutive angles supplementary
d) Opposite angles are congruent
e) Diagonals bisect each other
f) One pair of sides are parallel and congruent

11. **a)** Corollary 42a - If a quadrilateral is a parallelogram, then opposite angles are congruent.
b) Theorem 43 - If a quadrilateral is a parallelogram, then its diagonals bisect each other.
c) Theorem 41 - If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent.
d) Theorem 41
e) Theorem 42 - If a quadrilateral is a parallelogram, then any pair of consecutive angles is supplementary.
f) Definition of Parallelogram - a quadrilateral with both pairs of opposite sides parallel.

12.

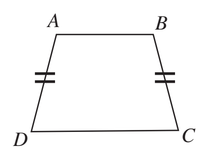
a) 18	**g)** 12; 4
b) 105	**h)** 120
c) 12	**i)** 15
d) 30	**j)** 16
e) 115	**k)** 79
f) 30	

13. True
14. True
15. True
16. False, the quadrilateral could be a kite.



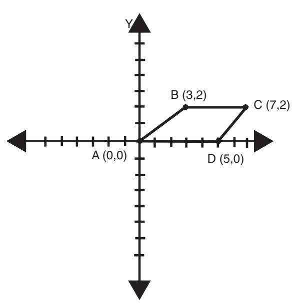
$\triangle ABC \cong \triangle ADC$

17. True
18. False, the quadrilateral could be an isosceles trapezoid.



$\overline{AB} \parallel \overline{DC}$

19.



$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AB = \sqrt{(3-0)^2 + (2-0)^2}$$
$$AB = \sqrt{3^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

$$BC = \sqrt{(7-3)^2 + (2-2)^2}$$
$$BC = \sqrt{4^2 + 0^2} = \sqrt{16} = 4$$

$$CD = \sqrt{(5-7)^2 + (0-2)^2}$$
$$CD = \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$DA = \sqrt{(5-0)^2 + (0-0)^2}$$
$$DA = \sqrt{5^2 + 0^2} = \sqrt{25} = 5$$

The figure is not a parallelogram. No sides are congruent.

8. **a)** right; congruent
b) congruent; perpendicular; bisect interior
c) parallelogram; rhombus; rectangle

9. **a)** Sometimes True **f)** Never True
b) Always True **g)** Sometimes True
c) Always True **h)** Never True
d) Always True **i)** Always True
e) Sometimes True **j)** Sometimes True

10. $\overline{AD}$ and $\overline{CB}$
$\overline{DC}$ and $\overline{BA}$
$\overline{DB}$ and $\overline{AC}$
$\overline{DE} \cong \overline{BE} \cong \overline{AE} \cong \overline{CE}$

11. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$
$\overline{DE} \cong \overline{BE}$; $\overline{AE} \cong \overline{CE}$

12. ∠AED; ∠DEC; ∠BEC; ∠AEB

13. ∠BDC; ∠ACD; ∠ACB; ∠DBC; ∠DBA; ∠DAC; ∠BAC

14. 90 **15.** 16 **16.** 50 **17.** Rectangle **18.** Rhombus **19.** Square

20. **a)** DU = 6
b) RU = 18
c) m∠GRD = 70
d) m∠UDG = 130

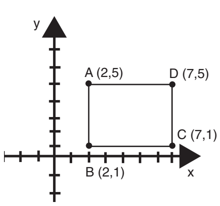
21. **a)** m∠DGR = 37
b) m∠GUR = 43

22. **a)**
DG = DU
3x + 6 = 4x – 10
16 = x

DG = 3(16) + 6
DG = 48 + 6
DG = 54
so, GA = 2 · 54 = 108

b) m∠URA = 45

23.

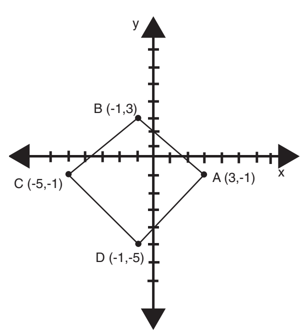


$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$AC = \sqrt{(7-2)^2 + (1-5)^2}$$
$$AC = \sqrt{5^2 + (-4)^2} = \sqrt{25+16} = \sqrt{41}$$

$$BD = \sqrt{(7-2)^2 + (5-1)^2}$$
$$BD = \sqrt{5^2 + 4^2} = \sqrt{25+16} = \sqrt{41}$$

$AC = BD$ *So,* $\overline{AC} \cong \overline{BD}$

24.



$$\text{Slope of } \overline{BA} = \frac{Rise}{Run} = \frac{-4}{4} = -1$$

$$\text{Slope of } \overline{BC} = \frac{Rise}{Run} = \frac{-4}{-4} = 1$$

Use either $\frac{-4}{4}$ *or* $\frac{-4}{-4}$ *to find* $D(-1,-5)$

11. $d = \sqrt{(x_2 \square x_1)^2 + (y_2 \square y_1)^2}$

$AB = \sqrt{(4 \square 1)^2 + (2 \square 9)^2}$ $\qquad$ $CD = \sqrt{(8 \square 5)^2 + (9 \square 2)^2}$

$AB = \sqrt{3^2 + (\square 7)^2}$ $\qquad$ $CD = \sqrt{3^2 + 7^2}$

$AB = \sqrt{9 + 49}$ $\qquad$ $CD = \sqrt{9 + 49}$

$AB = \sqrt{58}$ $\qquad$ $CD = \sqrt{58}$

$BC = \sqrt{(5 \square 4)^2 + (2 \square 2)^2}$ $\qquad$ $DA = \sqrt{(1 \square 8)^2 + (9 \square 9)^2}$

$BC = \sqrt{1^2 + 0^2}$ $\qquad$ $DA = \sqrt{(\square 7)^2 + 0^2}$

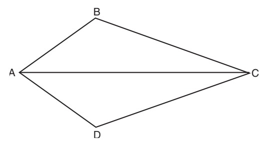
$BC = \sqrt{1}$ $\qquad$ $DA = \sqrt{49 + 0}$

$BC = 1$ $\qquad$ $DA = \sqrt{49}$

$DA = 7$

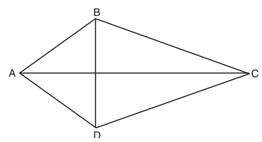
ABCD is **NOT** a kite. $\overline{AB}$ and $\overline{CD}$ are opposite sides.

12. Corollary 54a - If a quadrilateral is a kite, then the symmetry diagonal bisects the angles to which it is drawn.



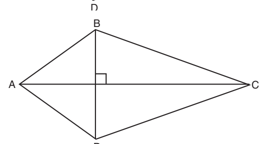
$\overline{AC}$ bisects ∠DAB and ∠DCB

13. Corollary 54b - If a quadrilateral is a kite, then the symmetry diagonal bisects the other diagonal.



$\overline{AC}$ bisects $\overline{BD}$

14. Corollary 54c - If a quadrilateral is a kite, then the diagonals are perpendicular to each other.



$\overline{AC}$ □ $\overline{BD}$

15.

STATEMENT	REASONS
1. ABCD is a kite with diagonals $\overline{AC}$ and $\overline{BD}$ intersecting at point E.	1. Given
2. Assume △AEB ≅ △CED	2. Indirect proof assumption
3. $\overline{AB}$ ≅ $\overline{CD}$	3. C.P.C.T.C.
4. However, $\overline{AB}$ ≇ $\overline{CD}$	4. Definition of Kite - A quadrilateral that has two pairs of consecutive sides, but opposite sides are not congruent.
5. Our assumption that △AEB ≅ △CED leads to a contradiction of the given. So we must conclude our assumption is false and △AEB ≇ △CED.	5. Redutio Ao Absurdum

16.

STATEMENT	REASONS
1. ABCD is a kite.	1. Given
2. ∠A ≅ ∠C	2.Theorem 54 - If a quadrilateral is a kite, then the pair of opposite angles, formed by the two pairs of non-congruent sides are congruent.
3. Assume ∠B ≅ ∠D	3. Indirect proof assumption
4. ABCD is a parallelogram.	4. Theorem 46 - If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
5. $\overline{AB}$ ≅ $\overline{CD}$ (or $\overline{BC}$ ≅ $\overline{DA}$)	5. Theorem 41 - If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent.
6. However, $\overline{AB}$ ≇ $\overline{CD}$ (or $\overline{BC}$ ≇ $\overline{DA}$)	6. Definition of kite - a quadrilateral that has two pairs of consecutive congruent sides, but opposite sides are not congruent.
7. Our assumption that ∠B and ∠D leads to a contradiction of the given. So we must conclude our assumption is false and ∠B ≇ ∠D.	7. Reductio Ad Absurdum

4. **a)** $\frac{360}{n}$, $\frac{360}{12}$, $\frac{\cancel{12}\cdot 30}{\cancel{12}}$, 30

b) $\frac{360}{n}$, $\frac{360}{24}$, $\frac{\cancel{24}\cdot 15}{\cancel{24}}$, 15

c) $\frac{360}{n}$, $\frac{360}{8}$, $\frac{\cancel{8}\cdot 45}{\cancel{8}}$, 45

d) $\frac{360}{n}$, $\frac{360}{10}$, $\frac{\cancel{10}\cdot 36}{\cancel{10}}$, 36

e) $\frac{360}{n}$, $\frac{360}{15}$, $\frac{\cancel{15}\cdot 24}{\cancel{15}}$, 24

5. **a)** $\frac{360}{30}$, $\frac{12\cdot\cancel{30}}{\cancel{30}}$, 12

b) $\frac{360}{22\frac{1}{2}}$, $\frac{360}{\frac{45}{2}}$, $\frac{720}{45}$, $\frac{\cancel{9}\cdot 8\cdot 2\cdot\cancel{5}}{\cancel{9}\cdot\cancel{5}}$, 16

c) $\frac{360}{32\frac{8}{11}}$, $\frac{360}{\frac{360}{11}}$, $\frac{\cancel{360}\cdot 11}{\cancel{360}}$, 11

d) $\frac{360}{8}$, $\frac{\cancel{8}\cdot 45}{\cancel{8}}$, 45

e) $\frac{360}{14\frac{2}{?}}$, $\frac{360}{\frac{72}{5}}$, $\frac{5\cdot 360}{72}$, $\frac{5\cdot 5\cdot\cancel{72}}{\cancel{72}}$, 25

6.
$$(n-2)\cdot 180$$
$$(8-2)\cdot 180$$
$$6\cdot 180$$
$$1080$$
$$1080-1000$$
$$80^o$$

7.
$$(n-2)\cdot 180 = 2\cdot 360$$
$$180n-360 = 720$$
$$180n = 1080$$
$$n = \frac{1080}{180}$$
$$n = \frac{6\cdot\cancel{180}}{\cancel{180}}$$
$$n = 6$$

8.
$$\frac{(n-2)\cdot 180}{n} = 8\cdot\frac{360}{n}$$
$$\frac{n}{1}\cdot\frac{(n-2)\cdot 180}{n} = \frac{n}{1}\cdot\frac{8}{1}\cdot\frac{360}{n}$$
$$180n-360 = 8\cdot 360$$
$$180n = 8\cdot 360+360$$
$$180n = 9\cdot 360$$
$$n = \frac{9\cdot 2\cdot\cancel{180}}{\cancel{180}}$$
$$n = 18$$

9.
$$(n-2)\cdot 180 = 1350$$
$$180n-360 = 1350$$
$$180n = 1710$$
$$n = \frac{1710}{180}$$
$$n = \frac{\cancel{2}\cdot\cancel{5}\cdot\cancel{3}\cdot\cancel{3}\cdot 19}{\cancel{2}\cdot 2\cdot\cancel{3}\cdot\cancel{3}\cdot\cancel{5}}$$
$$n = \frac{19}{2}\ or\ 9\frac{1}{2}$$

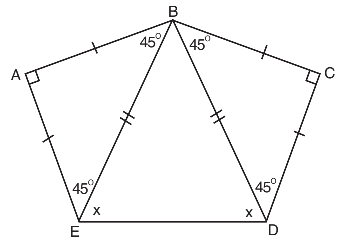
The number of sides cannot be a fraction. The sum of the measures of the interior angles cannot be 1350°.

10.
$$\frac{(n-2)\cdot 180}{n} = 152$$
$$(n-2)\cdot 180 = 152n$$
$$180n-360 = 152n$$
$$28n = 360$$
$$n = \frac{360}{28}$$
$$n = \frac{2\cdot\cancel{2}\cdot\cancel{2}\cdot 3\cdot 3\cdot 5}{\cancel{2}\cdot\cancel{2}\cdot 7}$$
$$n = \frac{90}{7} = 12\frac{6}{7}$$

The number of sides cannot be a fraction. One angle of the regular polygon cannot measure 152°.

11.
$$(n-2)\cdot 180 = (10)(360)$$
$$180n-360 = (10)(360)$$
$$180n = (10)(360)+360$$
$$180n = (11)(360)$$
$$n = \frac{(11)(2)(\cancel{180})}{\cancel{180}}$$
$$n = 22$$

12.
$$(n-2)\cdot 180$$
$$(5-2)\cdot 180$$
$$3\cdot 180$$
$$540$$



$$m\angle ABC = m\angle CDE + 15$$
$$m\angle ABC = x + 45 + 15$$
$$m\angle ABC = x + 60$$

$$540 = 90 + (x+60) + 90 + (x+45) + (x+45)$$
$$540 = 330 + 3x$$
$$210 = 3x$$
$$70 = x$$

So, m∠ABC = 70 + 60 = 130°

13.
$$(n-2)\cdot 180 = 30\left(\frac{360}{n}\right) + 60$$
$$180n - 360 = \frac{(30)(360)}{n} + 60$$
$$180n^2 - 360n = (30)(360) + 60n$$
$$180n^2 - 420n - 10800 = 0$$
$$\frac{1}{60}(180n^2 - 420n - 10800 = 0)$$
$$3n^2 - 7n - 180 = 0$$
$$(3n+20)(n-9) = 0$$
$$3n + 20 = 0 \quad \text{or} \quad n - 9 = 0$$
$$3n = -20 \qquad n = 9$$
$$n = \frac{-20}{3}$$
not possible

14.

Number of Sides	Sum of Measures of Interior Angles
4	360
10	1440
16	2520
22	3600
34	5760
50	8640
.	.
.	.
.	.
100	17640
.	.
.	.
.	.
1000	179,640

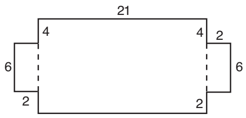
The sum increases infinitely. The greatest possible sum is infinity. (theoretically)

15.

Number of sides	Measure of each exterior angle
4	90
10	36
18	20
36	10
72	5
180	2
360	1
720	1/2
.	.
.	.
.	.
1440	1/4

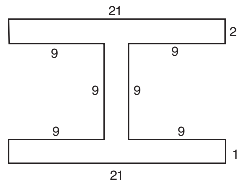
The measure of each exterior angle gets smaller and smaller approaching zero. The least possible measure of an exterior angle is the “smallest number” greater than zero.

11.



$A = b \cdot h$	$A = b \cdot h$	$A = b \cdot h$	$A = 12 + 252 + 12$
$A = 2 \cdot 6$	$A = 21 \cdot 12$	$A = 2 \cdot 6$	$A = 276$ Square Units
$A = 12$	$A = 252$	$A = 12$	

12.

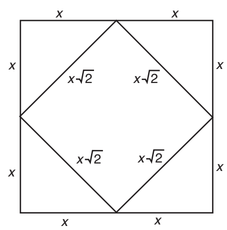


$A = b \cdot h$	$A = b \cdot h$	$A = b \cdot h$	$A = 42 + 27 + 21$
$A = 21 \cdot 2$	$A = 3 \cdot 9$	$A = 21 \cdot 1$	$A = 90$ Square Units
$A = 42$	$A = 27$	$A = 21$	

13. Area of Large Rectangle = b x h = 12 x 6 = 72 Square Units
Area of Small Rectangle = b x h = 5 x 4 = 20 Square Units
Area of Shaded Region = 72 – 20 = 52 Square Units

14.

$$5 = x\sqrt{2}$$
$$\frac{5}{\sqrt{2}} = x$$
$$\frac{5\sqrt{2}}{2} = x$$



Area of Large Square: $(x + x) \cdot (x + x) = A$

$$\left(\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}\right) \cdot \left(\frac{5\sqrt{2}}{2} + \frac{5\sqrt{2}}{2}\right) = A$$
$$\frac{10\sqrt{2}}{2} \cdot \frac{10\sqrt{2}}{2} = A$$
$$5\sqrt{2} \cdot 5\sqrt{2} = A$$
$$25 \cdot 2 = A$$
$$50 = A$$

Area of Small Square: 5 x 5 = 25 Square Units

Area of Shaded Region: 50 – 25 = 25 Square Units

15.

$A = b \cdot h$	$A = b \cdot h$	$A = b \cdot h$ (non-shaded)
$A = 21 \cdot 18$	$A = 8 \cdot 12$	$A = 16 \cdot 10$
$A = 378$	$A = 96$	$A = 160$

Area of Figure = 378 + 96 = 474 Square Units
Area of Shaded Region = 474 – 160 = 314 Square Units

16. The figure on the left is 5 x 6 or 30 small squares. In terms of "m", the area is $30 \div 4$ or $7\,{}^1\!/_2$ square "m"s. There are 9 unshaded small squares. In terms of "m", the area is $9 \div 4$ or $2\,{}^1\!/_4$ square "m"s. The shaded region is $7\frac{1}{2} - 2\frac{1}{4} = \frac{15}{2} - \frac{9}{4} = \frac{30}{4} - \frac{9}{4} = \frac{21}{4} = 5\frac{1}{4}$ square "m"s.

17. One square yard is 3 feet x 3 feet or 9 square feet
Area of Room - b x h = 21 x 13 = 273 square feet
Area of Room = $273 \div 9 = \frac{273}{9} = \frac{\cancel{3} \cdot 91}{\cancel{3} \cdot 3} = 30\frac{1}{3}$ square yards

The cost would be: $\frac{91}{3} \cdot \frac{18}{1} = \frac{91 \cdot \cancel{3} \cdot 6}{\cancel{3}} = 91 \cdot 6 = \$546.00$

18. The area of the larger lot is 4 times the area of the smaller lot.

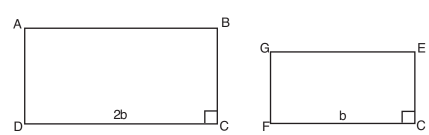
19. Area of GECF = ${}^1\!/_3$ · area of ABCD

$$FC \bullet CE = \frac{1}{3} \bullet DC \bullet CB$$

Let b be the length of FC

$$b \bullet CE = \frac{1}{3} \bullet 2b \bullet CB$$
$$\frac{CE}{CB} = \frac{2\cancel{b}}{3\cancel{b}} = \frac{2}{3}$$
$$CE \text{ is } \frac{2}{3} CB$$

Example:



8. Area △MNQ $= \frac{1}{2}(NQ)(MP)$

$= \frac{1}{2}(NP + PQ)(MP)$

$= \frac{1}{2}(10 + 4)(3)$

$= \frac{1}{2}(14)(3) = \frac{\cancel{2} \cdot 7 \cdot 3}{\cancel{2}} = 21 \text{ units}^2$

9. Area △RST $= \frac{1}{2}(RS)(TU)$

$= \frac{1}{2}(10)(2.1)$

$= \frac{\cancel{2} \cdot 5 \cdot 2.1}{\cancel{2}}$

$= 10.5 \text{ units}^2$

10. a) $MR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$MR = \sqrt{(-1-(-1))^2 + (-1-4)^2}$

$MR = \sqrt{(-1+1)^2 + (-5)^2}$

$MR = \sqrt{0^2 + (-5)^2}$

$MR = \sqrt{0 + 25}$

$MR = \sqrt{25}$

$MR = 5$

$NR = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$NR = \sqrt{(-1-(-2))^2 + (-1-(-1))^2}$

$NR = \sqrt{(-1+2)^2 + (-1+1)^2}$

$NR = \sqrt{1^2 + 0^2}$

$NR = \sqrt{1}$

$NR = 1$

Area △MRN $= \frac{1}{2}(MR)(NR)$

$= \frac{1}{2}(4)(1)$

$= \frac{4}{2} \text{ units}^2$ or 2 units^2

b) $NQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$NQ = \sqrt{(3-(-2))^2 + (-1-(-1))^2}$

$NQ = \sqrt{(3+2)^2 + (-1+1)^2}$

$NQ = \sqrt{5^2 + 0^2}$

$NQ = \sqrt{25}$

$NQ = 5$

Area △MQN $= \frac{1}{2}(NQ)(MR)$

$= \frac{1}{2}(5)(4)$

$= \frac{20}{2}$ or 10 units^2

c) $TQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$TQ = \sqrt{(3-1)^2 + (-1-(-1))^2}$

$TQ = \sqrt{2^2 + (-1+1)^2}$

$TQ = \sqrt{4 + 0}$

$TQ = \sqrt{4}$

$TQ = 2$

Area △MTQ $= \frac{1}{2}(TQ)(MR)$

$= \frac{1}{2}(2)(4)$

$= \frac{8}{2} = 4 \text{ units}^2$

d) $RQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$RQ = \sqrt{(3-(-1))^2 + (-1-(-1))^2}$

$RQ = \sqrt{(3+1)^2 + (-1+1)^2}$

$RQ = \sqrt{4^2 + 0^2}$

Area △MRQ $= \frac{1}{2}(TQ)(MR)$

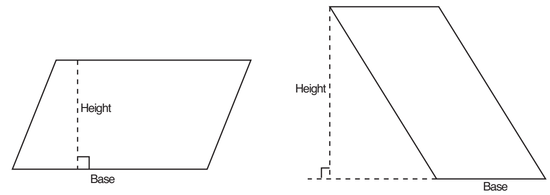
$= \frac{1}{2}(4)(4)$

$= \frac{16}{2} = 8 \text{ units}^2$

Unit V — Other Polygons
Part B — Areas of Polygons

p. 507 – Lesson 3 — Parallelogram

1. Theorem 59 - "If you have a parallelogram, then the area inside the parallelogram is the product of the measures of any base and the corresponding altitude."



Area = Base x Height

2. a) $A = b \cdot h$
$A = 9 \cdot 5$
$A = 45 \text{ units}^2$

AD, BC, CF, and FD are not needed.

b) $A = b \cdot h$
$A = 80 \cdot 20\sqrt{6}$
$A = 1600\sqrt{6} \text{ units}^2$

SU, SV, VT, TU, and SR are not needed.

3. a) $A = b \cdot h$
$54 = 18 \cdot h$
$\frac{54}{18} = h$
$3 \text{ units} = h$

b) Only one - the side parallel to the given side

4. $A = b \cdot h$
$126 = (5x) \cdot (4x)$
$126 = 20x^2$
$\frac{126}{20} = \frac{\cancel{2} \cdot 63}{\cancel{2} \cdot 10} = x^2$
$\sqrt{\frac{63}{10}} = \frac{\sqrt{63}}{\sqrt{10}} = \frac{\sqrt{9}\sqrt{7}}{\sqrt{10}} = \frac{3\sqrt{7}}{\sqrt{10}} = \frac{3\sqrt{7}\sqrt{10}}{\sqrt{10}\sqrt{10}} = \frac{3\sqrt{70}}{10} = x$

Base is $5x$ or $\frac{5 \cdot 3\sqrt{70}}{10} = \frac{5 \cdot 3\sqrt{70}}{5 \cdot 2} = \frac{3\sqrt{70}}{2}$

Height is $4x$ or $\frac{4 \cdot 3\sqrt{70}}{10} = \frac{2 \cdot 2 \cdot 3\sqrt{70}}{5 \cdot 2} = \frac{6\sqrt{70}}{5}$

5. Altitude of parallelogram is: $10\sqrt{2} = x\sqrt{2}$
$10 = x$

Area of parallelogram is: $b \cdot h$
$15 \cdot 10$
150 units^2

6. Altitude of parallelogram is: $\frac{x\sqrt{3}}{2}$ where $x = 10$
$\frac{10\sqrt{3}}{2}$
$5\sqrt{3}$

Area of parallelogram is: $b \cdot h$
$40 \cdot 5\sqrt{3}$
$200\sqrt{3} \text{ units}^2$

7. Altitude of parallelogram is: $\frac{x}{2}$ where $x = 10$
$\frac{10}{2} = 5$

Area of parallelogram is: $b \cdot h$
$40 \cdot 5$

8. Altitude of parallelogram is: $3\sqrt{2}$

Base of parallelogram is: $a^2 + b^2 = c^2$

$$a^2 + \left(3\sqrt{2}\right)^2 = \left(5\sqrt{2}\right)^2$$
$$a^2 + \left(9 \cdot 2\right) = 25 \cdot 2$$
$$a^2 + 18 = 50$$
$$a^2 = 32$$
$$a = \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$$

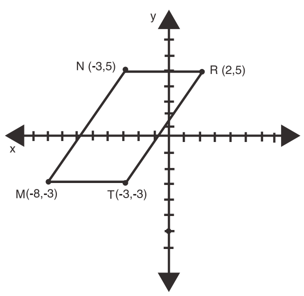
Base is $12\sqrt{2} - 4\sqrt{2} = 8\sqrt{2}$

Area of parallelogram is: $b \cdot h$

$$8\sqrt{2} \cdot 3\sqrt{2}$$
$$24 \cdot 2$$
$$48 \text{ units}^2$$

9. The area is doubled.
10. The area is multiplied by four (quadrilateral).
11. The area is multiplied by three (tripled).
12. The area is multiplied by nine.
13. The area is increased by 25%.

14.



Base is from (–8, –3) to (–3, –3), a horizontal line.

$$= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$
$$= \sqrt{\left(-8-(-3)\right)^2 + \left(-3-(-3)\right)^2}$$
$$= \sqrt{\left(-8+3\right)^2 + \left(-3+3\right)^2}$$
$$= \sqrt{\left(-5\right)^2 + 0^2} = \sqrt{25} = 5$$

Length of base is 5 units.

Altitude is from (–3, –3) to (–3, 5), a vertical line.

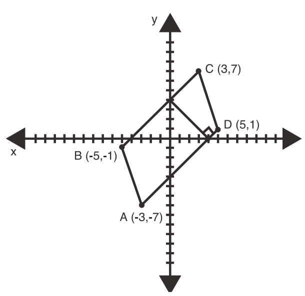
$$= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$
$$= \sqrt{\left(-3-(-3)\right)^2 + \left(-3-5\right)^2}$$
$$= \sqrt{\left(-3+3\right)^2 + \left(-3-5\right)^2}$$
$$= \sqrt{(0)^2 + (-8)^2} = \sqrt{64} = 8$$

Measure of altitude is 8 units.

Area of Parallelogram is b x h.

5 x 8 = 40 units^2

15.



Find slope of AD: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-(-7)}{5-(-3)} = \dfrac{1+7}{5+3} = \dfrac{8}{8} = 1$

Slope of altitude of parallelogram: –1 (line perpendicular to $\overline{AD}$)

From (0, 4) to (4, 0) slope is $\dfrac{0-4}{4-0} = \dfrac{-4}{4} = -1$

Length of segment from (0, 4) to (4, 0):

$$= \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$
$$= \sqrt{\left(4-0\right)^2 + \left(0-4\right)^2}$$
$$= \sqrt{\left(4\right)^2 + \left(-4\right)^2}$$
$$= \sqrt{16+16}$$
$$= \sqrt{32} = \sqrt{16 \cdot 2} = 4\sqrt{2}$$

$$AD = \sqrt{\left(x_2 - x_1\right)^2 + \left(y_2 - y_1\right)^2}$$
$$AD = \sqrt{\left(5-(-3)\right)^2 + \left(1-(-7)\right)^2}$$
$$AD = \sqrt{\left(5+3\right)^2 + \left(1+7\right)^2}$$
$$AD = \sqrt{8^2 + 8^2}$$
$$AD = \sqrt{64+64}$$
$$AD = \sqrt{128} = \sqrt{64 \cdot 2} = 8\sqrt{2}$$

Area is $\left(8\sqrt{2}\right)\left(4\sqrt{2}\right) = 32 \cdot 2 = 64 \text{ units}^2$

p. 513 – Lesson 5 — Regular Polygons

1.
$A = \frac{1}{2} \cdot s \cdot a \cdot n$
$A = \frac{1}{2} \cdot 4\sqrt{3} \cdot 3 \cdot 3$
$A = \frac{\cancel{2} \cdot 2 \cdot \sqrt{3} \cdot 3 \cdot 3}{\cancel{2}}$
$A = 2 \cdot 3 \cdot 3 \cdot \sqrt{3}$
$A = 18\sqrt{3}$ units^2

2.
$A = \frac{1}{2} \cdot s \cdot a \cdot n$
$A = \frac{1}{2} \cdot 16 \cdot 13 \cdot 5$
$A = \frac{\cancel{2} \cdot 8 \cdot 13 \cdot 5}{\cancel{2}}$
$A = 40 \cdot 13$
$A = 520$ units^2

3.
$A = \frac{1}{2} \cdot s \cdot a \cdot n$
$A = \frac{1}{2} \cdot 10 \cdot 8.5 \cdot 6$
$A = \frac{\cancel{2} \cdot 5 \cdot 8.5 \cdot 6}{\cancel{2}}$
$A = 30 \cdot 8.5$
$A = 255$ units^2

4.
$A = \frac{1}{2} \cdot s \cdot a \cdot n$
$A = \frac{1}{2} \cdot 10\sqrt{2} \cdot 5\sqrt{2} \cdot 4$
$A = \frac{10\cancel{\sqrt{2}} \cdot 5\cancel{\sqrt{2}} \cdot 4}{\cancel{2}}$
$A = 50 \cdot 4$
$A = 200$ units^2

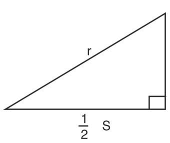
5.
$A = \frac{1}{2} \cdot s \cdot a \cdot n$
$A = \frac{1}{2} \cdot 3 \cdot 4.5 \cdot 8$
$A = \frac{3 \cdot 4.5 \cdot \cancel{2} \cdot 4}{\cancel{2}}$
$A = 12 \cdot 4.5$
$A = 54$ units^2

6.
$A = \frac{1}{2} \cdot s \cdot a \cdot n$
$A = \frac{1}{2} \cdot 16 \cdot 8\sqrt{3} \cdot 6$
$A = \frac{\cancel{2} \cdot 8 \cdot 8\sqrt{3} \cdot 6}{\cancel{2}}$
$A = 64\sqrt{3} \cdot 6$
$A = 384\sqrt{3}$ units^2

7.
$m\angle 2 = \frac{1}{2} \cdot 60$
$m\angle 2 = 30$
$m\angle 1 = 60$

8.
$m\angle 2 = \frac{1}{2} \cdot 90$
$m\angle 2 = 45$
$m\angle 1 = 45$

9.
$m\angle 2 = \frac{1}{2} \cdot 108$
$m\angle 2 = 54$
$m\angle 1 = 36$

10.


a) This is a 30-60-90 triangle

r = 12

$a = \frac{r}{2}$
$= \frac{12}{2}$
$= 6$

$\frac{1}{2}s = \frac{x\sqrt{3}}{2}$
$s = x\sqrt{3}$
$s = 12\sqrt{3}$

$P = 3 \cdot s$
$= 3 \cdot 12\sqrt{3}$
$= 36\sqrt{3}$

$A = \frac{1}{2} \cdot a \cdot P$
$= \frac{1}{2} \cdot 6 \cdot 36\sqrt{3}$
$= \frac{\cancel{2} \cdot 3 \cdot 36\sqrt{3}}{\cancel{2}}$
$= 108\sqrt{3}$ units^2

b) This is a 30-60-90 triangle

$s = 10\sqrt{3}$
$\frac{1}{2}s = 5\sqrt{3}$

$\frac{r \cdot \sqrt{3}}{2} = 5\sqrt{3}$
$r \cdot \sqrt{3} = 10\sqrt{3}$
$r = 10$

$P = 3 \cdot s$
$= 3 \cdot 10\sqrt{3}$
$= 30\sqrt{3}$

$a = \frac{1}{2}r$
$= \frac{1}{2} \cdot 10$
$= 5$

$A = \frac{1}{2} \cdot a \cdot P$
$= \frac{1}{2} \cdot 5 \cdot 30\sqrt{3}$
$= \frac{5 \cdot \cancel{2} \cdot 15\sqrt{3}}{\cancel{2}}$
$= 75\sqrt{3}$ units^2

c) This is a 30-60-90 triangle

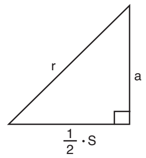
$P = 24\sqrt{3}$

$\frac{1}{3}P = s$
$\frac{1}{3} \cdot 24\sqrt{3} = s$
$8\sqrt{3} = s$

$4\sqrt{3} = \frac{1}{2}s$
$4\sqrt{3} = \frac{r \cdot \sqrt{3}}{2}$
$8\sqrt{3} = r\sqrt{3}$
$8 = r$

$a = \frac{1}{2}r$
$= \frac{1}{2} \cdot 8$
$= 4$

$A = \frac{1}{2} \cdot a \cdot P$
$= \frac{1}{\cancel{2}} \cdot \cancel{4} \cdot 24\sqrt{3}$
$= 48\sqrt{3}$ units^2

11. 

a) This is a 45-45-90 triangle

$s = 16$

$\frac{1}{2}s = a$, $\frac{1}{2} \cdot 16 = a$, $8 = a$

$r = a\sqrt{2}$, $r = 8\sqrt{2}$

$P = 4s$, $P = 4 \cdot 16$, $P = 64$

$A = \frac{1}{2} \cdot a \cdot P = \frac{1}{2} \cdot 8 \cdot 64 = 4 \cdot 64 = 256 \text{ units}^2$

b) This is a 45-45-90 triangle

$r = 8\sqrt{2}$

$r = a\sqrt{2}$, $8\sqrt{2} = a\sqrt{2}$, $8 = a$

$\frac{1}{2}s = 8$, $s = 16$

$P = 4 \cdot s$, $P = 4 \cdot 16$, $P = 64$

$A = \frac{1}{2} \cdot a \cdot P = \frac{1}{2} \cdot 8 \cdot 64 = 4 \cdot 64 = 256 \text{ units}^2$

c) This is a 45-45-90 triangle

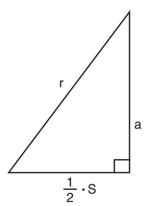
$a = 5\sqrt{2}$

$r = a\sqrt{2}$, $r = 5\sqrt{2} \cdot \sqrt{2}$, $r = 10$

$\frac{1}{2}s = a$, $\frac{1}{2}s = 5\sqrt{2}$, $s = 10\sqrt{2}$

$P = 4 \cdot s$, $P = 4 \cdot 10\sqrt{2}$, $P = 40\sqrt{2}$

$A = \frac{1}{2} \cdot a \cdot P = \frac{1}{2} \cdot 5\sqrt{2} \cdot 40\sqrt{2} = \frac{5\sqrt{\cancel{2}} \cdot 40\sqrt{\cancel{2}}}{\cancel{2}} = 200 \text{ units}^2$

12. 

a) This is a 30-60-90 triangle

$r = 10$

$a = \frac{r\sqrt{3}}{2}$, $a = \frac{10\sqrt{3}}{2}$, $a = 5\sqrt{3}$

$\frac{1}{2}s = \frac{1}{2}r$, $s = r$, $s = 10$

$P = 6s$, $P = 6 \cdot 10$, $P = 60$

$A = \frac{1}{2} \cdot a \cdot P = \frac{1}{2} \cdot 5\sqrt{3} \cdot 60 = \frac{5\sqrt{3} \cdot \cancel{2} \cdot 30}{\cancel{2}} = 150\sqrt{3} \text{ units}^2$

b) This is a 30-60-90 triangle

$a = 9\sqrt{3}$

$9\sqrt{3} = \frac{r\sqrt{3}}{2}$, $18\sqrt{3} = r\sqrt{3}$, $18 = r$

$\frac{1}{2}s = \frac{1}{2}r$, $s = r$, $s = 18$

$P = 6s$, $P = 6 \cdot 18$, $P = 108$

$A = \frac{1}{2} \cdot a \cdot P = \frac{1}{2} \cdot 9\sqrt{3} \cdot 108 = \frac{9\sqrt{3} \cdot \cancel{2} \cdot 54}{\cancel{2}} = 486\sqrt{3} \text{ units}^2$

c) This is a 30-60-90 triangle

$s = 6$, $\frac{1}{2}s = \frac{1}{2}6$, $\frac{1}{2}s = 3$

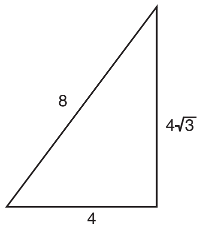
$\frac{1}{2}s = \frac{1}{2}r$, $3 = \frac{1}{2}r$, $6 = r$

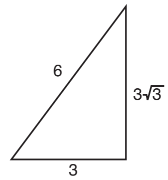
$a = \frac{r\sqrt{3}}{2}$, $a = \frac{6\sqrt{3}}{2}$, $a = 3\sqrt{3}$

$P = 6s$, $P = 6 \cdot 6$, $P = 36$

$A = \frac{1}{2} \cdot a \cdot P = \frac{1}{2} \cdot 3\sqrt{3} \cdot 36 = \frac{3\sqrt{3} \cdot \cancel{2} \cdot 3 \cdot 6}{\cancel{2}} = 54\sqrt{3} \text{ units}^2$

13.





Area of outside hexagon

$$A = \frac{1}{2} \cdot a \cdot P$$
$$= \frac{1}{2} \cdot 4\sqrt{3} \cdot (6 \cdot 8)$$
$$= \frac{4\sqrt{3} \cdot 48}{2}$$
$$= \frac{4\sqrt{3} \cdot \cancel{2} \cdot 24}{\cancel{2}}$$
$$= 96\sqrt{3} \text{ units}^2$$

Area of inside hexagon

$$A = \frac{1}{2} \cdot a \cdot P$$
$$= \frac{1}{2} \cdot 3\sqrt{3} \cdot (6 \cdot 6)$$
$$= \frac{3\sqrt{3} \cdot 36}{2}$$
$$= \frac{3\sqrt{3} \cdot \cancel{2} \cdot 18}{\cancel{2}}$$
$$= 54\sqrt{3} \text{ units}^2$$

Area of shaded region $96\sqrt{3} - 54\sqrt{3} = 42\sqrt{3}$ units^2

14. Area of pentagon

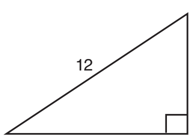
$$A = \frac{1}{2} \cdot a \cdot P$$
$$= \frac{1}{2} \cdot 5.5 \cdot (5 \cdot 8)$$
$$= \frac{5.5 \cdot 40}{2}$$
$$= \frac{5.5 \cdot \cancel{2} \cdot 20}{\cancel{2}}$$
$$= 110 \text{ units}^2$$

Area of white triangle

$$A = \frac{1}{2} \cdot a \cdot b$$
$$= \frac{1}{2} \cdot 8 \cdot 5.5$$
$$= \frac{\cancel{2} \cdot 4 \cdot 5.5}{\cancel{2}}$$
$$= 22 \text{ units}^2$$

Area of shaded region $110 - 22 = 88$ units^2

15.



This is a 30-60-90 triangle. The shaded region is made up of 6 identical triangles.

Altitude is $\frac{x}{2}$ where x = 12

$$\frac{12}{2} = 6$$

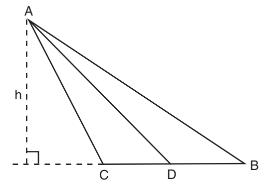
Base is $\frac{x\sqrt{3}}{2}$ where x = 12

$$\frac{12\sqrt{3}}{2} = 6\sqrt{3}$$

Area of one triangle:
$$A = \frac{1}{2} \cdot 6\sqrt{3} \cdot 6$$
$$= \frac{6\sqrt{3} \cdot \cancel{2} \cdot 3}{\cancel{2}}$$
$$= 18\sqrt{3} \text{ units}^2$$

Area of shaded region: $6 \cdot 18\sqrt{3} = 108\sqrt{3}$ units^2

p. 516 – Lesson 1 — Using Areas in Proofs



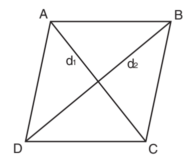
1. Theorem 62 - "If you have a median of a triangle, then the median separates the points inside the triangle into two polygonal regions with the same area."

   Point D is the midpoint of $\overline{CB}$, so $\overline{CD} \cong \overline{BD}$.

   Area $\triangle$ADC $= \frac{1}{2}(CD) \cdot h$; Area $\triangle$ADB $= \frac{1}{2}(DB) \cdot h$

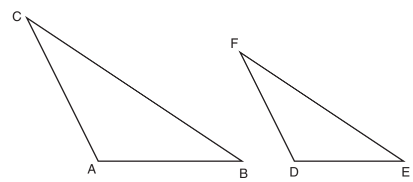
   Since CD = BD, $\frac{1}{2}(CD) \cdot h = \frac{1}{2}(DB) \cdot h$

   and, Area $\triangle$ADC = Area $\triangle$ADB



2. Theorem 63 - "If you have a rhombus, then the area enclosed by that rhombus, is equal to one-half the product of the measure of the diagonals of the rhombus."

   $A = \frac{1}{2}(AC)(BD)$ or $A = \frac{1}{2}(d_1)(d_2)$



3. Theorem 64 - "If you have two similar polygons, then the ratio of the areas of the two polygons is equal to the square of the ratio of any pair of corresponding sides."

   $\triangle ABC \sim \triangle DEF$

   $$\frac{\text{Area } \triangle ABC}{\text{Area } \triangle DEF} = \frac{(CB)^2}{(FE)^2} = \frac{(AB)^2}{(DE)^2} = \frac{(AC)^2}{(DF)^2}$$

4. A = s^2 A = s^2 Ratio of Areas is $\frac{16}{64} = \frac{1 \cdot \cancel{16}}{4 \cdot \cancel{16}} = \frac{1}{4}$

   A = 4^2 A = 8^2

   A = 16 units^2 A = 64 units^2

   Using Theorem 63: The polygons are similar.
   The ratio of corresponding sides is 4:8 or 1:2.
   The square of the ratio of corresponding sides is 1^2:2^2 or 1:4.

5. Theorem 63: $\frac{13^2}{20^2} = \frac{169}{400}$

6. BM = 8   MC = 8

   Area $\triangle AMB = \frac{1}{2} \cdot 8 \cdot 5 = \frac{8 \cdot 5}{2} = \frac{\cancel{2} \cdot 4 \cdot 5}{\cancel{2}} = 20$ units^2

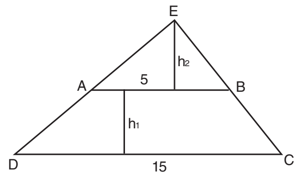
   Area $\triangle ABC = \frac{1}{2} \cdot 16 \cdot 5 = \frac{80}{2} = 40$ units^2

   Using Theorem 62, we can simply find the area of ABC and divide the answer by 2.

   Area $\triangle ABC = \frac{1}{2} \cdot 16 \cdot 5 = \frac{16 \cdot 5}{2} = \frac{\cancel{2} \cdot 8 \cdot 5}{\cancel{2}} = 40$ units^2

   $40 \div 2 = 20,$ the area of each smaller triangle.

14. The two triangles formed by extending the legs of trapezoid ABCD are △DEC and △AEB. By theorem 63:



$$\frac{\text{Area }\triangle AEB}{\text{Area }\triangle DEC}=\frac{(AE)^2}{(DE)^2}\ or\ \frac{(EB)^2}{(EC)^2}\ or\ \frac{(AB)^2}{(DC)^2}$$

Since AB = 5 and DC = 15, we will use the ratio $\frac{AB}{DC}$.

$$\text{Area }\triangle AEB=\frac{1}{2}(AB)\cdot(h_2)$$

$$\text{Area }\triangle DEC=\frac{1}{2}(DC)\cdot(h_1+h_2)$$

Find h:

$$A=\frac{1}{2}\cdot h_1(b_1+b_2)$$
$$100=\frac{1}{2}\cdot h_1(15+5)$$
$$200=h_1(20)$$
$$10=h_1$$

$$\frac{\frac{1}{2}(AB)\cdot(h_2)}{\frac{1}{2}(DC)\cdot(h_1+h_2)}=\left(\frac{AB}{DC}\right)^2$$
$$\frac{\frac{1}{2}(5)\cdot(h_2)}{\frac{1}{2}(15)\cdot(10+h_2)}=\left(\frac{5}{15}\right)^2$$
$$\frac{5\cdot h_2}{15(10+h_2)}=\frac{25}{225}$$
$$\frac{h_2}{3(10+h_2)}=\frac{1}{9}$$
$$3(10+h_2)=9h_2$$
$$30+3h_2=9h_2$$
$$30=6h_2$$
$$5=h_2$$

$$\text{Area }\triangle AEB=\frac{1}{2}(5)\cdot(5)=\frac{25}{2}\text{ or }12\frac{1}{2}\text{ units}^2$$
$$\text{Area }\triangle DEC=\frac{1}{2}(15)\cdot(10+5)=\frac{1}{2}\cdot15\cdot15=\frac{225}{2}\text{ or }112\frac{1}{2}\text{ units}^2$$

15. **a)** $\left(\frac{2\sqrt{3}}{6\sqrt{3}}\right)^2=\frac{\cancel{2}\cancel{\sqrt{3}}\cdot\cancel{2}\cancel{\sqrt{3}}}{\cancel{6}_3\cancel{\sqrt{3}}\cdot\cancel{6}_3\cancel{\sqrt{3}}}=\frac{1}{3}\cdot\frac{1}{3}=\frac{1}{9}$

b) $\left(\frac{2\sqrt{3}}{3\sqrt{5}}\right)^2=\frac{2\sqrt{3}\cdot2\sqrt{3}}{3\sqrt{5}\cdot3\sqrt{5}}=\frac{4\cdot3}{9\cdot5}=\frac{4\cdot\cancel{3}}{3\cdot\cancel{3}\cdot5}=\frac{4}{15}$

16. **a)**
$$\frac{81}{121}=\frac{x^2}{11^2}$$
$$\frac{81}{121}=\frac{x^2}{121}$$
$$81=x^2$$
$$9=x$$

b)
$$\frac{81}{121}=\frac{x^2}{(22)^2}$$
$$81\cdot22\cdot22=121\cdot x^2$$
$$\frac{81\cdot22\cdot22}{121}=x^2$$
$$\frac{81\cdot2\cdot\cancel{11}\cdot2\cdot\cancel{11}}{\cancel{11}\cdot\cancel{11}}=x^2$$
$$9^2\cdot2^2=x^2$$
$$\sqrt{9^2\cdot2^2}=x$$
$$\sqrt{9^2}\cdot\sqrt{2^2}=x$$
$$9\cdot2=x$$
$$18=x$$

17. **a)**
$$\frac{5}{9}=\left(\frac{x}{y}\right)^2$$
$$\left(\frac{5}{9}\right)^{1/2}=\left(\left(\frac{x}{y}\right)^2\right)^{1/2}$$
$$\sqrt{\frac{5}{9}}=\frac{x}{y}$$
$$\frac{\sqrt{5}}{\sqrt{9}}=\frac{\sqrt{5}}{3}=\frac{x}{y}$$

b) The two regular octagons are similar polygons. The angles are congruent. So the ratio of a pair of corresponding angles is 1:1.

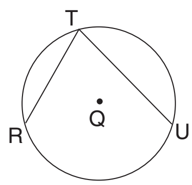
Unit VI — Circles

Part A — Fundamental Terms

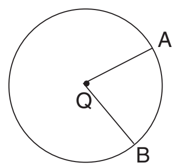
p. 526 – Lesson 2 — Arcs and Angles

1. $\widehat{AB}$, $\widehat{BC}$, $\widehat{CD}$, $\widehat{DE}$, $\widehat{EA}$ (Answers may vary. Here are other answers: $\widehat{CE}$, $\widehat{DA}$, $\widehat{EB}$)
2. $\widehat{ABD}$, $\widehat{ABE}$, $\widehat{EAC}$ (Answers may vary. Here are other answers: $\widehat{EAD}$, $\widehat{DAC}$, $\widehat{CAB}$, $\widehat{BCE}$)
3. $\widehat{ABC}$, $\widehat{AEC}$, $\widehat{BCD}$, $\widehat{BAD}$
4. ∠AQB, ∠AQE, ∠AQD (Answers may vary. Here are other answers: ∠BQE, ∠EQC, ∠EQD, ∠BQC, ∠DQC)
5. ∠BAC, ∠EAC (Answers may vary. Here are other answers: ∠BAE, ∠ABD)

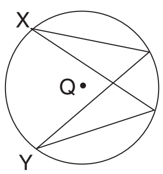
6. (Answers and diagrams may vary)
Inscribed Angle: ∠RTU
Minor Arc: $\widehat{RU}$ (also $\widehat{RT}$ and $\widehat{UT}$)



7. (Answers and diagrams may vary)
Central Angle: ∠AQB
Minor Arc: $\widehat{AB}$

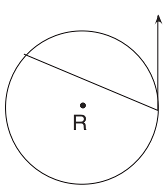


8. (Diagrams may vary)

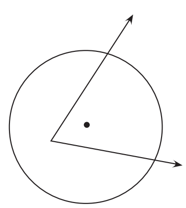


9. **a)** ∠UYV; ∠UXV **b)** ∠XQV; ∠WQV **c)** ∠WVX **d)** ∠YUX; ∠YVX

10.



11.



12. **a)** m∠AOB = 75
b) m$\widehat{AE}$ = 75
c) m$\widehat{CD}$ = 20
d) OE = 5
e) m$\widehat{ACE}$ = 285
f) m∠COE = 125

13. **a)** m∠WOZ = 34

$$4x + (3x + 10) + x + 2x + 10 = 360$$
$$10x + 20 = 360$$
$$10x = 340$$
$$x = 34$$

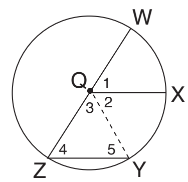
b) m$\widehat{XYZ}$ = 4x + (2x + 10)
= 6x + 10
= 6(34) + 10
= 204 + 10
= 214

c) m∠WOX = 3x + 10
= 3(34) + 10
= 102 + 10
= 112

d) m∠YOZ = 2x + 10
= 2(34) + 10
= 68 + 10
= 78

14. **a)** $\widehat{ADC}$ (or $\widehat{ACB}$) **b)** $\widehat{DAC}$ (or $\widehat{DBC}$) **c)** $\widehat{ABD}$ (or $\widehat{ACD}$) **d)** $\widehat{BAC}$ (or $\widehat{BDC}$)

15. Given: $\overline{WZ}$ is a diameter of $\odot Q$.
$m\widehat{WX} = m\widehat{XY} = n$
Prove: $m\angle Z = n$



STATEMENT	REASONS
1. $\overline{WZ}$ is a diameter of $\odot Q$	1. Given
2. $m\widehat{WX} = m\widehat{XY} = n$	2. Given
3. $m\angle 1 = m\widehat{WX} = n$ $m\angle 2 = m\widehat{XY} = n$	3. Definition of the measure of a central angle and the measure of its intercepted arc.
4. $m\angle 1 = m\angle 2$	4. Substitution
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	5. A straight line contains 180°.
6. $m\angle 4 + m\angle 5 + m\angle 3 = 180$	6. Theorem 25 - If you have any given triangle, then the sum of the measures of its angles is 180°.
7. $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5 + m\angle 3$	7. Substitution
8. $m\angle 1 + m\angle 2 = m\angle 4 + m\angle 5$	8. Subtraction Property of Equality
9. Draw $\overline{QY}$	9. Postulate 2 - For any two different points, there is exactly one line containing them.
10. $\overline{QY} \cong \overline{QZ}$	10. Radii of the same circle are congruent.
11. $\angle 4 \cong \angle 5$	11. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
12. $m\angle 4 = m\angle 5$	12. Definition of Congruent Angles
13. $m\angle 1 + m\angle 1 = m\angle 4 + m\angle 4$	13. Substitution
14. $2m\angle 1 = 2m\angle 4$	14. Properties of Algebra
15. $m\angle 1 = m\angle 4$	15. Multiplication Property of Equality
16. $n = m\angle 4$ (or $m\angle Z$)	16. Substitution

Unit VI — Circles
Part A — Fundamental Terms

p. 530 – Lesson 3 — Circle Relationships

1. One external; no internal
2. Two external; no internal
3. No external; no internal
4. No external; no internal
5. Two external; one internal
6. Two external; two internal
7. Point W is in the interior of both circles. Point X is in the exterior of the inner circle and in the interior of the outer circle. Point Y is in the exterior of the inner circle and in the interior of the outer circle. Point Z is in the exterior of both circles.
8. CD = CE + EF + FD
CD = 10 + 12 + 3
CD = 25
9. 63
10. 63
11. 85
12. 85
13. 212
14. 212
15. 275
16. 275

17. $\overset{\frown}{JK}$ and $\overset{\frown}{PN}$; $\overset{\frown}{KT}$ and $\overset{\frown}{NM}$
$\overset{\frown}{TSJ}$ and $\overset{\frown}{MRP}$; $\overset{\frown}{TJK}$ and $\overset{\frown}{MRN}$

18. No. Congruent figures must have the same size (in this case, length, if stretching out straight) and shape.

19. **a)** must
b) must
c) can be but need not be
d) must
e) cannot be
f) must
g) must
h) can be but need not be
i) must

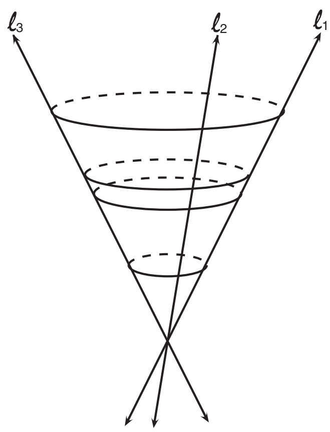
20. infinitely many

21. infinitely many

22. yes; yes; no

23. yes; yes; no

24.



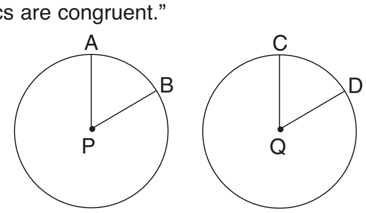
Unit VI — Circles

Part B — Angle and Arc Relationships

p. 534 – Lesson 1 — Theorem 65 and 66

1. **a)** Theorem 65 - “In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent.”

b)



c) Given: ⊙P ≅ ⊙Q; ∠P ≅ ∠Q

d) Prove: $\overset{\frown}{AB} \cong \overset{\frown}{CD}$

e)

STATEMENT	REASONS
1. ⊙P ≅ ⊙Q	1. Given
2. ∠P ≅ ∠Q	2. Given
3. m∠P = m∠Q	3. Definition of Congruent Angles
4. m∠P = m$\overset{\frown}{AB}$	4. Definition of Arc Measure
5. m∠Q = m$\overset{\frown}{CD}$	5. Definition of Arc Measure
6. m$\overset{\frown}{AB}$ = m$\overset{\frown}{CD}$	6. Substitution
7. $\overset{\frown}{AB} \cong \overset{\frown}{CD}$	7. Definition of Congruent Arcs

6. Corollary 67b - "If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary."
x = 180 – 85 or 95   y = 180 – 110 or 70

7. **a)** $\overset{\frown}{XZ}$ **b)** $\overset{\frown}{NP}$ **c)** $\overset{\frown}{AC}$ **d)** $\overset{\frown}{OMQ}$ (or $\overset{\frown}{ONQ}$) **e)** $\overset{\frown}{XZ}$ **f)** $\overset{\frown}{NP}$

8. **a)** $m\angle Y = 70^\circ$ **b)** $m\angle O = 40^\circ$ **c)** $m\angle B = 55^\circ$ **d)** $m\angle MNQ = 40^\circ$ **e)** $m\angle W = 70^\circ$ **f)** $m\angle QPM = 40^\circ$

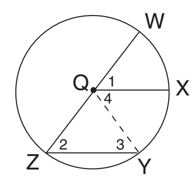
9. ∠MNQ, ∠MOQ, ∠MPQ

10. The angles are congruent by Corollary 67c: "If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent."

11. ∠YXW, ∠WZY

12. The angles are congruent by Corollary 67c: "If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent."

13. $m\angle U = 15^\circ$, $m\angle V = 30^\circ$, $m\angle UXV = 180 - (15 + 30)$
180 - 45
135°



14. Given: $\overline{WZ}$ is a diameter of ⊙Q; $\overline{QX} \parallel \overline{ZY}$
Prove: $\overset{\frown}{WX} \cong \overset{\frown}{XY}$

STATEMENT	REASONS
1. $\overline{WZ}$ is a diameter of ⊙Q	1. Given
2. $\overline{QX} \parallel \overline{ZY}$	2. Given
3. Draw $\overline{QY}$	3. Postulate 2 - 1st Assumption - For any two different points, there is exactly one line containing them.
4. ∠1 ≅ ∠2	4. Postulate 11 - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
5. $\overline{QZ} \cong \overline{QY}$	5. Radii of the same circle are congruent.
6. ∠2 ≅ ∠3	6. Theorem 33 - If two sides of a triangle are congruent then the angles opposite them are congruent.
7. ∠3 ≅ ∠4	7. Theorem 16 - If two parallel lines are cut by a transversal, the alternate interior angles are congruent.
8. ∠1 ≅ ∠4	8. Transitive Property of Angle Congruence
9. $\overset{\frown}{WX} \cong \overset{\frown}{XY}$	9. Theorem 65 - If two central angles are congruent, then their intercepted arcs are congruent.

15. $m\overset{\frown}{RS} + m\overset{\frown}{RT} + m\overset{\frown}{ST} = 360$
$(2x + 8) + (4x - 20) + (3x + 12) = 360$

$9x = 360$
$x = 40$

$m\overset{\frown}{RS} = 2x + 8 = 2(40) + 8 = 80 + 8 = 88$

$m\overset{\frown}{RT} = 4x - 20 = 4(40) - 20 = 160 - 20 = 140$

$m\overset{\frown}{ST} = 3x + 12 = 3(40) + 12 = 120 + 12 = 132$

$m\angle R = \frac{1}{2} \cdot m\overset{\frown}{ST} = \frac{1}{2} \cdot 132 = 66$

$m\angle S = \frac{1}{2} \cdot m\overset{\frown}{RT} = \frac{1}{2} \cdot 140 = 70$

$m\angle T = \frac{1}{2} \cdot m\overset{\frown}{RS} = \frac{1}{2} \cdot 88 = 44$

Unit VI — Circles

Part B — Angle and Arc Relationships

p. 544 – Lesson 3 — Theorem 68

1. $\angle ABG$, $\angle CBG$
$\angle ABF$, $\angle CBF$
$\angle ABE$, $\angle CBE$
$\angle ACE$

2. $\widehat{BHG}$, $\widehat{BEG}$
$\widehat{BGF}$, $\widehat{BEF}$
$\widehat{BFE}$, $\widehat{BDE}$
$\widehat{BFD}$, $\widehat{BD}$

3. $m\angle VYW = \frac{1}{2}\cdot m\widehat{WY}$
$\frac{1}{2}\cdot 82$
$\frac{1\cdot 41\cdot \cancel{2}}{\cancel{2}}$
41

$m\angle VYX = \frac{1}{2}\cdot m\widehat{XY}$
$\frac{1}{2}\cdot\left[m\left(\widehat{XW}+\widehat{WY}\right)\right]$
$\frac{1}{2}\cdot\left[96+82\right]$
$\frac{1}{2}\cdot 178$
$\frac{1\cdot 89\cdot \cancel{2}}{\cancel{2}}$
89

$m\angle WYX = \frac{1}{2}\cdot m\widehat{XW}$
$\frac{1}{2}\cdot 96$
$\frac{1\cdot 48\cdot \cancel{2}}{\cancel{2}}$
48

$m\angle UYX = \frac{1}{2}\cdot m\widehat{XZY}$
$\frac{1}{2}\cdot\left[360-\left(m\widehat{XW}+m\widehat{WY}\right)\right]$
$\frac{1}{2}\cdot\left[360-\left(82+96\right)\right]$
$\frac{1}{2}\cdot\left[360-178\right]$
$\frac{1}{2}\cdot 182$
$\frac{1\cdot 91\cdot \cancel{2}}{\cancel{2}}$
91

4. **a)** $m\angle CAD = \frac{1}{2}\cdot m\widehat{AC}$
$\frac{1}{2}\cdot 102$
$\frac{1\cdot 51\cdot \cancel{2}}{\cancel{2}}$
51

b) $m\widehat{BC} = 360 - m\widehat{AC} - m\widehat{AEB}$
$360-102-200$
58

c) $m\angle ABC = \frac{1}{2}\cdot m\widehat{AC}$
$\frac{1}{2}\cdot 102$
$\frac{1\cdot 51\cdot \cancel{2}}{\cancel{2}}$
51

d) $m\angle BAF = \frac{1}{2}\cdot m\widehat{BEA}$
$\frac{1}{2}\cdot 200$
$\frac{1\cdot 100\cdot \cancel{2}}{\cancel{2}}$
100

e) $m\angle BAC = \frac{1}{2}\cdot m\widehat{BC}$
$\frac{1}{2}\cdot 58$
$\frac{1\cdot 29\cdot \cancel{2}}{\cancel{2}}$
29

f) $m\angle ACB = \frac{1}{2}\cdot m\widehat{BEA}$
$\frac{1}{2}\cdot 200$
$\frac{1\cdot 100\cdot \cancel{2}}{\cancel{2}}$
100

5. **a)** $m\angle DAC = \frac{1}{2}\cdot m\widehat{AD}$
$\frac{1}{2}\cdot 80$
$\frac{1\cdot 40\cdot \cancel{2}}{\cancel{2}}$
40

b) $m\angle AFD = \frac{1}{2}\cdot m\widehat{AD}$
$\frac{1}{2}\cdot 80$
$\frac{1\cdot 40\cdot \cancel{2}}{\cancel{2}}$
40

c) $m\angle DEF = \frac{1}{2}\cdot m\widehat{DAF}$
$\frac{1}{2}\cdot 180$
$\frac{1\cdot 90\cdot \cancel{2}}{\cancel{2}}$
90

d) $m\angle DFE = \frac{1}{2}\cdot m\widehat{DE}$
$\frac{1}{2}\cdot 70$
$\frac{1\cdot 35\cdot \cancel{2}}{\cancel{2}}$
35

e) $m\angle AFD = 180 - m\widehat{AD}$
$180-80$
100

f) $m\angle BAF = \frac{1}{2}\cdot m\widehat{AF}$
$\frac{1}{2}\cdot 100$
$\frac{1\cdot 50\cdot \cancel{2}}{\cancel{2}}$
50

g) $m\angle EDF = \frac{1}{2}\cdot m\widehat{EF}$
$\frac{1}{2}\cdot\left(180-70\right)$
$\frac{1}{2}\cdot 110$
$\frac{1\cdot 55\cdot \cancel{2}}{\cancel{2}}$
55

3.
$$x = \frac{1}{2}(230-60) = \frac{1}{2}\cdot 170 = \frac{\not 2\cdot 85}{\not 2} = 85$$

4.
$$y = 360-90 = 270$$
$$x = \frac{1}{2}(270-90) = \frac{1}{2}\cdot 180 = \frac{\not 2\cdot 90}{\not 2} = 90$$

5.
$$35 = \frac{1}{2}\left[(360-90-x)-x\right]$$
$$35 = \frac{1}{2}\left[270-2x\right]$$
$$35 = \frac{\not 2\cdot 135}{\not 2} - \frac{\not 2\cdot x}{\not 2}$$
$$35 = 135-x$$
$$-100 = -x$$
$$100 = x$$

6.
$$x = \frac{1}{2}(250-110) = \frac{1}{2}\cdot 140 = \frac{\not 2\cdot 70}{\not 2} = 70$$

7.
$$30 = \frac{1}{2}(x-80)$$
$$30 = \frac{1}{2}x - \frac{\not 2\cdot 40}{\not 2}$$
$$30 = \frac{1}{2}x-40$$
$$70 = \frac{1}{2}x$$
$$140 = x$$

8.
$$x-180 = \frac{1}{2}\left(x-(90+a)\right)$$
$$x-180 = \frac{1}{2}\left(x-(90-a)\right)$$
$$2x-360 = x-90-a$$
$$x-270 = -a$$
$$x = 270-a$$

9.
$$x = \frac{1}{2}(280-80) = \frac{1}{2}\cdot 200 = \frac{\not 2\cdot 100}{\not 2} = 100$$

10.
$$y = 360-x$$
$$44 = \frac{1}{2}(y-x)$$
$$44 = \frac{1}{2}\left((360-x)-x\right)$$
$$88 = 360-2x$$
$$-272 = -2x$$
$$136 = x$$
$$y = 360-136$$
$$y = 224$$

11.
$$100+a+m\overset{\frown}{BC} = 360$$
$$m\overset{\frown}{BC} = 360-100-a = 260-a$$
$$x = \frac{1}{2}\left[(260-a)-a\right] = \frac{1}{2}\cdot(260-2a) = \frac{\not 2\cdot 130}{\not 2} - \frac{\not 2\cdot a}{\not 2} = 130-a$$

12.
$$m\overset{\frown}{AB} = 180-80 = 100$$
$$x = \frac{1}{2}(100-80) = \frac{1}{2}\cdot(20) = \frac{\not 2\cdot 10}{\not 2} = 10$$

13.
$$m\angle ACE = \frac{1}{2}\left(m\overset{\frown}{AE} - m\overset{\frown}{AD}\right)$$
$$18 = \frac{1}{2}\cdot\left(92-m\overset{\frown}{AD}\right)$$
$$36 = 92-m\overset{\frown}{AD}$$
$$-56 = -m\overset{\frown}{AD}$$
$$56 = m\overset{\frown}{AD}$$

14.
$$m\overset{\frown}{BD} = 180-m\overset{\frown}{AD} = 180-56 = 124$$

15.
$$m\overset{\frown}{BE} = 180-92 = 88$$
$$m\angle EFB = \frac{1}{2}\left(m\overset{\frown}{AD}+m\overset{\frown}{BE}\right) = \frac{1}{2}\cdot(56+88) = \frac{1}{2}\cdot(144) = \frac{\not 2\cdot 72}{\not 2} = 72$$

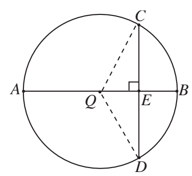
16.
$$m\angle DAC = \frac{1}{2}m\overset{\frown}{AD} = \frac{1}{2}\cdot(56) = \frac{\not 2\cdot 28}{\not 2} = 28$$

17.
$$m\angle ADC = 180-m\angle ACD-m\angle DAB = 180-18-28 = 180-46 = 134$$

2. continued

STATEMENT	REASONS
19. $\widehat{ADB}$ is a semicircle	19. Definition of Semicircle
20. $m\widehat{ADB} = 180^{\circ}$	20. Definition of Semicircle
21. $m\widehat{ACB} = m\widehat{ADB}$	21. Substitution
22. $m\widehat{AC} + m\widehat{CB} = m\widehat{AD} + m\widehat{DB}$	22. Substitution
23. $m\widehat{CB} = m\widehat{DB}$	23. Definition of Congruent Arcs
24. $m\widehat{AC} = m\widehat{AD}$	24. Addition (Subtraction) Property of Equality
25. $\widehat{AC} \cong \widehat{AD}$	25. Definition of Congruent Arcs
26. $\overline{AB}$ bisects $\widehat{CAD}$	26. Definition of Arc Bisector

3. "If the midpoints of the two arcs of a circle determined by a chord are joined by a line segment, then the line segment is the perpendicular besector of the chord."



Given: $\widehat{CD}$ and $\widehat{CAD}$ are two arcs determined by chord $\overline{CD}$.

Point A and point B are midpoints of $\widehat{CD}$ and $\widehat{CAD}$ joined by $\overline{AB}$.

Prove: $\overline{AB} \perp \overline{CD}$; $\overline{AB}$ bisects $\overline{CD}$

STATEMENT	REASONS
1. $\widehat{CD}$ and $\widehat{CAD}$ are two arcs determined by chord $\overline{CD}$.	1. Given
2. Point A and point B are midpoints of $\widehat{CD}$ and $\widehat{CAD}$ joined by $\overline{AB}$.	2. Given
3. $\widehat{CB} \cong \widehat{DB}$	3. Definition of midpoint
4. $\widehat{AC} \cong \widehat{AD}$	4. Definition of midpoint
5. $m\widehat{CB} = m\widehat{DB}$	5. Definition of Congruent Arcs
6. $m\widehat{AC} = m\widehat{AD}$	6. Definition of Congruent Arcs
7. $m\widehat{AC} + m\widehat{CB} + m\widehat{BD} + m\widehat{DA} = 360$	7. Postulate 8 - First Assumption - The set of all points on a circle can be put into a one-to-one correspondence with the real numbers from 0 to 360, inclusive, with the exception of any one point which may be paired with 0 and 360.
8. $m\widehat{AC} + m\widehat{CB} + m\widehat{CB} + m\widehat{AC} = 360$	8. Substitution
9. $2m\widehat{AC} + 2m\widehat{CB} = 360$	9. Properties of Algebra - Collect like Terms
10. $m\widehat{AC} + m\widehat{CB} = 180$	10. Multiplication Property of Equality
11. $m\widehat{AC} + m\widehat{CB} = m\widehat{ACB}$	11. Postulate 8 - Fourth Assumption - Arc Addition Assumption
12. $m\widehat{ACB} = 180$	12. Substitution
13. $\widehat{ACB}$ is a semicircle	13. Definition of Semicircle - "...a semicircle is the intercepted arc of a central angle of 180."
14. $\overline{AB}$ is a diameter	14. Definition of Semicircle - An arc of a circle is a semicircle, if and only if, it is an arc whose endpoints are the endpoints of a diameter of a circle.
15. $\overline{AB}$ passes through point Q, the center of the circle	15. Definition of diameter
16. Draw radius $\overline{QC}$	16. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
17. Draw $\overline{QD}$	17. Postulate 2
18. $\overline{QC} \cong \overline{QD}$	18. Radii of the same circle are congruent
19. $m\angle CQB = m\widehat{CB}$	19. Definition of Measure of a Central Angle - the measure of a minor arc and the measure of a central angle are equal.
20. $m\angle DQB = m\widehat{DB}$	20. Definition of Measure of a Central Angle
21. $m\angle CQB = m\angle DQB$	21. Substitution
22. $\angle CQB \cong \angle DQB$	22. Definition of Congruent Angles
23. $\overline{QE} \cong \overline{QE}$	23. Reflexive Property of Congruent Segments
24. $\triangle CQE \cong \triangle DQE$	24. Postulate 13 - Triangle Congruence - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent. (SAS Congruence Postulate)
25. $\angle QEC \cong \angle QED$	25. C.P.C.T.C.
26. $\angle QEC$ and $\angle QED$ are supplementary angles	26. Theorem 10 - If the exterior sides of two adjacent angles are opposite rays, then the two angles are supplementary.
27. $\angle QEC$ and $\angle QED$ are right angles	27. Corollary 10b - If two angles are supplementary and congruent, then each angle is a right angle.
28. $\overline{AB} \perp \overline{CD}$	28. Definition of Perpendicular Lines (Segments)
29. $\overline{CE} \cong \overline{DE}$	29. C.P.C.T.C.
30. Point E is the midpoint of $\overline{CD}$	30. Definition of Midpoint of a Line Segment
31. $\overline{AB}$ is the bisector of $\overline{CD}$	31. Definition of Bisector of a Line Segment. Q.E.D.

4. A line segment is the perpendicular bisector of chord of a circle if and only if the line segment joins the midpoints of the two arcs determined by the chord.
5. $\overline{PR}$
6. Point P
7. Point Q
8. $\overset{\frown}{RX}$
9. $\overset{\frown}{TY}$
10. $\overline{XY}$
11. $\overline{QU}$
12. $\overline{XY}$
13. No
14. x = 9 Theorem 73
15. x = 34° Corollary 73a

16.
$$(BX)^2 = (BC)^2 + (CX)^2$$
$$(7)^2 = (4)^2 + (CX)^2$$
$$49 = 16 + (CX)^2$$
$$33 = (CX)^2$$
$$\pm\sqrt{33} = CX$$
(CX cannot be negative)
$$\sqrt{33} = CX$$

$$CX + CY = XY$$
$$CX = CY$$
$$\sqrt{33} + \sqrt{33} = XY$$
$$2\sqrt{33} = XY$$

17. Draw radius $\overline{QD}$

QD = QA

QD = 13

$$DM = \frac{1}{2} \cdot DC$$
$$DM = \frac{1}{2} \cdot 24$$
$$DM = 12$$

$$(QD)^2 = (DM)^2 + (QM)^2$$
$$(13)^2 = (12)^2 + (QM)^2$$
$$169 = 144 + (QM)^2$$
$$25 = (QM)^2$$
$$\pm\sqrt{25} = QM \text{ (QM cannot be negative)}$$
$$5 = QM$$

18. Draw radius $\overline{QY}$
$$CY = \frac{1}{2} \cdot XY$$
$$CY = \frac{1}{2} \cdot 18$$
$$CY = 9$$

$$(QY)^2 = (QC)^2 + (CY)^2$$
$$(QY)^2 = (9)^2 + (9)^2$$
$$(QY)^2 = 81 + 81$$
$$(QY)^2 = 162$$
$$QY = \pm\sqrt{162} \text{ (QY cannot be negative)}$$
$$QY = \sqrt{81} \cdot \sqrt{2}$$
$$QY = 9\sqrt{2}$$

19.
$$m\overset{\frown}{UW} = 360 - 288$$
$$= 72$$

$$m\overset{\frown}{VW} = \frac{1}{2} \cdot m\overset{\frown}{UW}$$
$$= \frac{1}{2} \cdot 72$$
$$= \frac{1 \cdot \cancel{2} \cdot 36}{\cancel{2}}$$
$$= 36$$

$$m\angle VQW = m\overset{\frown}{VW}$$
$$m\angle VQW = 36$$

20. BX = 7 Theorem 73

DY = 8 Theorem 73

$$m\overset{\frown}{BE} = 58$$
$$m\overset{\frown}{BE} = m\overset{\frown}{AE}$$
$$m\overset{\frown}{BF} = 180 - \left(m\overset{\frown}{BE} + m\overset{\frown}{AE}\right)$$
$$= 180 - (58 + 58)$$
$$= 180 - 116$$
$$= 64$$

21.
$$m\overset{\frown}{BH} = 180 - m\overset{\frown}{GB}$$
$$= 180 - 140 \quad mAD = 64$$
$$= 40$$

$$m\overset{\frown}{BH} = m\overset{\frown}{AH} \qquad \text{Corollary 73a}$$
$$m\overset{\frown}{AB} = m\overset{\frown}{BH} + m\overset{\frown}{AH}$$
$$m\overset{\frown}{AB} = 40 + 40$$
$$m\overset{\frown}{AB} = 80$$

If AB = 18, then HB = 9 (Corollary 73a)

QI = 12 (Given)

Draw $\overline{QB}$

$$(QB)^2 = (IB)^2 + (QI)^2$$
$$(QB)^2 = (9)^2 + (12)^2$$
$$(QB)^2 = 81 + 144$$
$$(QB)^2 = 225$$
$$QB = 15$$

$$(QB)^2 = (JC)^2 + (QJ)^2$$
$$(15)^2 = (JC)^2 + (10)^2$$
$$225 = (JC)^2 + 100$$
$$125 = (JC)^2$$
$$\pm\sqrt{125} = JC \text{ (JC cannot be negative)}$$
$$\sqrt{25} \cdot \sqrt{5} = JC$$
$$5\sqrt{5} = JC$$

So, QB = 15

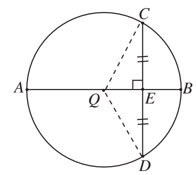
$$CD = 2 \cdot JC = 2 \cdot 5\sqrt{5} = 10\sqrt{5}$$

Unit VI — Circles

Part C — Line and Segment Relationships

p. 560 – Lesson 2 — Theorem 74 & 75

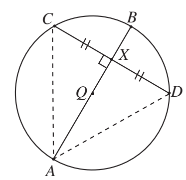
1. Theorem 74 - "If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord."



Given: $\overline{AB}$ is a diameter of ⊙Q; $\overline{AB}$ bisects chord $\overline{CD}$
Prove: $\overline{AB} \perp \overline{CD}$

STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of ⊙Q	1. Given
2. $\overline{AB}$ bisects chord $\overline{CD}$	2. Given
3. Point E is the midpoint of $\overline{CD}$	3. Definition of Line Segment Bisector
4. $\overline{CE} \cong \overline{DE}$	4. Definition of Midpoint
5. $\overline{QE} \cong \overline{QE}$	5. Reflexive Property for Congruent Line Segments
6. Draw $\overline{QC}$	6. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
7. Draw $\overline{QD}$	7. Postulate 2
8. $\overline{QC} \cong \overline{QD}$	8. Radii of the same circle are congruent
9. △QEC ≅ △QED	9. Postulate 13 - Triangle Congruence - "If the three sides of one triangle are congruent to the three sides of another triangle, then the two triangles are congruent." (SSS Congruence Assumption)
10. ∠QEC ≅ ∠QED	10. C.P.C.T.C.
11. $\overline{AB} \perp \overline{CD}$	11. Corollary 10d - "If two congruent angles form a linear pair, then the intersecting lines forming the angles are perpendicular."

2. Theorem 75 - "If a chord of a circle is a perpendicular bisector of another chord of that circle, then the original chord must be a diameter of the circle."



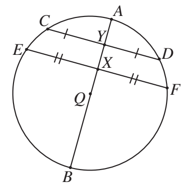
Given: Chord $\overline{AB}$ bisects chord $\overline{CD}$ at point X.
$\overline{AB} \perp \overline{CD}$
Prove: $\overline{AB}$ is a diameter of the circle.

STATEMENT	REASONS
1. Chord $\overline{AB}$ bisects chord $\overline{CD}$ at point X	1. Given
2. $\overline{AB} \perp \overline{CD}$	2. Given
3. Assume $\overline{AB}$ is not a diameter of ⊙Q	3. Indirect Proof Assumption
4. There must exist a chord $\overline{EF}$ through point Q, the center of the circle, and point X, the midpoint of chord $\overline{CD}$	4. Postulate 2 - For any two different points, there is exactly one line containing them.
5. $\overline{EF}$ is a diameter	5. Definition of Diameter - A line segment which is a chord of a circle, and passes through the center of that circle.
6. $\overline{EF} \perp \overline{CD}$	6. Theorem 74 - If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord.
7. However, $\overline{AB} \perp \overline{CD}$ at point X	7. Given
8. $\overline{EF}$ and $\overline{AB}$ cannot both be perpendicular to $\overline{CD}$ at point X	8. Theorem 6 - If, in a plane, there is a point on a line, then there is exactly one perpendicular to the line through that point.
9. Our Assumption must be false. $\overline{AB}$ must be a diameter	9. Reductio Ad Absurdum

3. $PN = \frac{1}{2} \cdot 12$
$PN = 6$

$(QN)^2 = (PQ)^2 + (PN)^2$
$(QN)^2 = (3)^2 + (6)^2$
$(QN)^2 = 9 + 36$
$(QN)^2 = 45$
$QN = \pm\sqrt{45}$ (QN cannot be negative)
$= \sqrt{9} \cdot \sqrt{5}$
$= 3\sqrt{5}$

4. AB = 18 Theorem 75
If a chord ($\overline{AB}$) of a circle is a perpendicular bisector of another chord ($\overline{MN}$) of that circle, then the original chord must be a diameter of the circle.



15. Given: $\overline{AB}$ is a diameter of ⊙Q
$\overline{AB}$ bisects $\overline{CD}$; $\overline{AB}$ bisects $\overline{EF}$
Prove: $\overline{CD} \parallel \overline{EF}$

STATEMENT	REASONS
1. $\overline{AB}$ is a diameter of ⊙Q	1. Given
2. $\overline{AB}$ bisects $\overline{CD}$	2. Given
3. AB ⊥ CD	3. Theorem 74 - If a diameter of a circle bisects a chord that is not a diameter, then that diameter is perpendicular to that chord.
4. $\overline{AB}$ bisects $\overline{EF}$	4. Given
5. $\overline{AB} \perp \overline{EF}$	5. Theorem 74
6. CD ∥ EF	6. Theorem 22 - If two lines are perpendicular to a third line, then the two lines are parallel.

16. Two chords of a circle that are not diameters are parallel to each other if and only if a diameter of the circle bisects the two chords.

17. Proof of Theorem 75

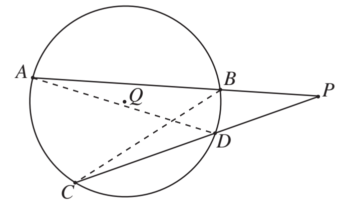
STATEMENT	REASONS
1. Chord $\overline{AB}$ bisects chord $\overline{CD}$ at point E	1. Given
2. Point E is the midpoint of $\overline{CD}$	2. Definition of Line Segment Bisector
3. $\overline{CE} \cong \overline{DE}$ at point E	3. Definition of Midpoint
4. AB ⊥ CD	4. Given
5. ∠AEC is a right angle	5. Definition of Perpendicular Lines (Segments)
6. ∠AED is a right angle	6. Definition of Perpendicular Lines (Segments)
7. ∠AEC ≅ ∠AED	7. Theorem 11 - If you have right angles, then those right angles are congruent.
8. $\overline{AE} \cong \overline{AE}$	8. Reflexive Property for Congruent Line Segments
9. Draw AC	9. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
10. Draw $\overline{AD}$	10. Postulate 2
11. △AEC ≅ △AED	11. Postulate 13 - Triangle Congruence - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle then the two triangles are congruent. (SAS Congruence Assumption)
12. ∠CAB ≅ ∠DAB	12. C.P.C.T.C.
13. $m\angle CAB = \frac{1}{2} m\overset{\frown}{BC}$	13. Theorem 67 - If you have an inscribed angle of a circle, then the measure of the angle is one-half the measure of the intercepted arc.
14. $m\angle DAB = \frac{1}{2} m\overset{\frown}{BD}$	14. Theorem 67
15. $m\angle CAB = m\angle DAB$	15. Definition of Congruent Angles
16. $\frac{1}{2} m\overset{\frown}{BC} = \frac{1}{2} m\overset{\frown}{BD}$	16. Substitution
17. $m\overset{\frown}{BC} = m\overset{\frown}{BD}$	17. Multiplication Property of Equality
18. ∠ACD ≅ ∠ADC	18. C.P.C.T.C.
19. $m\angle ACD = \frac{1}{2} m\overset{\frown}{DA}$	19. Theorem 67
20. $m\angle ADC = \frac{1}{2} m\overset{\frown}{AC}$	20. Theorem 67
21. $m\angle ACD = m\angle ADC$	21. Definition of Congruent Angles
22. $\frac{1}{2} m\overset{\frown}{DA} = \frac{1}{2} m\overset{\frown}{AC}$	22. Substitution
23. $m\overset{\frown}{DA} = m\overset{\frown}{AC}$	23. Multiplication Property of Equality
24. $m\overset{\frown}{BC} + m\overset{\frown}{BD} + m\overset{\frown}{DA} + m\overset{\frown}{AC} = 360$	24. Postulate 8 - First Assumption - The set of all points on a circle can be put into a one-to-one correspondence with the real numbers from 0 to 360, inclusive, with the exception of any one point which may be paired with 0 and 360.
25. $m\overset{\frown}{BC} + m\overset{\frown}{BC} + m\overset{\frown}{AC} + m\overset{\frown}{AC} = 360$	25. Substitution
26. $2m\overset{\frown}{BC} + 2m\overset{\frown}{AC} = 360$	26. Properties of Algebra - Collect Like Terms
27. $m\overset{\frown}{BC} + m\overset{\frown}{AC} = 180$	27. Multiplication Property of Equality
28. $m\overset{\frown}{BC} + m\overset{\frown}{AC} = m\overset{\frown}{ACB}$	28. Postulate 8 - Fourth Assumption - Arc Addition Assumption
29. $m\overset{\frown}{ACB} = 180$	29. Substitution
30. $\overset{\frown}{ACB}$ is a semicircle	30. Definition of Semicircle - "...a semicircle is the intercepted arc of a central angle of 180°"
31. $\overline{AB}$ is a diameter of the circle	31. Definition of Semicircle - An arc of a circle is a semicircle, if and only if, it is an arc whose endpoints are the endpoints of a diameter of a circle.

Unit VI — Circles

Part C — Line and Segment Relationships

p. 568 – Lesson 4 — Theorem 77 & 78

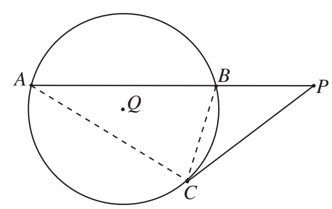
1. Theorem 77 - "If two secant segments are drawn to a circle from a single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment."



Given: $\overline{PA}$ and $\overline{PC}$ are secants of ⊙Q.
Prove: AP · BP = CP · DP

STATEMENT	REASONS
1. $\overline{PA}$ and $\overline{PC}$ are secants of ⊙Q	1. Given
2. Draw $\overline{AD}$	2. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
3. Draw $\overline{CB}$	3. Postulate 2
4. ∠A ≅ ∠C	4. Corollary 67c - If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.
5. ∠P ≅ ∠P	5. Reflexive Property of Angle Congruence
6. △APD ~ △CPB	6. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar. (AA)
7. $\frac{AP}{CP} = \frac{DP}{BP}$	7. Definition of Similarity - Two polygons are similar if and only if corresponding angles are congruent and corresponding sides are in proportion.
8. AP · BP = CP · DP	8. Multiplication Property of Equality (Multiply both sides by CP · PB)

2. Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle from a single point outside the circle, then the length of the tangent segment is the mean proportional between the length of the secant segment and its external segment."



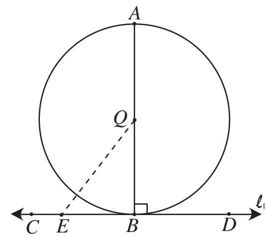
Given: $\overline{PA}$ is a secant segment of ⊙Q.
$\overline{PC}$ is a tangent segment to ⊙Q.
Prove: $\frac{AP}{CP} = \frac{DP}{BP}$ ($\overline{CP}$ mean proportional between $\overline{AP}$ and $\overline{BP}$.)

STATEMENT	REASONS
1. $\overline{PA}$ is a secant segment of ⊙Q.	1. Given
2. $\overline{PC}$ is a tangent segment to ⊙Q.	2. Given
3. Draw chord AC	3. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
4. Draw chord $\overline{BC}$	4. Postulate 2
5. m∠PCB = $\frac{1}{2}$ m$\overset{\frown}{BC}$	5. Theorem 68 - If you have an angle formed by a secant and a tangent at the point of tangency, then the measure of that angle is one-half the measure of its intercepted arc.
6. m∠PAC = $\frac{1}{2}$ m$\overset{\frown}{BC}$	6. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one-half the measure of the intercepted arc.
7. m∠PCB = m∠PAC	7. Substitution
8. ∠PCB ≅ ∠PAC	8. Definition of Congruent Angles
9. ∠P ≅ ∠P	9. Reflexive Property for Congruent Angles
10. △APC ~ △CPB	10. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar. (AA)
11. $\frac{AP}{CP} = \frac{DP}{BP}$	11. Definition of Similarity - Two polygons are similar if and only if corresponding angles are congruent and corresponding sides are in proportion.
12. $\overline{CP}$ is the mean proportional between $\overline{AP}$ and $\overline{BP}$	12. Definition of Mean Proportional

Unit VI — Circles

Part C — Line and Segment Relationships

p. 571 – Lesson 5 — Theorem 79



1. Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

Given: ⊙Q with diameter $\overline{AB}$.
$\overleftrightarrow{CD} \perp \overline{AB}$ at point B
Prove: $\overleftrightarrow{CD}$ is tangent to ⊙Q.

STATEMENT	REASONS
1. ⊙Q with diameter $\overline{AB}$	1. Given
2. $\overleftrightarrow{CD} \perp \overline{AB}$ at point B	2. Given
3. Choose any point E on $\overleftrightarrow{CD}$ and draw $\overline{QE}$.	3. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
4. QB < QE	4. A segment is the shortest segment from a point to a line if and only it it is the segment perpendicular to the line.
5. Point E lies in the exterior of ⊙Q	5. If a point is in the exterior of a circle, then the measure of the segment joining the point to the center of the circle is greater than the measure of the radius.
6. $\overleftrightarrow{CD}$ is tangent to ⊙Q at point B	6. Definition of Tangent

2. Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

Given: Line *m* is perpendicular to $\overline{PA}$ at point A.
Prove: Line *m* is tangent to ⊙Q.

Proof: Let point B be any point on line *m* other than point A. SInce $\overline{PA} \perp m$, △QAB is a right triangle with hypotenuse $\overline{QB}$. This means QB > QA and point B must be in the exterior of ⊙Q. Therefore, point B cannot lie on the circle and point A is the only point on line *m* that is on the circle. It follows that line *m* is tangent to the circle.

3. 90 degrees

4. Tangent; Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

5. 6

6. 42 degrees

7. $3\sqrt{7}$

$$\frac{3}{FG} = \frac{FG}{21}$$
$$(FG)^2 = 63$$
$$FG = \pm\sqrt{63} \text{ (FG cannot be negative)}$$
$$FG = \sqrt{9 \cdot 7}$$
$$FG = 3\sqrt{7}$$

8. 35

$$JP + PI + IF + FH + HQ + QK$$
$$6 + 6 + 2 + 3 + 9 + 9$$
$$12 + 5 + 18$$
$$35$$

9. 48 degrees

10. Tangent; Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

11. $2\sqrt{7}$

$$\frac{FE}{2} = \frac{14}{FE}$$
$$FE^2 = 28$$
$$FE = \pm\sqrt{28} \text{ (FE cannot be negative)}$$
$$FE = \sqrt{4 \cdot 7} \text{ or } 2\sqrt{7}$$

12. $\sqrt{391}$ Draw $\overline{PM} \perp$ to $\overline{QC}$ forming right $\triangle PMQ$. $\overline{PM} \cong \overline{DC}$.

$$(PM)^2 + (MQ)^2 = (PQ)^2$$
$$(PM)^2 + (9-6)^2 = (6+2+3+9)^2$$
$$(PM)^2 + (3)^2 = (20)^2$$
$$(PM)^2 + 9 = 400$$
$$(PM)^2 = 391$$
$$PM = \pm\sqrt{391} \text{ (PM cannot be negative)}$$
$$PM = \sqrt{391}$$

13. 42 degrees

14. 8 6 + 2

15. P; Q Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line is tangent to the circle at that endpoint."

16. In the plane of ⊙Q, the line *m* is tangent to ⊙Q at point A, if and only if, the line *m* is perpendicular to diameter $\overline{PA}$ at point A.

5. $\overline{PX} \cong \overline{PY}$ (Theorem 80)

△PYX is isosceles by definition of an isosceles triangle (two sides are congruent).

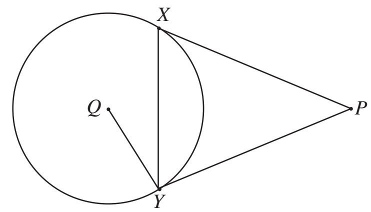
∠PYZ ≅ ∠PXY (Theorem 33)

$\overline{QY} \perp \overline{PY}$ (Corollary 68a)

∠QYP is a right angle, so m∠QYP = 90.

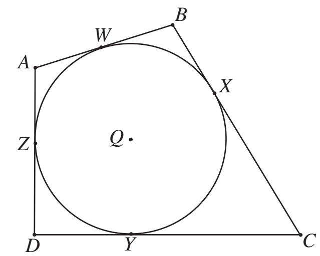
SInce m∠XYQ = 10 degrees, m∠XYP = 80 degrees. (90 - 10 = 80)

The sum of the measures of the angles of a triangle is 180.



m∠XYP + m∠YXP + m∠P = 180
80 + 80 + m∠P = 180
160 + m∠P = 180
m∠P = 20

6. Label the points of tangency W, X, Y, and Z.



Part 1:
AW = AZ, BW = BX, CX = CY and DY = DZ (Theorem 80)
AB = AW + BW and DC = DY + CY.
AB + DC = AW + BW + DY + CY

Part 2:
AD = AZ + DZ and BC = BX + CX
AD + BC = AZ + DZ + BX + CX
Substituting AW for AZ, BW for BX, DY for DZ, and CY for CX, we have
AD + BC = AW + DY + BW + CY
Using the Commutative Property for Addition, we have
AD +BC = AW + BW + DY + CY
Therefore, AB + DC = AD + BC

7. RX = RZ and TY = TZ (Theorem 80)
PX = PR + RX
PY = PT + TY
PX + PY = PR + RX + PT + TY
RT = RZ + TZ
RT = RX +TY (Substituting RX for RZ and TY for TZ)
Since PX + PY = PR + RX + TY + PT (Using the Commutative Property of Addition)
we can substitute RT for RX +TY to get
PX + PY = PR + RT + PT

8. $\overline{QB} \perp \overline{AB}$ and $\overline{QC} \perp \overline{AC}$ (Corollary 68a)
∠QBA and ∠QCA are right angles.

m∠QBA = m∠QCA = 90
m∠QBA + m∠BQA + y = 180
90 + 80 + y = 180
170 + y = 180
y = 10

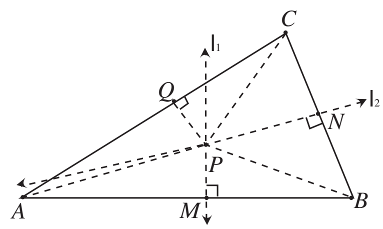
Therefore, W = 10 (Corollary 80a)
m∠QCA + w + z = 180
90 + 10 + z = 180
100 + z = 180
z = 80

m∠BQD = 100 (180 – 80 = 100)
x = 100 (measure of an arc is the same as the measure of its central angle)

Unit VI — Circles

Part D — Circles Concurrency

p. 581 – Lesson 1 — Theorem 83



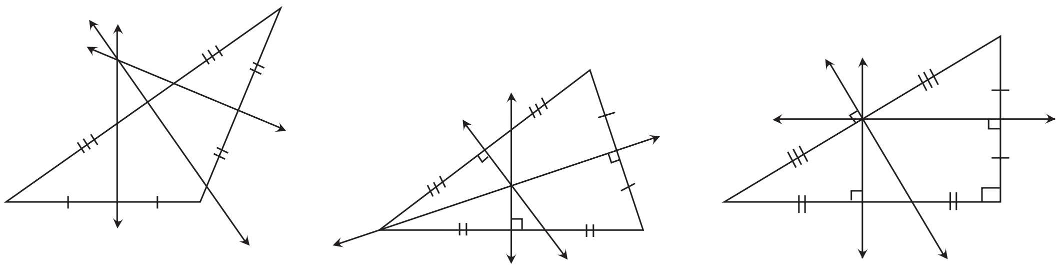
1.

Given: △ABC; $\overline{AB}$ is a chord of some circle

Prove: △ABC is cyclic

STATEMENT	REASONS
1. △ABC	1. Given
2. $\overline{AB}$ is a chord of some circle.	2. Given
3. Locate point M on $\overline{AB}$ as the midpoint of $\overline{AB}$.	3. Theorem 4 - If you have a given line segment, then that segment has exactly one midpoint.
4. Draw ℓ_1 perpendicular to $\overline{AB}$ at point M.	4. Theorem 6 - If, in a plane, there is a point on a line, then there is exactly one perpendicular to the line through that point.
5. ℓ_1 contains a diameter of the circle which has $\overline{AB}$ as a chord.	5. Theorem 75 - If a chord of a circle (ℓ_1) is a perpendicular bisector of another chord ($\overline{AB}$) of that circle, then the original chord (ℓ_1) must be a diameter of the circle.
6. $_1$ must pass through the center of the circle.	6. Definition of Diameter
7. Locate point N on $\overline{BC}$ as the midpoint of $\overline{BC}$.	7. Theorem 4
8. Draw ℓ_2 perpendicular to $\overline{BC}$ at point N.	8. Theorem 6
9. Call the intersection of ℓ_1 and ℓ_2 point P.	9. Postulate 5 - If two different lines intersect, the intersection is a unique point.
10. Draw $\overline{PA}$.	10. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
11. Draw $\overline{PB}$.	11. Postulate 2
12. Draw $\overline{PC}$.	12. Postulate 2
13. ∠PMB is a right angle.	13. Definition of Perpendicular Lines
14. △PMB is a right triangle.	14. Definition of Right Triangle
15. ∠PMA is a right angle.	15. Definition of Perpendicular Lines
16. △PMA is a right triangle.	16. Definition of Right Triangle
17. $\overline{MA} \cong \overline{MB}$	17. Definition of Midpoint of a Line Segment
18. $\overline{PM} \cong \overline{PM}$	18. Reflexive Property for Congruent Line Segments
19. △PMA ≅ △PMB	19. Postulate Corollary 13b - If the two legs of a right triangle are congruent to the two legs of another right triangle, then the two right triangles are congruent. (LL)
20. $\overline{PA} \cong \overline{PB}$	20. C.P.C.T.C.
21. ∠PNB is a right angle.	21. Definition of Perpendicular Lines
22. △PNB is a right triangle.	22. Definition of Right Triangle
23. $\overline{NC} \cong \overline{NB}$	23. Definition of Midpoint of a Line Segment
24. $\overline{PN} \cong \overline{PN}$	24. Reflexive Property for Congruent Line Segments
25. △PNB = △PNC	25. Postulate Corollary 13b
26. $\overline{PB} \cong \overline{PC}$	26. C.P.C.T.C.
27. PA = PB	27. Definition of Congruent Line Segments
28. PB = PC	28. Definition of Congruent Line Segments
29. PA = PB = PC	29. Transitive Property of Equality
30. △ABC is cyclic.	30. Using $\overline{PA}$, $\overline{PB}$ and $\overline{PC}$ as radii, draw a circle with point P as the center passing through points A, B and C. (Q.E.D.)

14.



15. MB = MA (Corollary 83a)

In $\triangle$APM, $(AP)^2 + (MP)^2 = (MA)^2$

$$(AP)^2 + (12)^2 = (13)^2$$

$$(AP)^2 + 144 = 169$$

$$(AP)^2 = 25$$

$$AP = \pm\sqrt{25} \quad \text{(AP cannot be negative)}$$

$$AP = 5$$

Since $\overline{MP}$ bisects $\overline{AC}$ and AC = AP + PC, then AC = 5 + 5 = 10

16. Always

17. Always

18. Never

19. Sometimes

20. Always (Recall theorem 67. So, the right angle must intercept a semicircle. This leads to the hypotenuse being the diameter of the circle.)

21. 180 – 34 = 146

$$m\angle YXZ + m\angle YZX = 68$$

$$\frac{1}{2}(m\angle YXZ + m\angle YZX) = 34$$

$$m\angle PXZ + m\angle PZX = 34$$

22. The midpoint of a line segment is given by: $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

The midpoint of $\overline{AB}$: $\left(\frac{0+16}{2}, \frac{0+0}{2}\right)$, $\left(\frac{16}{2}, \frac{0}{2}\right)$, $(8, 0)$

The midpoint of $\overline{BC}$: $\left(\frac{16+12}{2}, \frac{0+8}{2}\right)$, $\left(\frac{28}{2}, \frac{8}{2}\right)$, $(14, 4)$

The midpoint of $\overline{AC}$: $\left(\frac{0+12}{2}, \frac{0+8}{2}\right)$, $\left(\frac{12}{2}, \frac{8}{2}\right)$, $(6, 4)$

Slope of $\overline{AB}$: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0-0}{16-0} = \frac{0}{16} = 0$

Slope of line perpendicular to $\overline{AB}$: Undefined

Slope of $\overline{BC}$: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{12-16} = \frac{8}{-4} = -2$

Slope of line perpendicular to $\overline{BC}$: $\frac{1}{2}$

Slope of $\overline{AC}$: $\frac{y_2 - y_1}{x_2 - x_1} = \frac{8-0}{12-0} = \frac{8}{12} = \frac{\cancel{4} \cdot 2}{\cancel{4} \cdot 3} = \frac{2}{3}$

Slope of line perpendicular to $\overline{AC}$: $-\frac{3}{2}$

10. The inscribed square is a rhombus, so diagonal $\overline{AC}$ bisects $\angle BAD$. $m\angle BAD = 90$, so $m\angle BAC = 45$.
$\triangle AEQ$ is therefore a 45-45-90 triangle. Let AE and QE equal x. AQ is then $x\sqrt{2}$.

$$\frac{QE}{QA} = \frac{x}{x\sqrt{2}} = \frac{x\cdot\sqrt{2}}{x\sqrt{2}\cdot\sqrt{2}} = \frac{x\sqrt{2}}{x\cdot 2} = \frac{\sqrt{2}}{2}$$

11. **a)** 120
b) 90
c) 60
d) 45

12.

STATEMENT	REASONS
1. Quadrilateral XYWZ is cyclic.	1. Given
2. $\overline{ZY}$ is a diameter of ⊙Q.	2. Given
3. $\overline{XY}$ II $\overline{ZW}$	3. Given
4. $\angle XYZ \cong \angle WZY$	4. Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are conruent.
5. $\angle ZXY$ is a right angle.	5. Corollary 67a - If you have an angle inscribed in a semicircle, then that angle must be a right angle.
6. $\triangle ZXY$ is a right triangle.	6. Definition of Right Triangle
7. $\angle YWZ$ is a right angle.	7. Corollary 67a
8. $\triangle YWZ$ is a right triangle.	8. Definition of Right Triangle
9. $\triangle ZXY \cong \triangle YWZ$	9. Postulate Corollary 13e - If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and an acute angle of another right triangle, then the two right triangles are congruent. (HA)
10. $\overline{XZ} \cong \overline{WY}$	10. C.P.C.T.C.
11. $\overset{\frown}{XZ} \cong \overset{\frown}{WY}$	11. Theorem 81 - If two chords of a circle are congruent, then their minor arcs are congruent.
12. $\overline{XY} \cong \overline{WZ}$	12. C.P.C.T.C.
13. $\overset{\frown}{XY} \cong \overset{\frown}{WZ}$	13. Theorem 81 - If two chords of a circle are congruent, then their minor arcs are congruent.

13. **a)** $m\angle AQB = 90$

b)
$$(AQ)^2 + (BQ)^2 = (AB)^2$$
$$(10)^2 + (10)^2 = (AB)^2$$
$$100 + 100 = (AB)^2$$
$$200 = (AB)^2$$
$$\pm\sqrt{200} = AB \quad \text{(AB cannot be negative)}$$
$$\sqrt{200} = AB$$
$$\sqrt{100\cdot 2} = AB$$
$$10\sqrt{2} = AB$$

c) Distance from point Q to AB $= \frac{1}{2}\cdot AB$
$$= \frac{1}{2}\cdot 10\sqrt{2}$$
$$= 5\sqrt{2}$$