Geometry: A Complete Course

(with Trigonometry)

Module D – Solutions Manual

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VideoText Interactive

Geometry: A Complete Course (with Trigonometry)
Module D–Solutions Manual

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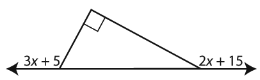
14. $2x + 25 = 180 - (3x - 5) + 100$
$2x + 25 = 280 - 3x + 5$
$5x = 260$
$x = 52$

15. m∠ADC = 180 – 76 – 28
m∠ADC = 76
m∠DCB = 90 – 28
m∠DCB = 62
m∠ADC = m∠DCB + m∠B
$76 = 62 + x°$
$14 = x°$ or m∠A + m∠C + m∠B = 180
$76 + 90 + x° = 180$
$x = 14$

16. 

$180 - (3x + 5) + 180 - (2x + 15) = 90$
$180 - 3x - 5 + 180 - 2x - 15 = 90$
$360 - 5x - 20 = 90$
$-5x = -250$
$x = 50$

The acute angles are: 180 - (3x + 5)
180 - 155 = 25
180 - (2x + 15)
180 - 115 = 65

17. $x + 2x + 3x = 180$
$6x = 180$
$x = 30$ $2x = 60$ $3x = 90$
Exterior Angles are:
150 120 90
150 : 120 : 90
5 : 4 : 3

18.

Statement	Reason
1. ∠A ≅ ∠D	**1.** Given
2. ∠B ≅ ∠E	**2.** Given
3. m∠A = m∠D	**3.** Definition of Congruent Angles
4. m∠B = m∠E	**4.** Definition of Congruent Angles
5. m∠A + m∠B + m∠C = 180	**5.** Theorem 25: If you have any given triangle, then the sum of the measures of its angles is 180.
6. m∠D + m∠E + m∠F = 180	**6.** Theorem 25
7. m∠A + m∠B + m∠C = m∠D + m∠E + m∠F	**7.** Substitution
8. m∠C = m∠F	**8.** Subtraction Property of Equality
9. ∠C ≅ ∠F	**9.** Definition of Congruent Angles – Q. E. D.

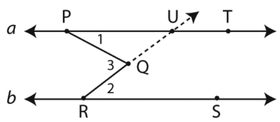
19.

Statement	Reason
1. $\overline{CE}$ bisects ∠BCD	**1.** Given
2. ∠DCE ≅ ∠ECB	**2.** Definition of Angle Bisector
3. m∠DCE = m∠ECB	**3.** Definition of Congruent Angles
4. m∠DCB = m∠A + m∠B	**4.** Theorem 26: If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
5. m∠DCB = m∠DCE + m∠ECB	**5.** Postulate 7 – Protractor Postulate: Angle Addition Assumption (Fourth)
6. m∠DCB = m∠ECB + m∠ECB	**6.** Substitution
7. m∠DCB = 2 • m∠ECB	**7.** Properties of Algebra
8. 2 • m∠ECB = m∠A + m∠B	**8.** Substitution
9. ∠A ≅ ∠B	**9.** Given
10. m∠A = m∠B	**10.** Definition of Congruent Angles
11. 2 • m∠ECB = m∠B + m∠B	**11.** Substitution
12. 2 • m∠ECB = 2 • m∠B	**12.** Properties of Algebra
13. m∠ECB = m∠B	**13.** Multiplication Property for Equations
14. ∠ECB ≅ ∠B	**14.** Definition of Congruent Angles
15. $\overline{CE} \parallel \overline{AB}$	**15.** Theorem 20: If two lines are cut by a transversal so that alternate interior angles are congruent, then the two lines are parallel. – Q.E.D.

20.

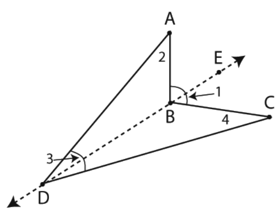
Statement	Reason
1. $\overline{CE} \parallel \overline{AD}$	**1.** Given
2. ∠3 ≅ ∠ADE	**2.** Postulate 11: If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3. m∠3 = m∠ADE	**3.** Definition of Congruent Angles
4. m∠ADE = m∠1 + m∠2	**4.** Theorem 26: If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
5. m∠3 = m∠1 + m∠2	**5.** Substitution

21.



Statement	Reason
1. Extend $\overline{RQ}$ to intersect line *a* at point U and form △PQU	1. Postulate 2: For any 2 points, there is exactly one line containing them.
2. *a* ∥ *b*	**2.** Given
3. ∠2 ≅ ∠PUQ	**3.** Theorem 16: If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
4. m∠2 = m∠PUQ	**4.** Definition of Congruent Angles
5. m∠3 = m∠1 + m∠PUQ	**5.** Theorem 26: If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
6. m∠3 = m∠1 + m∠2	**6.** Substitution - Q.E.D.

22.



Statement	Reason
1. Draw $\overleftrightarrow{DB}$. Label point E on $\overleftrightarrow{DB}$ for convenience in naming angles.	1. Postulate 2: For any 2 points, there is exactly one line containing them.
2. m∠ABE = m∠ADE + m∠2	**2.** Theorem 26: If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
3. m∠CBE = m∠CDE + m∠4	**3.** Theorem 26
4. m∠ABE + m∠CBE = m∠ADE + m∠2 + m∠CDE + m∠4	4. Addition of Equality
5. m∠1 = m∠ABE + m∠CBE	**5.** Postulate 7 – Protractor Postulate: Angle Addition Assumption (Fourth)
6. m∠3 = m∠ADE + m∠CDE	**6.** Postulate 7 – Protractor Postulate: Angle Addition Assumption (Fourth)
7. m∠1 = m∠2 + m∠3 + m∠4	**7.** Substitution – Q.E.D.

4. $\frac{5}{x} = \frac{8}{15}$

$x \cdot 8 = 5 \cdot 15$

$x \cdot 8 \cdot \frac{1}{8} = 75 \cdot \frac{1}{8}$

$x = \frac{75}{8}$ or $9\frac{3}{8}$

5. $\frac{3-x}{x+1} = \frac{2}{1}$

$(x+1) \cdot 2 = (3-x) \cdot 1$

$2x + 2 = 3 - x$

$3x = 1$

$\frac{1}{3} \cdot 3x = 1 \cdot \frac{1}{3}$

$x = \frac{1}{3}$

6. $\frac{x+3}{10} = \frac{3x-2}{8}$

$10(3x-2) = (x+3)8$

$30x - 20 = 8x + 24$

$22x = 44$

$\frac{1}{22} \cdot 22x = 44 \cdot \frac{1}{22}$

$x = 2$

7. $\frac{2x-3}{3} = \frac{3x-7}{2}$

$3 \cdot (3x-7) = (2x-3) \cdot 2$

$9x - 21 = 4x - 6$

$5x = 15$

$\frac{1}{5} \cdot 5x = 15 \cdot \frac{1}{5}$

$x = 3$

8. $\frac{x-3}{2} = \frac{2}{x}$

$2 \cdot 2 = (x-3) \cdot x$

$4 = x^2 - 3x$

$0 = x^2 - 3x - 4$

$0 = (x-4)(x+1)$

$0 = x - 4$ or $0 = x + 1$

$4 = x$ or $-1 = x$
(both answers check!)

9. $\frac{x+2}{6} = \frac{6}{x+2}$

$6 \cdot 6 = (x+2)(x+2)$

$36 = x^2 + 4x + 4$

$0 = x^2 = 4x - 32$

$0 = (x+8)(x-4)$

$0 = x + 8$ or $0 = x - 4$

$-8 = x$ or $4 = x$
(both answers check!)

10. a. $8 \cdot 14 = 7 \cdot 16$ — Means – Extremes Product

b. $\frac{8}{7} = \frac{16}{14}$ — Multiply both sides by $\frac{8 \cdot 16}{7 \cdot 14}$ to get reciprocal of each ratio.

c. $\frac{7}{14} = \frac{8}{16}$ — Multiply both sides by $\frac{8}{14}$, i.e. switch means.

d. $\frac{16}{8} = \frac{14}{7}$ — Multiply both sides by $\frac{16}{7}$, i.e. switch extremes.

e. $\frac{7+8}{8} = \frac{14+16}{16}$ — Denominator addition.

f. $\frac{7-8}{8} = \frac{14-16}{16}$ — Denominator subtraction.

g. $\frac{7}{8} = \frac{7+14}{8+16}$ — Numerator – Denominator Sum

11. a. $y \cdot 6 = x \cdot 7$ — Means – Extremes Product

b. $\frac{y}{x} = \frac{7}{6}$ — Multiply both sides by $\frac{y \cdot 7}{x \cdot 6}$ to get reciprocal of each ratio.

c. $\frac{x}{6} = \frac{y}{7}$ — Multiply both sides by $\frac{y}{6}$, i.e. switch means.

d. $\frac{7}{y} = \frac{6}{x}$ — Multiply both sides by $\frac{7}{x}$, i.e. switch extremes.

e. $\frac{x+y}{y} = \frac{6+7}{7}$ — Denominator addition.

f. $\frac{x-y}{y} = \frac{6-7}{7}$ — Denominator subtraction.

g. $\frac{x}{y} = \frac{x+6}{y+7}$ — Numerator – Denominator Sum

12. a. $x \cdot 19 = 13 \cdot y$ — Means – Extremes Product

b. $\frac{x}{13} = \frac{y}{19}$ — Multiply both sides by $\frac{x \cdot y}{13 \cdot 19}$ to get reciprocal of each ratio.

c. $\frac{13}{19} = \frac{x}{y}$ — Multiply both sides by $\frac{x}{19}$, i.e. switch means.

d. $\frac{y}{x} = \frac{19}{13}$ — Multiply both sides by $\frac{y}{13}$, i.e. switch extremes.

e. $\frac{13+x}{x} = \frac{19+y}{y}$ — Denominator addition.

f. $\frac{13-x}{x} = \frac{19-y}{y}$ — Denominator subtraction.

g. $\frac{13}{x} = \frac{13+19}{x+y}$ — Numerator – Denominator Sum

13. $\frac{b}{y}$ **14.** $\frac{b}{a}$ **15.** $\frac{x}{a}$ **16.** $\frac{u}{v}$ **17.** $\frac{d}{c}$ **18.** $\frac{u+2v}{v}$

19. $\frac{\text{made}}{\text{attempts}} = \frac{117}{156} = \frac{x}{100}$

$156 \cdot x = 117 \cdot 100$

$x = \frac{117 \cdot 100}{156}$

$x = \frac{\not{3} \cdot 3 \cdot \not{13} \cdot \not{2} \cdot \not{2} \cdot 5 \cdot 5}{\not{2} \cdot \not{2} \cdot \not{3} \cdot \not{13}}$

$x = 3 \cdot 25 = 75$

75%

20. $\frac{1 \text{ cement}}{2 \text{ sand}} = \frac{x}{15 \text{ sand}}$

$2 \cdot x = 1 \cdot 15$

$x = \frac{15}{2}$ or $7\frac{1}{2}$ lbs.

21. $\frac{8 \text{ hits}}{9 \text{ games}} = \frac{x}{108 \text{ games}}$

$9\ x = 8 \cdot 108$

$x = \frac{8 \cdot \not{9} \cdot 12}{\not{9}}$

$x = 96$ hits

22. $\frac{2 \text{ butter}}{3 \text{ sugar}} = \frac{x}{4\frac{1}{2} \text{ sugar}}$

$3x = 2 \cdot 4.5$

$x = \frac{2 \cdot 4.5}{3}$ 1.5

$x = 2 \cdot 1.5$

$x = 3$

23. $\frac{9}{x} = \frac{x}{16}$

$x \cdot x = 9 \cdot 16$

$x^2 = 144$

$x = \sqrt{144}$

$x = 12$
(principal root)

24. $\frac{2}{x} = \frac{x}{18}$

$x \cdot x = 2 \cdot 18$

$x^2 = 36$

$x = \sqrt{36}$

$x = 6$
(principal root)

25. $\frac{4}{6} = \frac{10}{x}$

$6 \cdot 10 = 4 \cdot x$

$60 = 4x$

$15 = x$

26. $\frac{\frac{1}{2}}{\frac{1}{3}} = \frac{\frac{1}{4}}{x}$

$\frac{1}{3} \cdot \frac{1}{4} = \frac{1}{2} \cdot x$

$\frac{1}{12} = \frac{1}{2} x$

$\frac{2}{1} \cdot \frac{1}{12} = \frac{2}{1} \cdot \frac{1}{2} x$

$\frac{1}{6} = x$

27. $\frac{2}{7} = \frac{7}{x}$

$7 \cdot 7 = 2 \cdot x$

$49 = 2x$

$\frac{49}{2} = x$

28. $\frac{1}{9} = \frac{9}{x}$

$9 \cdot 9 = 1\,x$

$81 = x$

29. $\frac{\frac{1}{5}}{\frac{1}{11}} = \frac{\frac{1}{11}}{x}$

$\frac{1}{11} \cdot \frac{1}{11} = \frac{1}{5} \cdot x$

$\frac{1}{121} = \frac{1}{5} \cdot x$

$\frac{5}{1} \cdot \frac{1}{121} = \frac{5}{1} \cdot \frac{1}{5} \cdot x$

$\frac{5}{121} = x$

30. $\frac{(a+c+e+g\ldots)}{(b+d+f+h\ldots)}$

31. Always true

32. Sometimes true – Only if terms are added within the ratio itself and not across the equal sign do new ratios form a proportion.

33. No. $\frac{a}{a+b} \neq \frac{c+b}{d}$

$\frac{3}{9} = \frac{4}{12}$

$\frac{3+9}{9} \neq \frac{4+9}{12}$

11. $\triangle CEF \sim \triangle AED$

To find x or CE

$$\frac{CE}{AE} = \frac{CF}{AD}$$

$$\frac{x}{17} = \frac{5}{21}$$

$$17 \bullet 5 = x \bullet 21$$

$$85 = 21 \bullet x$$

$$\frac{85}{21} = x$$

To find Y or EF

$$\frac{EF}{ED} = \frac{CF}{AD}$$

$$\frac{y}{14} = \frac{5}{21}$$

$$14 \bullet 5 = y \bullet 21$$

$$\frac{14 \bullet 5}{21} = y$$

$$\frac{2 \bullet \cancel{7} \bullet 5}{3 \bullet \cancel{7}} = \frac{10}{3} = y$$

12. $\triangle DEF \sim \triangle BCF$

To find x or DF

$$\frac{DF}{BF} = \frac{DE}{BC}$$

$$\frac{x}{12} = \frac{8}{18}$$

$$12 \bullet 8 = x \bullet 18$$

$$\frac{12 \bullet 8}{18} = \frac{\cancel{2} \bullet 2 \bullet \cancel{3} \bullet 2 \bullet 2 \bullet 2}{\cancel{2} \bullet \cancel{3} \bullet 3} = x$$

$$\frac{16}{3} = x$$

To find Y or CF

$$\frac{CF}{EF} = \frac{DE}{BC}$$

$$\frac{y}{7} = \frac{18}{8}$$

$$7 \bullet 18 = y \bullet 8$$

$$\frac{7 \bullet 18}{8} = y$$

$$\frac{7 \bullet \cancel{2} \bullet 3 \bullet 3}{\cancel{2} \bullet 2 \bullet 2} = y$$

$$\frac{63}{4} = y$$

15.

Statement	Reason
1. $\triangle DEF$ with altitudes $\overline{BE}$ and $\overline{AD}$	**1.** Given
2. $\overline{BE} \perp \overline{AC}$ $\overline{AD} \perp \overline{BC}$	**2.** Definition of Altitude
3. $\angle ADC$ is a right angle $\angle BEC$ is a right angle	**3.** Definition of perpendicular
4. $\triangle ADC$ is a right triangle $\triangle BEC$ is a right triangle	**4.** Definition of right triangle
5. $\angle C \cong \angle C$	**5.** Reflexive property for congruent angles
6. $\triangle ADC \sim \triangle BEC$	**6.** Postulate corollary 12 - b - If an acute angle of one right angle is congruent to an acute angle of another right triangle, then the triangles are similar.
7. $\frac{AD}{BE} = \frac{AC}{BC}$	**7.** The measures of corresponding sides of similar triangles are proportional.
8. $\frac{AD}{AC} = \frac{BE}{BC}$	**8.** Multiplication property for equations (multiply both sides by $\frac{BE}{AC}$ (i. e. , switch the means)

16.

Statement	Reason
1. $\overline{AB} \parallel \overline{ED}$	**1.** Given
2. $\angle A \cong \angle E$ $\angle B \cong \angle D$	**2.** Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
3. $\triangle ABC \sim \triangle EDC$	**3.** Postulate Corollary 12 a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar.
4. $\frac{AB}{ED} = \frac{BC}{DC}$	**4.** The measures of corresponding sides of similar triangles are proportional.

17.

Statement	Reason
1. $\overline{AB} \parallel \overline{DE}$; $\overline{BC} \parallel \overline{EF}$	**1.** Given
2. $\angle BAC \cong \angle EDF$ $\angle ACB \cong \angle DFE$	**2.** Postulate 11 - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
3. $\triangle ABC \sim \triangle DEF$	**3.** Postulate Corollary 12 a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar.
4. $\frac{AB}{DE} = \frac{AC}{DF}$	**4.** If two polygons are similar, then corresponding sides are proportional.
5. (DE)(AC) = (AB)(DF)	**5.** Means - Extremes product for proportions - Q. E. D.

25. **a)**

$$\frac{138}{69} = \frac{144}{z}$$

$$69 \bullet 144 = 138 \bullet z$$

$$\frac{69 \bullet 144}{138} = z$$

$$\frac{\cancel{3} \bullet \cancel{23} \bullet \cancel{2} \bullet 2 \bullet 3 \bullet 2 \bullet 2 \bullet 3}{\cancel{2} \bullet \cancel{3} \bullet \cancel{23}} = z$$

$$72 = z$$

b)

$$\frac{72}{135} = \frac{69}{y}$$

$$135 \bullet 69 = 72 \bullet y$$

$$\frac{135 \bullet 69}{72} = y$$

$$\frac{\cancel{3} \bullet \cancel{3} \bullet 3 \bullet 5 \bullet 3 \bullet 23}{\cancel{3} \bullet \cancel{3} \bullet 2 \bullet 2 \bullet 2} = y$$

$$\frac{1035}{8} = y = 129\ \frac{3}{8}$$

c)

$$\frac{135}{45} = \frac{1035}{8}$$

$$\frac{45}{1} \bullet \frac{1035}{8} = 135 \bullet x$$

$$\frac{\frac{45 \bullet 1035}{8}}{135} = x$$

$$\frac{\cancel{3} \bullet \cancel{3} \bullet \cancel{5} \bullet \cancel{3} \bullet 5 \bullet 3 \bullet 23}{2 \bullet 2 \bullet 2 \bullet \cancel{3} \bullet \cancel{3} \bullet \cancel{3} \bullet \cancel{5}} = x$$

$$\frac{345}{8} = x$$

$$43\frac{1}{8} = x$$

Total Length of frontage on Surry Road. 138' + 69' + 129 3/8 + 43 1/8' = 379 1/2

$\frac{138}{379.5}$ is approximately .3636 or 36.36%

$\frac{69}{379.5}$ is approximately .1818 or 18.18%

$\frac{129.375}{379.5}$ is approximately .3409 or 34.09%

$\frac{43.125}{379.5}$ is approximately .1136 or 11.36%

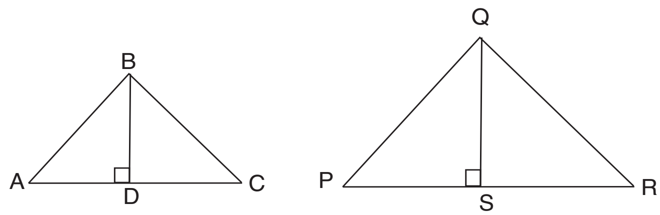
Unit IV — Triangles

Part D — Similarity – Part 2 (Triangles and Their Parts)

p. 360 – Lesson 3 — Theorem 29

1. **a)** Theorem 29 - If two triangles are similar, then the measures of corresponding altitudes are in the same ratio as the measures of corresponding sides.

   b)



   c) Given: $\triangle ABC \sim \triangle PQR$
   $\overline{BD}$ is an altitude of $\triangle ABC$
   $\overline{QS}$ is an altitude of $\triangle PQR$

   d) Prove: $\frac{BD}{QS} = \frac{BA}{QP}$

Statement	Reason
1. $\triangle ABC \sim \triangle PQR$	**1.** Given
2. $\angle A \cong \angle P$	**2.** Corresponding angles of similar polygons are congruent.
3. $\overline{BD}$ is an altitude of $\triangle ABC$ $\overline{QS}$ is an altitude of $\triangle PQR$	**3.** Definition of altitude of a triangle.
4. $\overline{BD} \perp \overline{AC}$	**4.** Definition of altitude of a triangle.
5. $QS \perp PR$	**5.** Definition of altitude of a triangle.
6. $\angle ADB$ is a right angle	**6.** Definition of perpendicular lines
7. $\angle PSQ$ is a right angle	**7.** Definition of perpendicular lines
8. $\angle ADB \cong \angle PSQ$	**8.** Theorem 11 - If you have right angles, then those right angles are congruent.
9. $\triangle ADB \sim \triangle PSQ$	**9.** Corollary 12a - If two angles of one triangle are congruent to two corresponding angles of an other triangle, then the triangles are similar. (Angle-Angle)
10. $\frac{BD}{QS} = \frac{BA}{QP}$	**10.** If two polygons are similar, then corresponding sides are proportional.

15.

$$\frac{BC}{EF} = \frac{MB}{NE}$$

$$\frac{15}{10} = \frac{MB}{5}$$

$$10 \bullet MB = 15 \bullet 5$$

$$MB = \frac{15 \bullet 5}{10}$$

$$MB = \frac{3 \bullet 5 \bullet \cancel{5}}{2 \bullet \cancel{5}}$$

$$MB = \frac{15}{2}$$

16.

$$\frac{DE}{AB} = \frac{NE}{MB}$$

$$\frac{9}{12} = \frac{NE}{15}$$

$$12 \bullet NE = 9 \bullet 15$$

$$NE = \frac{9 \bullet 15}{12}$$

$$NE = \frac{3 \bullet 3 \bullet \cancel{3} \bullet 5}{2 \bullet 2 \bullet \cancel{3}}$$

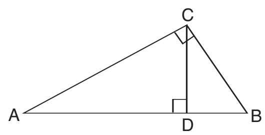
$$NE = \frac{45}{4}$$

17.

Statement	Reason
1. $\triangle ABC \sim \triangle DEF$	**1.** Given
2. AG = GB	**2.** Given
3. G is the midpoint of $\overline{AB}$	**3.** Definition of midpoint
4. $\overline{CG}$ is a median	**4.** Definition of median
5. DH = HE	**5.** Given
6. H is the midpoint of $\overline{DE}$	**6.** Definition of midpoint
7. $\overline{FH}$ is a median	**7.** Definition of median
8. $\frac{CG}{FH} = \frac{AB}{DE}$	**8.** Corollary 29b - If two triangles are similar, then the measures of corresponding medians are in the same ratio as the measures of corresponding sides.
9. $\frac{CG}{FH} = \frac{BC}{EF}$	**9.** Corollary 29b - If two triangles are similar, then the measures of corresponding medians are in the same ratio as the measures of corresponding sides.
10. $\frac{AB}{DE} = \frac{BC}{EF}$	**10.** Substitution
11. AB = AG + GB DE = DH + HE	**11.** Postulate 6 - Ruler - Fourth Assumption - Segment Addition.
12. AB = AG + AG DE = DH + DH	**12.** Substitution
13. $AB = 2 \cdot AG$ $DE = 2 \cdot DH$	**13.** Properties of arithmetic
14. $\frac{2 \bullet AG}{2 \bullet DH} = \frac{BC}{EF}$	**14.** Substitution
15. $\frac{AG}{DH} = \frac{BC}{EF}$	**15.** Properties of arithmetic - Q. E. D

4. **a)** Corollary 30c - If you have the altitude to the hypotenuse of a right triangle, then the product of the measures of the hypotenuse and the altitude is equal to the product ofthe measures of the legs.

b)



c) Given: Right △ ABC with right angle at C.
$\overline{CD} \perp \overline{AB}$

d) Prove: AB • CD = CB • AC

e) Proof:

Statement	Reason
1. Right △ ABC with right angle at C.	**1.** Given
2. $\overline{CD} \perp \overline{AB}$	**2.** Given
3. $\overline{CD}$ is an altitude	**3.** Definition of an altitude
4. △ ACD ∽ △ ABC ∽ △ CBD	**4.** Theorem 30 - If you have the altitude to the hypotenuse of a right triangle, then it forms two triangles that are similar to each other, and to the original triangle.
5. $\frac{AB}{CB} = \frac{AC}{CD}$	**5.** In similar polygons, the measures of corresponding sides are proportional.
6. AB • CD = CB • AC	**6.** Means - Extremes product property.

5. **a)** $\sqrt{12} = \sqrt{2 \cdot 2 \cdot 3} = \sqrt{2^2 \cdot 3} = \sqrt{2^2} \cdot \sqrt{3} = 2\sqrt{3}$

b) $\sqrt{48} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} = \sqrt{2^2 \cdot 2^2 \cdot 3} = \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{3} = 2 \cdot 2 \cdot \sqrt{3} = 4\sqrt{3}$

c) $\sqrt{\frac{3}{5}} = \frac{\sqrt{3}}{\sqrt{5}} = \frac{\sqrt{3} \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}} = \frac{\sqrt{15}}{\sqrt{5^2}} = \frac{\sqrt{15}}{5}$

d) $\frac{2}{3\sqrt{7}} = \frac{2 \cdot \sqrt{7}}{3\sqrt{7} \cdot \sqrt{7}} = \frac{2\sqrt{7}}{3 \cdot \sqrt{7^2}} = \frac{2\sqrt{7}}{3 \cdot 7} = \frac{2\sqrt{7}}{21}$

e) $\sqrt{\frac{128}{8}} = \sqrt{\frac{2 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}}} = \sqrt{16} = 4$

6.

a). $\frac{9}{x} = \frac{x}{16}$

$x \cdot x = 9 \cdot 16$

$x^2 = 144$

$x = 12$

b) $\frac{2}{x} = \frac{x}{8}$

$x \cdot x = 2 \cdot 8$

$x^2 = 16$

$x = 4$

c) $\frac{4}{x} = \frac{x}{5}$

$x \cdot x = 4 \cdot 5$

$x^2 = 4 \cdot 5$

$x = \sqrt{4 \cdot 5}$

$x = 2\sqrt{5}$

d) $\frac{\sqrt{3}}{x} = \frac{x}{\sqrt{5}}$

$x \cdot x = \sqrt{3} \cdot \sqrt{5}$

$x^2 = \sqrt{15}$

$x = \sqrt{\sqrt{15}}$

$x = \sqrt{15^{1/2}}$

$x = ((15)^{1/2})^{1/2}$

$x = 15^{1/4}$

$x = \sqrt[4]{15}$

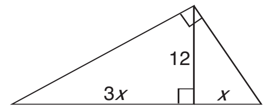
24.

$6 \bullet 8 = 10 \bullet CD$

$48 = 10 \bullet CD$

$4.8 = CD$

25.



$$\frac{3x}{12} = \frac{12}{x}$$

$12 \bullet 12 = (3x)(x)$

$144 = 3x^2$

$48 = x^2$

$\sqrt{48} = x$

$\sqrt{16 \bullet 3} = x$

$\sqrt{16} \bullet \sqrt{3} = x$

$4\sqrt{3} = x$

Length of hypotenuse is 4x or $16\sqrt{3}$.

26.

$x(2x + 2) = 30 \bullet 40$

$2x^2 + 2x = 1200$

$x^2 + x = 600$

$x^2 + x - 600 = 0$

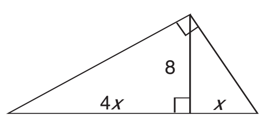
$(x + 25)(x - 24) = 0$

$x + 25 = 0 \quad x - 24 = 0$

$x = -25 \quad x = 24$

Answer cannot be negative

27.



$$\frac{4x}{8} = \frac{8}{x}$$

$8 \bullet 8 = 4x \bullet x$

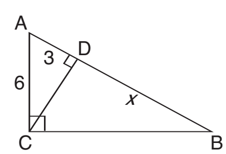
$64 = 4x^2$

$16 = x^2$

$4 = x$

Length of hypotenuse is 5 x or 20.

28.



$$\frac{3}{6} = \frac{6}{3 + x}$$

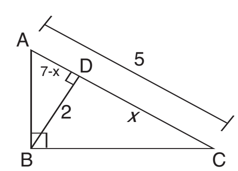
$6 \bullet 6 = 3(3 + x)$

$36 = 9 + 3x$

$27 = 3x$

$9 = x$

29.



$$\frac{5 - x}{2} = \frac{2}{x}$$

$(2)(2) = (5 - x)x$

$4 = 5x - x^2$

$x^2 - 5x + 4 = 0$

$(x - 1)(x - 4) = 0$

$x - 1 = 0$ or $x - 4 = 0$

$x = 1$ or $x = 4$

6. a). In the triangle, with hypotenuse "x", the unlabled side has a length determined by the equation 24 - 2 • ? = 14.

$2 \bullet ? = 10 \therefore ? = 5.$

$x^2 = 5^2 + 12^2$

$x^2 = 25 + 144$

$x^2 = 169$

$x = 13$

b)

$12 - 4 = 8$

$x^2 = 8^2 + 6^2$

$x^2 = 64 + 36$

$x^2 = 100$

$x = 10$

c)

$18\sqrt{2} - 16\sqrt{2} = 2\sqrt{2}$

$x^2 = 8^2 + (2\sqrt{2})^2$

$x^2 = 64 + 4 \bullet 2$

$x^2 = 64 + 8$

$x^2 = 72$

$x = \sqrt{72}$

$x = \sqrt{36 \bullet 2}$

$x = 6\sqrt{2}$

d)

Sides of square are 11 by the principles of a 45 - 45 - 90 triangle.

$x^2 + 11^2 = 12^2$

$x^2 + 121 = 144$

$x^2 = 23$

$x = \sqrt{23}$

6. e)

$x^2 = 10^2 + (\frac{x}{2})^2$

$x^2 = 100 + \frac{x^2}{4}$

$\frac{3x^2}{4} = 100$

$x^2 = \frac{100}{1} \bullet \frac{4}{3}$

$x = \sqrt{\frac{100 \bullet 4}{3}}$

$x = \frac{\sqrt{100} \bullet \sqrt{4}}{\sqrt{3}}$

$x = \frac{10 \bullet 2}{\sqrt{3}}$

$x = \frac{20\sqrt{3}}{\sqrt{3} \bullet \sqrt{3}}$

$x = \frac{20\sqrt{3}}{3}$

7. a)

$a^2 + b^2 = c^2$

$2^2 + 3^2 \stackrel{?}{=} 4^2$

$4 + 9 \neq 16$

Not sides of right triangle

b)

$a^2 + b^2 = c^2$

$(\sqrt{2})^2 + (\sqrt{3})^2 \stackrel{?}{=} (\sqrt{5})^2$

$2 + 3 = 5$

yes, sides of right triangle

c)

$a^2 + b^2 = c^2$

$1^2 + 1^2 \stackrel{?}{=} 2^2$

$1 + 1 \neq 4$

Not sides of right triangle

d)

$a^2 + b^2 = c^2$

$(\frac{1}{4})^2 + (\frac{1}{5})^2 \stackrel{?}{=} (\frac{1}{3})^2$

$\frac{1}{16} + \frac{1}{25} \stackrel{?}{=} \frac{1}{9}$

$\frac{25}{400} + \frac{16}{400} = \frac{41}{400} \; ; \; \frac{41}{400} \neq \frac{1}{9}$

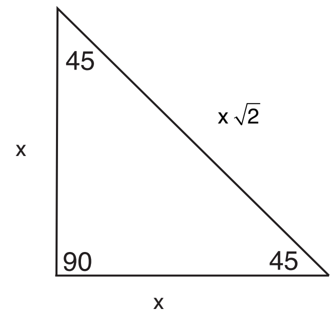
Not sides of right triangle

8. a) $y = 6,\ z = 6\sqrt{2}$

b) $x = 10,\ z = 10\sqrt{2}$

c) $y = 2.5,\ z = 2.5\sqrt{2}$

d) $x = \sqrt{2},\ z = \sqrt{2} \bullet \sqrt{2} = 2$



e)

$$7\sqrt{2} = x\sqrt{2}$$

$$7 = x$$

$$7 = y$$

f)

$$8 = x\sqrt{2}$$

$$\frac{8}{\sqrt{2}} = x$$

$$\frac{8\ \sqrt{2}}{\sqrt{2} \bullet \sqrt{2}} = x$$

$$\frac{8\ \sqrt{2}}{2} = x$$

$$4\sqrt{2} = x \quad 4\sqrt{2} = y$$

9. a)

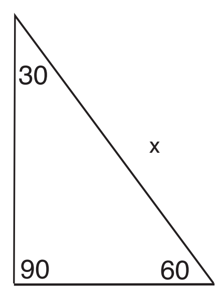
$$m = \frac{q}{2}$$

$$4 = \frac{q}{2}$$

$$8 = q$$

$$n = \sqrt{3} \cdot m$$

$$n = 4\sqrt{3}$$



b)

$$m = \frac{q}{2}$$

$$\frac{2}{3} = \frac{q}{2}$$

$$\frac{4}{3} = q$$

$$n = \sqrt{3} \bullet m$$

$$n = \frac{2}{3}\sqrt{3}$$

c)

$$m = \frac{q}{2}$$

$$m = \frac{6}{2} = 3$$

$$n = \sqrt{3} \bullet m$$

$$n = 3\sqrt{3}$$

d)

$$m = \frac{q}{2}$$

$$m = \frac{\frac{3}{4}}{2}$$

$$m = \frac{3}{4} \div \frac{2}{1} = \frac{3}{4} \bullet \frac{1}{2} = \frac{3}{8}$$

$$n = \sqrt{3} \bullet m$$

$$n = \frac{3\sqrt{3}}{8}$$

e)

$$n = \sqrt{3} \bullet m$$
$$5\sqrt{3} = \sqrt{3} \bullet m$$

$$\frac{5\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} = m$$

$$5 = m$$

$$m = \frac{q}{2}$$

$$5 = \frac{q}{2}$$

$$10 = q$$

f)

$$n = \sqrt{3} \bullet m$$

$$9 = \sqrt{3} \bullet m$$

$$\frac{9}{\sqrt{3}} = m$$

$$\frac{9\sqrt{3}}{3} = m$$
$$3\sqrt{3} = m$$

$$m = \frac{q}{2}$$
$$3\sqrt{3} = \frac{q}{2}$$
$$6\sqrt{3} = q$$

10. a)

$$6 = \frac{1}{2} \bullet AC$$

$$12 = AC$$

b)

$$6^2 + (BC)^2 = 12^2$$

$$36 + (BC)^2 = 144$$

$$(BC)^2 = 108$$

$$BC = \sqrt{108} = \sqrt{36 \bullet 3} = 6\sqrt{3}$$

c)

$$6 + 6\sqrt{3} + 6 + 6\sqrt{3} = \text{Perimeter}$$

$$12 + 12\sqrt{3} = \text{Perimeter}$$

11. a)

$$AC = \frac{1}{2} \bullet 12$$

$$AC = 6$$

b)

$$BC = \frac{x\sqrt{3}}{2}$$

$$BC = \frac{12\sqrt{3}}{2}$$

$$BC = 6\sqrt{3}$$

c) Since, AB = 12, then AC = 6 and so DC = 6

11. d) $AD = DC\sqrt{2}$

$AD = 6\sqrt{2}$

e) $60 = M\angle BAC$

f) $m\angle DAC = m\angle DAB - m\angle BAC$

$m\angle DAC = 105 - 60$

$m\angle DAC = 45$

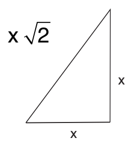
$m\angle CDA = 180 - m\angle DCA - m\angle DAC$

$m\angle CDA = 180 - 90 - 45$

$m\angle CDA = 180 - 135$

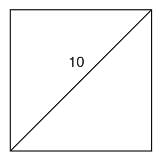
$m\angle CDA = 45$

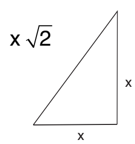
12. $x\sqrt{2}$



Diagonal is $10\sqrt{2}$

13.





$10 = x\sqrt{2}$

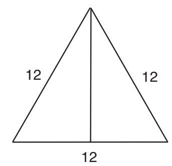
$\frac{10}{\sqrt{2}} = x$

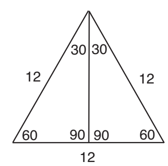
$\frac{10\sqrt{2}}{\sqrt{2} \bullet \sqrt{2}} = x$

$\frac{10\sqrt{2}}{2} = x$

$5\sqrt{2} = x$

14.

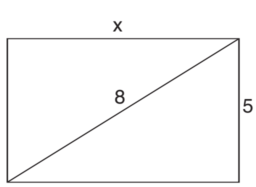




$\text{Altitude} = \frac{12\sqrt{3}}{2}$

$\text{Altitude} = 6\sqrt{3}$

15.



$8^2 = x^2 + 5^2$

$64 = x^2 + 25$

$39 = x^2$

$\sqrt{39} = x$

$\text{Perimeter} = 5 + \sqrt{39} + 5 + \sqrt{39}$

$= 10 + 2\sqrt{39}$

4.

$$(MN)^2 = (MQ)^2 + (QN)^2$$
$$(MN)^2 = 16^2 + 4^2$$
$$(MN)^2 = 256 + 16$$
$$(MN)^2 = 272$$
$$(PN)^2 = (MN)^2 + (PM)^2$$
$$(PN)^2 = 272 + 10^2$$
$$x^2 = 272 + 100$$
$$x^2 = 372$$
$$x = \sqrt{372}$$
$$x = \sqrt{4 \bullet 93}$$
$$x = 2\sqrt{93}$$

5.

$$(PU)^2 = (UV)^2 + (VP)^2$$
$$x^2 = 8^2 + 6^2$$
$$x^2 = 64 + 36$$
$$x^2 = 100$$
$$x = 10$$
$$(MU)^2 = (PU)^2 + (PM)^2$$
$$y^2 = x^2 + 4^2$$
$$y^2 = 100 + 16$$
$$y^2 = 116$$
$$y = \sqrt{4 \bullet 29}$$
$$y = 2\sqrt{29}$$

6.

Using 30 - 60 - 90 right triangle :

$QC = QD = QA = QB = 4$
$AF = FD = CG = GB = 2\sqrt{3}$

Since $QC = 4$ and $EC = 8$, $\triangle EQC$ is a 30 - 60 - 90 right triangle.
$EQ = 4\sqrt{3}$

Perimeter of ABCD :

$$AB + BC + CD + DA$$
$$(2 + 2) + (x + x) + (2 + 2) + (x + x)$$
$$4 + 2x + 4 + 2x$$
$$4 + 2(2\sqrt{3}) + 4 + 2(2\sqrt{3})$$
$$4 + 4\sqrt{3} + 4 + 4\sqrt{3}$$
$$8 + 8\sqrt{3} \text{ or } 8(1 + \sqrt{3})$$

7.

$$(HG)^2 = (GJ)^2 + (JH)^2$$
$$12^2 = 6^2 + x^2$$
$$144 = 36 + x^2$$
$$108 = x^2$$
$$\sqrt{108} = x$$
$$\sqrt{36 \bullet 3} = x$$
$$6\sqrt{3} = x$$

In a 30 - 60 - 90 triangle, the length of the side opposite the 30 angle is one - half the length of the hypotenuse, and the length of the side opposite the 60 angle is one - half the length of the hypotenuse multiplied by $\sqrt{3}$. In the given figure, the hypotenuse is 12, the leg opposite $\angle 1$ is 6, and the leg opposite $\angle 2$ is $6\sqrt{3}$. So, this triangle is a 30 - 60 - 90 right triangle, and the measure of $\angle 1$ is 30 .

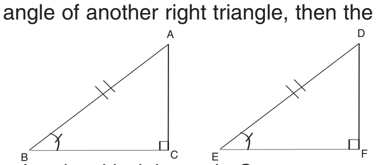
8.

$$(RQ)^2 = (QT)^2 + (RT)^2$$
$$6^2 = (4)^2 + (h)^2$$
$$6^2 - 4^2 = h^2$$
$$36 - 16 = h^2$$
$$20 = h^2$$
$$2\sqrt{5} = h$$
$$(VR)^2 = (RT)^2 + (TV)^2$$
$$x^2 = h^2 + 3^2$$
$$x^2 = 20 + 9$$
$$x^2 = 29$$
$$x = \sqrt{29}$$

Unit IV — Triangles
Part E — Congruence – Part 1 (General Geometric Relationship)

p. 397 – Lesson 3 — Congruence Postulate Corollaries

1. Postulate Corollary 13b - If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the triangles are congruent. (HA congruence corollary)



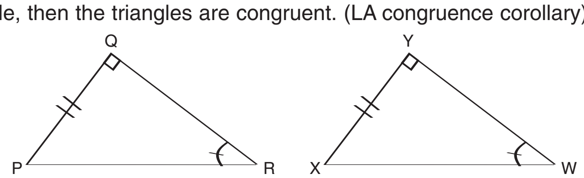
c) Given: △ ABC is a right triangle with right angle C
△ DEF is a right triangle with right angle F
$\overline{AB} \cong \overline{DE}$
∠ B ≅ E

d) Prove: △ ABC ≅ △ DEF

Statement	Reason
1. △ ABC is a right triangle with right angle C △ DEF is a right triangle with right angle F	**1.** Given
2. ∠ C ≅ ∠ F	**2.** Theorem 11 - If you have right angles, then those right right angles are congruent.
3. ∠ B ≅ ∠ E	**3.** Given
4. $\overline{AB}$ ≅ ∠ $\overline{DE}$	**4.** Given
5. △ ABC ≅ △ DEF	**5.** Postulate Corollary 13a - If two angles and a non-included side of one triangle are congruent to the corresponding angles and non- included side of another triangle, then the triangles are congruent.

2. Postulate Corollary 13c - If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent. (LA congruence corollary)

b)



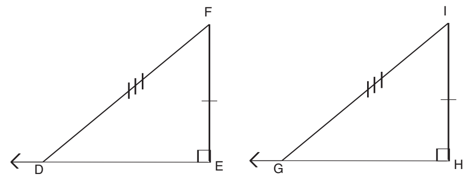
c) Given: △ PQR is a right triangle with right angle at Q
△ DEF is a right triangle with right angle at Y
$\overline{PQ} \cong \overline{XY}$
∠ R ≅ ∠ W (or ∠ P ≅ ∠ X)

d) Prove: △ PQR ≅ △ XYW

Case I : Congruent angle does NOT include congruent leg.

Statement	Reason
1. △ PQR is a right triangle with right angle at Q △ XYW is a right triangle with right angle at Y	**1.** Given
2. ∠ Q ≅ ∠ Y	**2.** Theorem 11 - If you have right angles, then those right right angles are congruent.
3. ∠ R ≅ ∠ W	**3.** Given
4. $\overline{PQ} \cong \overline{XY}$	**4.** Given
5. △ PQR ≅ △ XYW	**5.** Postulate Corollary 13a - If two angles and a non-included side of one triangle are congruent to the corresponding angles and non- included side of another triangle, then the triangles are congruent.

4. Postulate Corollary 13e - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the two triangles are congruent. (HL congruence corollary)



5. △ABC ≅ △DEF; HA congruence corollary 13b - If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the triangles are congruent.

6. △UVW ≅ △UXW; LL congruence corollary 13d - If two legs of one right triangle are congruent to the corresponding legs of another right triangle,then the triangles are congruent.

7. △MNJ ≅ △NMH; HL congruence corollary 13e - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles ar congruent.

8. △TRZ ≅ △TXZ; LA congruence corollary 13c - If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.

9. △TUV ≅ △TFX; LA congruence corollary 13c - If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.

10. △MNH ≅ △MNG; HA congruence corollary 13b - If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acut angle of another right triangle, then the triangles are congruent.

11. △PVQ ≅ △RUT; HA congruence corollary 13b - If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acut angle of another right triangle, then the triangles are congruent.

12. △APX ≅ △MQY; LL congruence corollary 13d - If two legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent.

13. ∠ D ≅ ∠ B
or
∠ A ≅ ∠ CEB

14. $\overline{AC} \cong \overline{EC}$
or
$\overline{DC} \cong \overline{BC}$

15. $\overline{AC} \cong \overline{EC}$ and ∠ A ≅ ∠ CED
or
$\overline{AC} \cong \overline{EC}$ and ∠ D ≅ ∠ B
or
$\overline{DC} \cong \overline{BC}$ and ∠ A ≅ ∠ CED
or
$\overline{DC} \cong \overline{BC}$ and ∠ D ≅ ∠ B

16. $\overline{AC} \cong \overline{EC}$ and
$\overline{DC} \cong \overline{BC}$

17.

Statement	Reason
1. ∠ ADC is a right angle	**1.** Given
2. ∠ CBA is a right angle	**2.** Given
3. ∠ ADC ≅ ∠ CBA	**3.** Theorem 11 - If you have right angles, then those right right angles are congruent.
4. $\overline{DC} \parallel \overline{BA}$	**4.** Given
5. ∠ ACD ≅ ∠ CAB	**5.** Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
6. $\overline{AC} \cong \overline{CA}$	**6.** Reflexive Property for congruence.
7. △ ADC ≅ △ CBA	**7.** Postulate Corollary 13a - AAS Postulate Corollary - If two angles anda non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.

Alternate Proof:

Steps 1 and 2 the same
Step 3: △ ADC and △CBA are right triangles because of the definition of right triangles
Step 4, 5, and 6 the same
Step 7: △ ADC ≅ △ CBA because of Postulate Corollary 13b - (HA congruence corollary) - If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the triangles are congruent.

18.

Statement	Reason
1. $\overline{DE} \perp \overline{AC}$	**1.** Given
2. $\overline{BF} \perp \overline{AC}$	**2.** Given
3. ∠ DEC is a right angle ∠ BFA is a right angle	**3.** Definition of perpendicular
4. △ DEC is a right triangle △ BFA is a right triangle	**4.** Definition of right triangle
5. ABCD is a parallelogram	**5.** Given
6. $\overline{DC} \parallel \overline{AB}$	**6.** Definiton of Parallelogram - a quadrilateral with two pairs of parallel sides.
7. ∠ DCE ≅ ∠ BAF	**7.** Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
8. $\overline{AE} \cong \overline{CF}$	**8.** Given
9. $\overline{EF} \cong \overline{FE}$	**9.** Reflexive Property for congruent segments
10. $\overline{AF} \cong \overline{CE}$	**10.** Postulate 6 - Ruler Fourth Assumption - Segment Addition postulate.
11. △ AFB ≅ △ CED	**11.** Postulate corollary 13c - If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent. (LA congruence corollary)

21.

Statement	Reason
1. $\overline{AD} \perp$ plane of $\triangle$ BDC, Plane M	**1.** Given
2. $\overline{AD} \perp \overline{BD}$; $\overline{AD} \perp \overline{DC}$	**2.** Definition of line perpendicular to a plane - A line is perpendicular to a plane at a given point if and only if it is perpendicular to every line in the plane that contains the given point.
3. $\angle$ ADB is a right angle $\angle$ ADC is a right angle	**3.** Definition of perpendicular lines
4. $\triangle$ ADB is a right triangle $\triangle$ ADC is a right triangle	**4.** Definition of right triangle
5. $\triangle$ BDC is an equilateral triangle	**5.** Given
6. $\overline{BD} \cong \overline{CD}$	**6.** Definition of Equilateral Triangle - A triangle with three congruent sides.
7. $\overline{AD} \cong \overline{AD}$	**7.** Reflexive Property of congruence for Line segment (i. e. , A line segment is congruent to itself).
8. $\triangle$ ADB $\cong$ $\triangle$ ADC	**8.** Postulate 13 - SAS Congruence Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the two triangles are congruent.

22.

Statement	Reason
1. $\overline{NQ} \perp \overline{QM}$; $\overline{QN} \perp \overline{NP}$	**1.** Given
2. $\angle$ MQN is a right angle $\angle$ PNQ is a right angle	**2.** Definition of perpendicular lines
3. $\triangle$ MQN is a right triangle $\triangle$ PNQ is a right triangle	**3.** Definition of right triangle
4. $\overline{MQ} \cong \overline{PN}$	**4.** Given
5. $\overline{QN} \cong \overline{NQ}$	**5.** Reflexive Property of congruence for Line segment (i. e. , A line segment is congruent to itself).
6. $\triangle$ MQN $\cong$ $\triangle$ PNQ	**6.** Postulate Corollary 13d - If two legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent. (LL congruence corollary)

23.

Statement	Reason
1. m $\angle$ GKH = 90	**1.** Given
2. $\angle$ GKH is a right angle	**2.** Definition of right triangle
3. $\angle$ GKH $\cong$ $\angle$ JKI	**3.** Theorem 15 - If two lines intersect, then the vertical angles - are congruent.
4. m $\angle$ GKH = m $\angle$ JKI	**4.** Definition of congruent angles
5. m $\angle$ JKI = 90	**5.** Substitution
6. $\angle$ JKI is a right angle	**6.** Definition of right angle
7. $\triangle$ GKH is a right triangle $\triangle$ JKI is a right triangle	**7.** Definition of right triangle
8. $\overline{GJ}$ and $\overline{HI}$ bisect each other at K	**8.** Given
9. $\overline{KH} \cong \overline{KI}$	**9.** Definition of bisector of line segment.
10. $\overline{GK} \cong \overline{JK}$	**10.** Definition of bisector of line segment.
11. $\triangle$ GKH $\cong$ $\triangle$ JKI	**11.** Postulate Corollary 14d - If two legs of one right triangle are congruent to the corresponding legs of another right triangle, then the triangles are congruent. (LL congruence corollary)

Unit IV — Triangles
Part F - Congruence Part 2 (Applications)

p. 406 – Lesson 2 — Using the Definitions of Congruence

1. HI **2.** m$\angle$HIG
3. HG **4.** m $\angle$G

$\angle$PNM

5.

Statement	Reason
1. $\overline{AC} \cong \overline{BC}$	**1.** Given
2. $\overline{DC}$ bisects $\angle$ACB	**2.** Given
3. $\angle$ACD $\cong$ $\angle$BCD	**3.** Definition of angle bisector
4. $\overline{DC} \cong \overline{DC}$	**4.** Reflexive Property for line segments
5. $\triangle$ ACD $\cong$ $\triangle$ BCD	**5.** Postulate 13 - SAS Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the two trianges are congruent.
6. $\overline{AD} \cong \overline{BD}$	**6.** C.P.C.T.C.

6.

Statement	Reason
1. $\overline{DB} \perp \overline{AC}$	**1.** Given
2. $\angle$AEB is a right angle $\angle$CEB is a right angle	**2.** Definition of perpendicular line segments
3. $\angle$AEB $\cong$ $\angle$CEB	**3.** Theorem 11 - If you have right angles, then those right angles are congruent.
4. $\overline{DB}$ bisects $\overline{AC}$	**4.** Given
5. $\overline{AE} \cong \overline{CE}$	**5.** Definition of segment bisector
6. $\overline{BE} \cong \overline{BE}$	**6.** Reflexive Property for line segments
7. $\triangle$ ABE $\cong$ $\triangle$ CBE	**7.** Postulate 13 - SAS Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the two trianges are congruent.
8. $\angle$ABD $\cong$ $\angle$CBD	**8.** C.P.C.T.C

7.

Statement	Reason
1. $\overline{BD}$ bisects $\angle$ABC	**1.** Given
2. $\angle$ABD $\cong$ $\angle$DBC	**2.** Definition of angle bisector
3. $\overline{BD} \cong \overline{BD}$	**3.** Reflexive Property for line segments
4. $\overline{BD}$ bisects $\angle$CDA	**4.** Given
5. $\angle$ADB $\cong$ $\angle$CDB	**5.** Definition of Angle bisector
6. $\triangle$ ABD $\cong$ $\triangle$ CBD	**6.** Postulate 13 - ASA Assumption - If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the two triangles are congruent.
7. $\angle$BAD $\cong$ $\angle$BCD	**7.** C.P.C.T.C

11.

Statement	Reason
1. $\angle 1 \cong \angle 3$	**1.** Given
2. $\angle$Q and $\angle$P are right angles	**2.** Given
3. $\angle Q \cong \angle P$	**3.** Theorem 11 - If you have right angles, then those right angles are congruent.
4. $\overline{SR} \cong \overline{RS}$	**4.** Reflexive Property for line segments.
5. $\triangle$ SQR $\cong$ $\triangle$ RPS	**5.** Postulate corollary 13 a - AAS Postulate - If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the two triangles are congruent.
6. $\overline{QR} \cong \overline{PS}$	**6.** C.P.C.T.C.

12.

Statement	Reason
1. $\angle$Q and $\angle$P are right angles	**1.** Given
2. $\triangle$ SQR is a right triangle $\triangle$ RPS is a right triangle	**2.** Definition of a right triangle.
3. $\overline{RS} \cong \overline{SR}$	**3.** Reflexive Property for line segments
4. $\overline{QS} \cong \overline{PR}$	**4.** Given
5. $\triangle$ SQR $\cong$ $\triangle$ RPS	**5.** Postulate corollary 13 e - H L congruence postulate - If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.
6. $\angle 4 \cong \angle 2$	**6.** C.P.C.T.C.

13.

Statement	Reason
1. $\overline{WY} \cong \overline{YW}$	**1.** Reflexive Property for line segments
2. $\overline{WX} \parallel \overline{YZ}$	**2.** Given
3. $\angle XWY \cong \angle ZYW$	**3.** Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
4. $\angle$XYW is a right angle $\angle$ZWY is a right angle	**4.** Given
5. $\triangle$ XYW is a right triangle $\triangle$ ZWY is a right triangle	**5.** Definition of a right triangle
6. $\triangle$ XYW $\cong$ $\triangle$ ZWY	**6.** Postulate Corollary 13 c - L A congruence postulate - If one leg and an acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.
7. $\angle X \cong \angle Z$	**7.** C.P.C.T.C.

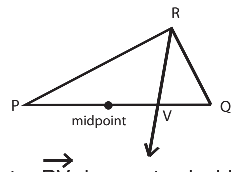
14. Draw auxiliary line segment ZX. This would allow $\triangle$ WXZ to be congruent to $\triangle$ YZX by Postulate 13 - SSS Assumption, and $\angle W \cong \angle Y$ by C.P.C.T.C.

Technically, there are two ways this could be proved. But, by drawing line segment WY, you would be required to use Angle Addition.

19. No

a) Theorem 4 - If you have a given line segment ($\overline{PQ}$), then that segment has exactly one midpoint.

b) Theorem 8 - If, in a half plane, there is a given angle, then that angle has exactly one bisector.



In the given triangle, the angle bisector $\overrightarrow{RV}$ does not coincide with the ray going through the midpoint of $\overline{PQ}$. So the angle bisector and the segment bisector are not necessarily the same

20. Yes

a) Postulate 5 - Intersection of lines or planes - If 2 different lines intersect, the intersection is a unique point.

b) Theorem 4 - If you have a given line segment, then that segment has exactly one midpoint.

These two points can always be the same point.

21.

3. Postulate 11 - Corresponding angles of parallel lines - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
4. Reflexive Property of Congruence for Angles
5. Postulate 11
6. Postulate 12 - AAA Similarity Assumption - If the three angles of one triangle are congruent to the three corresponding angles of another triangle, then the triangles are similar.
7. Corresponding sides of similar triangles are proportional.
9. Substitution of Equals (DB = PQ from Step 1)
10. Substitution of Equals (9 into 7)
12. Substitution of Equals (9 into 7)
13. Multiplication of Equality

22.

1. Postulate 6 - Ruler - Second Assumption - To every pair of points on a line, there corresponds exactly one real number, called the unique distance between the points.
2. Postulate 9 - Uniqueness of parallel lines - In a plane, through a point not on a given line, there is exactly one line parallel to a given line.
3. Postulate 11 - Corresponding angles of parallel lines - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
4. Reflexive property for angles.
5. Postulate 12 - AAA similarity Assumption - If the three angles of one triangle are congruent to the three corresponding angles of another triangle, then the triangles are similar.
9. Substitution of equals (1 into 8)
10. Substitution of equals (9 into 10)
11. Multiplication of Equality
12. Definition of Congruent Segments
14. Transitive property for congruent angles.
15. Definition of congruent segments
16. Postulate 13 - SAS Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the two triangles are congruent.
17. C.P.C.T.C
18. Postulate Corollary 12 a -AA Postualte Corollary for Similarity - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar.

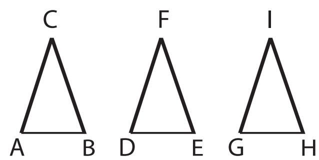
Unit IV — Triangles

Part F - Congruence Part 2 (Applications)

p. 414 – Lesson 3 — Theorem 32

1. **a)** Theorem 32 - If two triangles are congruent to a third triangle, then the two trianges are congruent to each other.

b)



c) Given: $\triangle ABC \cong \triangle DEF$
$\triangle GHI \cong \triangle DEF$

d) Prove: $\triangle ABC \cong \triangle GHI$

e)

Statement	Reason
1. $\triangle ABC \cong \triangle DEF$	**1.** Given
2. $\overline{AB} \cong \overline{DE}$, $\overline{BC} \cong \overline{EF}$ $\overline{AC} \cong \overline{DF}$; $\angle A \cong \angle D$, $\angle B \cong \angle E$, $\angle C \cong \angle F$,	**2.** Definition of congruent triangles
3. $\triangle GHI \cong \triangle DEF$	**3.** Given
4. $\overline{GH} \cong \overline{DE}$, $\overline{HI} \cong \overline{EF}$ $\overline{GI} \cong \overline{DF}$; $\angle G \cong \angle D$, $\angle H \cong \angle E$, $\angle I \cong \angle F$,	**4.** Definition of congruent triangles
5. $\overline{AB} \cong \overline{GH}$, $\overline{BC} \cong \overline{HI}$ $\overline{AC} \cong \overline{GI}$	**5.** Transitive Property for segment congruence
6. $\angle A \cong \angle G$, $\angle B \cong \angle H$ $\angle C \cong \angle I$	**6.** Transitive Property for angle congruence
7. $\triangle ABC \cong \triangle GHI$	**7.** Definition of congruent triangles - Two triangles are congruent if and only if there is a correspondence between the vertices such that each pair of corresponding sides and each pair of corresponding angles are congruent.

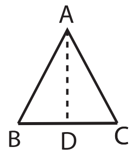
2.

Statement	Reason
1. $\overline{GC} \parallel \overline{BI}$	**1.** Given
2. $\angle GCA \cong \angle IBE$	**2.** Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
3. $\overline{AG} \parallel \overline{EI}$	**3.** Given
4. $\angle GAC \cong \angle IEB$	**4.** Theorem16
5. $\overline{AB} \cong \overline{EC}$	**5.** Given
6. $\overline{BC} \cong \overline{CB}$	**6.** Reflexive property for congruence - a line segment is congruent to itself.
7. $\overline{AC} \cong \overline{EB}$	**7.** Postulate 6 - Ruler - Fourth Assumption - Segment Addition
8. $\triangle BIE \cong \triangle CGA$	**8.** Postulate 13 - Third Assumption - ASA congruence Assumption - If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the two triangles are congruent
9. $\triangle AGC \cong \triangle FHD$	**9.** Given
10. $\triangle BIE \cong \triangle FHD$	**10.** Theorem 32 - If two triangles are congruent to a third triangle, then the two triangles are congruent to each other.

p. 416 – Lesson 4 — Theorem 33

1. **a)** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.

b)



c) Given : △ ABC with $\overline{AB} \cong \overline{AC}$

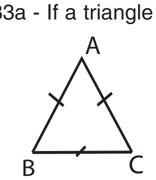
d) Prove: ∠ABC ≅ ∠ACB

e)

Statement	Reason
1. △ ABC with $\overline{AB} \cong \overline{AC}$	**1.** Given
2. Draw Auxilary $\overline{AD} \perp \overline{BC}$ as an altitude of △ ABC	**2.** Postulate 10 - Uniqueness of perpendicular lines - In a plane, through a point not on a line, there is exactly one line perpendicular to the given line.
3. ∠ADC is a right angle ∠ADB is a right angle	**3.** Definition of perpendicular
4. △ ADC is a right triangle △ ADB is a right triangle	**4.** Definition of right triangle
5. $\overline{AD} \cong \overline{AD}$	**5.** Reflexive Property of Congruence
6. △ ABD ≅ △ ADC	**6.** Posulate corollary 13e - If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and one leg of another right triangle, then the two right triangles are congruent.
7. ∠ABC ≅ ∠ACB	**7.** C.P.C.T.C.

2. **a)** Corollary 33a - If a triangle is equilateral, then it is equiangular.

b)



c) Given : △ ABC is Equilateral

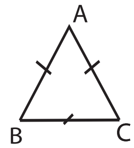
d) Prove: △ ABC is Equiangular

e)

Statement	Reason
1. △ ABC is Equilateral	**1.** Given
2. $\overline{AB} \cong \overline{AC} \cong \overline{BC}$	**2.** A triangle is equilateral, if and only if, all three sides are congruent.
3. ∠C ≅ ∠B ≅ ∠A	**3.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
4. △ ABC is Equiangular	**4.** A triangle is equiangular, if and only if, all of its angles are congruent.

3. **a)** Corollary 33b - If a triangle is equilateral, then the measure of each of its angles is 60.

b)



c) Given : △ ABC is Equilateral

d) Prove: m ∠ A = 60, m ∠ B = 60, m ∠ C = 60

e)

Statement	Reason
1. △ ABC is Equilateral	**1.** Given
2. △ ABC is Equiangular	**2.** Corollary 33a - If a triangle is equilateral, then it is equiangular.
3. ∠A ≅ ∠B ≅ ∠C	**3.** A triangle is equiangular, if and only if, all of its angles are congruent.
4. m ∠ A = m ∠ B = m ∠ C = 60	**4.** Corollary 25b - If all the angles of a triangle are congruent, then the measure of each angle is 60.

4.

4. Definition of congruent line segments
5. Postulate 2 - Uniqueness of lines - For any two different points, there is exactly one line containing them.
6. Theorem 10 - If the exterior sides of two adjacent angles are opposite rays, then the angles are supplementary.
7. Definition of supplementary angles.
8. Definition of right angle.
9. Substitution
12. Theorem 11 - If you have right angles, then those right angles are congruent.
13. Given
14. Postulate 13 - SAS Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the triangles are congruent.
15. C.P.C.T.C.
16. Given
21. Postulate Corollary 13 b - If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the triangles are congruent.

5. m ∠ E = m ∠ CDE = 15
Therefore, m ∠ DCE = 150 (180 - 15 - 15)
So, m ∠ ACB = 30 (180 - 150)
m ∠ A = 30 (∠ ACB ≅ ∠ A)
m ∠ B = 120 (180 - 30 - 30)
m ∠ BFD = 45 (180 - 120 - 15)

6. a) x b) y c) x (m ∠ A = x , AC = BC)

7.

AB = BC	AB = 4 (15/2)	BC = 6x - 15
4x = 6x - 15	AB = 60/2	BC = 6 (15/2) - 15
-2x = -15	AB = 30	BC = 90/2 - 15
x = 15/2		BC = 45 - 15 = 30

8.

DE = EF	DE = 4 (15)+ 15	EF = 2 (15) + 45	DF = 3 (15) + 15
4x + 15 = 2x + 45	DE = 60 + 15	EF = 30 + 45	DF = 45 + 15
2x = 30	DE = 75	EF = 75	DF = 60
x = 15			

9. m ∠ X = m ∠ Z | m ∠ X = 4x + 60 | m ∠ Y = 2x + 30 | m ∠ Z = 14x + 30

m ∠ X = m ∠ Z	m ∠ X = 4x + 60	m ∠ Y = 2x + 30	m ∠ Z = 14x + 30
4x + 60 = 14x + 30	m ∠ X = 4 • 3 + 60	m ∠ Y = 2 • 3 + 30	m ∠ Z = 14 • 3 + 30
30 = 10x	m ∠ X = 12 + 60	m ∠ Y = 6 + 30	m ∠ Z = 42 + 30
3 = x	m ∠ X = 72	m ∠ Y = 36	m ∠ Z = 72

10. a) 50 b) 40 c) 50 d) 40

11.

Statement	Reason
1. $\overline{AC} \cong \overline{AD}$	**1.** Given
2. ∠C ≅ ∠D	**2.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
3. $\overline{CB} \cong \overline{DE}$	**3.** Given
4. $\overline{BE} \cong \overline{EB}$	**4.** Reflexive property for congruence
5. $\overline{CE} \cong \overline{DB}$	**5.** Addition of Equals (common segment)
6. △ ACE ≅ △ ADB	**6.** Postulate 13 - SAS Assumption - If two sides and the included angle of another triangle, then the triangles are congruent.

12.

Statement	Reason
1. $\overline{AB} \cong \overline{AE}$	**1.** Given
2. ∠ABD ≅ ∠AEC	**2.** Theorem 30 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
3. $\overline{AC} \cong \overline{AD}$	**3.** Given
4.∠C ≅ ∠D	**4.** Theorem 30 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
5. △ ACE ≅ △ ADB	**5.** Postulate Corollary 13 - AAS Congruence Postulate - If two angles and a non-included side of one triangle are congruent to the corresponding two angles and non-included side of another triangle, then the two triangles are congruent.

13.

Statement	Reason
1. △ AEB is isosceles	**1.** Given
2. $\overline{AE} \cong \overline{BE}$	**2.** A triangle is an isoceles triangle, if and only if, it has at least two congruent sides angles.
3. ∠DBA ≅ ∠FAC	**3.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
4. $\overline{DA} \perp \overline{AC}$; $\overline{FC} \perp \overline{AC}$	**4.** Given
5. ∠DAB is a right angle ∠FCA is a right angle	**5.** Definition of perpendicular line segments.
6. ∠DAB ≅ ∠FCA	**6.** Theorem 11 - If you have right angles, then those right angles are congruent.
7. △DAB ∼ △FCA	**7.** Postulate Corollary 12a - AA Similarity Postulate - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar.
8. $\frac{CF}{AD} = \frac{AF}{BD}$	**8.** Definition of Similarity - Two polygons are said to be similar, if and only if, for some pairing of their vertices, the corresponding sides are in proportion.
9. AD • AF = CF • BD	**9.** Multiplication for Equality - (Multiply by AD • BD) - In a proportion, the product of the means equals the product of the extremes.

14. Statement | Reason

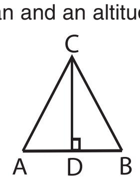
Statement	Reason
1. $\overline{AB} \cong \overline{AC}$	**1.** Given
2. $\angle C \cong \angle ABC$	**2.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
3. $\angle A \cong \angle CBD$	**3.** Given
4. $\triangle ABC \sim \triangle BCD$	**4.** Postulate Corollary 12a - AA Similarity Postulate - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar.
5. $\frac{AC}{BD} = \frac{BC}{CD}$	**5.** Definition of Similarity - Two polygons are said to be similar, if and only if, for some pairing of their vertices, the corresponding angles are congruent, and the corresponding sides are in proportion.

15.

Statement	Reason
1. $\triangle$ MNP with $\overline{MP} \cong \overline{NP}$	**1.** Given
2. $\angle M \cong \angle N$	**2.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
3. $\overline{RT} \perp \overline{PN}$; $\overline{RS} \perp \overline{PM}$	**3.** Given
4. $\angle$ RSM is a right angle $\angle$ RTN is a right angle	**4.** Definition of perpendicular line segment
5. $\angle RSM \cong \angle RTN$	**5.** Theorem 11 - If you have right angles, then those right angles are congruent.
6. $\triangle RSM \cong \triangle RTN$	**6.** Postulate Corollary 12a - AA Similarity Postulate - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar.
7. $\frac{RS}{RT} = \frac{RM}{RN}$	**7.** Definition of Similarity - Two polygons are said to be similar, if and only if, for some pairing of their vertices, the corresponding angles are congruent, and the corresponding sides are in proportion.
8. RT • RM = RS • RN	**8.** Multiplication for Equality - (Multiply by AD • BD) - In a proportion, the product of the means equals the product of the extremes.

16. **Part I:** If a median and an altitude of a triangle are the same segment, then the triangle is isosceles.

b)



c) Given : $\triangle$ ABC with altitude $\overline{CD}$ and median $\overline{CD}$.

d) Prove: $\triangle$ ABC is isosceles

e)

Statement	Reason
1. $\triangle$ ABC with altitude $\overline{CD}$	**1.** Given
2. $\overline{CD} \perp \overline{AB}$	**2.** Definition of Altitude
3. $\angle$ADC is a right angle $\angle$BDC is a right angle	**3.** Definition of Perpendicular
4. $\angle ADC \cong \angle BDC$	**4.** Theorem 11 - If you have right angles, then those right angles are congruent.

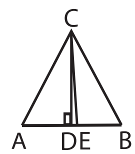
16. (continued)

Statement	Reason
5. $\overline{CD} \cong \overline{CD}$	**5.** Reflexive Property for line segments
6. $\overline{CD}$ is a median	**6.** Given
7. D is the midpoint of $\overline{AB}$	**7.** Definition of median
8. $\overline{AD} \cong \overline{BD}$	**8.** Definition of midpoint
9. $\triangle$ CAD $\cong$ $\triangle$ CBD	**9.** Postulate 13 - SAS Congruence Assumption - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.
10. $\overline{AC} \cong \overline{BC}$	**10.** C.P.C.T.C.
11. $\triangle$ ABC is an isosceles triangle	**11.** A triangle is isosceles if and only if it has at least two congruent sides.

Part II : If a triangle is isosceles, then a median and an altitude are the same segment.

Given: $\triangle$ ABC is isosceles with median $\overline{CE}$ and altitude $\overline{CD}$.

Prove: $\overline{CD} \cong \overline{CE}$



e)

Statement	Reason
1. $\triangle$ ABC is isosceles	**1.** Given
2. $\overline{AC} \cong \overline{BC}$	**2.** A triangle is isosceles if and only if it has at least two congruent sides.
3. $\overline{CE}$ is a median	**3.** Given
4. Point E is the midpoint of $\overline{AB}$	**4.** Definition of median of a triangle
5. $\overline{AE} \cong \overline{BE}$	**5.** Definition of midpoint
6. $\overline{CE} \cong \overline{CE}$	**6.** Reflexive Property for line segments
7. $\triangle$ AEC $\cong$ $\triangle$ BEC	**7.** Postulate 13 - SSS Congruence Assumption - If the three sides of one triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent.
8. $\angle$AEC $\cong$ $\angle$BEC	**8.** C.P.C.T.C.
9. $\angle$AEC and $\angle$BEC are Supplementary	**9.** Theorem 10 - If the exterior sides of two adjacent angles are opposite rays, then the two angles are supplementary.
10. $\angle$AEC is a right angle $\angle$BEC is a right angle	**10.** Corollary 10b - If two angles are supplementary and congruent, then each angle is a right angle.
11. $\overline{CE} \perp \overline{AB}$	**11.** Definition of perpendicular line segments
12. $\overline{CD}$ is an altitude	**12.** Given
13. $\overline{CD} \perp \overline{AB}$	**13.** Definition of Altitude
14. $\overline{CD} \cong \overline{CE}$	**14.** Postulate 10 - Uniqueness of perpendicular lines - In a plane through a point not on a given line, there is exactly one line (segment) perpendicular to the given line (segment).

17. The 6 faces of a cube are congruent squares. Since the lengths of all sides of a square are the same and all angles of a square are right angles, the triangles formed by diagonals of the faces, such as $\overline{DG}$ and $\overline{GE}$, form congruent right triangles by Postulate 13, SAS Assumption. All diagonals of the faces are congruent since C.P.C.T.C. . Connect point E to point D forming equilateral triangle DGE. It follows that each angle of $\triangle$ DGE is 60 by Corollary 33b.

18. Draw auxiliary $\overline{AD}$. Let $\overline{AD}$ be the angle bisector. Theorem 8 - If, in a half plane, there is a given angle, then that angle has exactly one bisector.

19. Given : $\triangle$ ABC with $\overline{AB} \cong \overline{AC}$

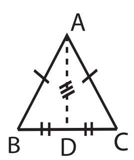
Prove: $\angle$ ABC $\cong$ $\angle$ ACB

Statement	Reason
1. $\triangle$ ABC with $\overline{AB} \cong \overline{AC}$	**1.** Given
2. Draw AD as the angle bisector of $\angle$BAC	**2.** Theorem 8 - If, in a half-plane, there is a given angle, then that angle has exactly one bisector.
3. $\angle$BAD $\cong$ $\angle$CAD	**3.** Definition of Angle Bisector
4. $\overline{AD} \cong \overline{AD}$	**4.** Reflexive property for congruence.
5. $\triangle$ ABD $\cong$ $\triangle$ ACD	**5.** Postulate 13 - SAS Assumption - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.
6. $\angle$ABC $\cong$ $\angle$ACB	**6.** C.P.C.T.C.

20. Draw Auxilary $\overline{AD}$, choosing point D as the midpoint of $\overline{BC}$. Then let $\overline{AD}$ be a median of $\triangle$ ABC.
Postulate 2 - Uniqueness of lines, planes, and spaces - For any two different points, there is exactly one line segment containing them.

21. **1. a)** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.

b)



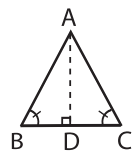
c) Given : $\triangle$ ABC with $\overline{AB} \cong \overline{AC}$

d) Prove: $\angle$ABC $\cong$ $\angle$ACB

e)

Statement	Reason
1. $\triangle$ ABC with $\overline{AB} \cong \overline{AC}$	**1.** Given
2. Choose point D on $\overline{BC}$ as the midpoint of $\overline{BC}$	**2.** Theorem 4 - If you have a given line segment, then that segment has exactly one midpoint.
3. Draw Auxilary $\overline{AD}$	**3.** Postulate 2 - Uniqueness of lines, planes, and spaces - For any two different points, there is exactly one line segment containing them.
4. $\overline{AD} \cong \overline{AD}$	**4.** Reflexive Property for Congruence.
5. $\overline{BD} \cong \overline{CD}$	**5.** Definition of midpoint.
6. $\triangle$ ABD $\cong$ $\triangle$ ACD	**6.** Postulate 13 - SSS Assumption - If the three sides of one triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent.
7. $\angle$ABC $\cong$ $\angle$ACB	**7.** C.P.C.T.C.

p. 423 – Lesson 5 — Theorem 34

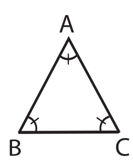


1. Given : $\triangle ABC$ with $\angle B \cong \angle C$
   Prove: $\overline{AB} \cong \overline{AC}$

Statement	Reason
1. $\triangle ABC$ with $\angle B \cong \angle C$	**1.** Given
2. Draw $\overline{AD}$ as an altitude of $\triangle ABC$	**2.** Posutlate 10 - Uniqueness of perpendicular lines - In a plane, through a point not on a line, there is exactly one line perpendicular to the given line.
3. $\overline{AD} \perp \overline{BC}$	**3.** Definition of altitude
4. $\angle ADB$ is a right angle $\angle ADC$ is a right angle	**4.** Definition of perpendicular
5. $\triangle ADB$ is a right triangle $\triangle ADC$ is a right triangle	**5.** Definition of right triangle
6. $\overline{AD} \cong \overline{AD}$	**6.** Reflexive property for congruence.
7. $\triangle ADB \cong \triangle ADC$	**7.** Postulate Corollary 13d - If an acute angle and a leg of one right triangle are congruent to the corresponding angle and leg of another right triangle, then the two right triangles are congruent.
8. $\overline{AB} \cong \overline{AC}$	**8.** C.P.C.T.C.

2. **a)** Corollary 34a - If a triangle is equiangular, then it is equilateral.

   b)



   c) Given : $\triangle ABC$ is Equiangular

   d) Prove: $\triangle ABC$ is Equilateral

   e)

Statement	Reason
1. $\triangle ABC$ is Equiangular	**1.** Given
2. $\angle A \cong \angle B \cong \angle C$	**2.** Definition of Equiangular Triangle
3. $\overline{BC} \cong \overline{AC} \cong \overline{AB}$	**3.** Theorem 34 - If two angles of triangle are congruent, then the sides opposite them are congruent.
4. $\triangle ABC$ is Equilateral	**4.** Definition of Equilateral Triangle, congruent.

3. Corollary 33a - If a triangle is equilateral, then it is equiangular.
4. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
5. Corollary 34a - If a triangle is equiangular, then it is equilateral.
6. Theorem 33 - Two sides of a triangle are congruent if and only if the angles opposite them are congruent.
   Theorem 34 - Two angles of a triangle are congruent if and only if the sides opposite them are congruent.
   Corollary 33a - A triangle is equilateral if and only if it is equiangular.
   Corollary 34a - A triangle is equiangular if and only if it is equilateral.

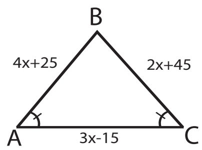
7. AB = 12
BC = 12
x = 60

8. XC = 6
m ∠ C = 86

9. HI = 3
m ∠ IJH = 20
m ∠ IKH = 10
m ∠ IKJ = 10
m ∠ HIK = 70
m ∠ JIK = 70

10. 9 **11.** 9 **12.** 4 **13.** 5
14. 80 **15.** 7 **16.** 40 **17.** 7

18.



4x + 25 = 2x + 45
2x = 20
x = 10

AB = 4 (10) + 25 = 65
BC = 2 (10) + 45 = 65
AC = 3 (10) - 15 = 15

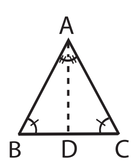
19. e)

Statement	Reason
1. ∠ADE ≅ ∠BED	**1.** Given
2. ∠ADE and ∠1 are supplementary ∠BED and ∠2 are supplementary	**2.** Theorem 10 - If the exterior sides of two adjacent angles are opposite rays, then the angles are supplementary.
3. ∠1 ≅ ∠2	**3.** Theorem 14 - If two angles are supplementary to the same angle or congruent angles, then they are congruent to each other.
4. $\overline{DC}$ ≅ $\overline{EC}$	**4.** Theorem 34 - If two angles of a triangle are congruent, then the sides opposite them are congruent.

20. Draw Auxilary $\overline{AD}$, as the angle bisector of ∠A.
Theorem 8 - If, in a half plane, there is a given angle, then that angle has exactly one bisector.

21. **a)** Theorem 34 - If two angles of a triangle are congruent, then the sides opposite them are congruent.

b)



c) Given : △ ABC with ∠B ≅ ∠C

d) Prove: $\overline{AB}$ ≅ $\overline{AC}$

e)

Statement	Reason
1. △ ABC with with ∠B ≅ ∠C	**1.** Given
2. Draw $\overline{AD}$ as the angle bisector of ∠BAC	**2.** Theorem 8 - If, in a half plane, there is a given angle, then that angle has exactly one bisector.
3. ∠BAD ≅ ∠CAD	**3.** Definition of Angle Bisector
4. $\overline{AD}$ ≅ $\overline{AD}$	**4.** Reflexive Property for Congruence.
5. △ ABD ≅ △ ACD	**5.** Postulate Corollary 13a - AAS Congruence Corollary - If two angles and a non-included side of one triangle are congruent to the corresponding two angles and non-included side of another triangle, then the two triangles are congruent.
6. $\overline{AB}$ ≅ $\overline{AC}$	**6.** C.P.C.T.C.

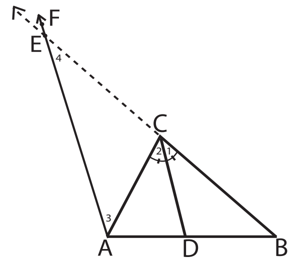
Unit IV — Triangles

Part F - Congruence Part 2 (Applications)

p. 428 – Lesson 6 — Theorem 35

1. a) Theorem 35 - If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that have the same ratio as the two other sides. (Angle Bisector Theorem)

b)



c) Given : $\overline{CD}$ bisects ∠ACB

d) Prove: $\frac{AB}{DB} = \frac{AC}{BC}$

e)

Statement	Reason
1. $\overline{CD}$ bisects ∠ACB	**1.** Given
2. ∠1 ≅ ∠2	**2.** Definition of angle bisector that angle has exactly one bisector.
3. Draw auxiliary line $\overleftrightarrow{AF}$ parallel to angle bisector $\overline{DC}$. Extended $\overline{BC}$ to intersect $\overleftrightarrow{AF}$ at point E.	**3.** Postulate 9 - In a plane, through a point (point A) not on a given line, there is exactly one line parallel to the given line. ($\overline{CD}$)
4. ∠1 ≅ ∠4	**4.** Postulate 11 - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
5. ∠2 ≅ ∠3	**5.** Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
6. ∠3 ≅ ∠4 (∠3 ≅ ∠2 , ∠2 ≅ ∠1 , ∠1 ≅ ∠4) 5 ⟶ 2 ⟶ 4	**6.** Transitive Property for congruent angles.
7. $\overline{CE} \cong \overline{CA}$	**7.** Theorem 34 - If two angles of a triangle are congruent, then the sides opposite them are congruent.
8. CE = CA	**8.** Definition of segment congruence.
9. $\frac{BD}{DA} = \frac{BC}{CE}$	**9.** Theorem 28 - If a line is parallel to one side of a triangle, and intersects the other two sides in different points, then it divides the other two sides proportionally.
10. $\frac{BD}{DA} = \frac{BC}{CA}$	**10.** Substitution
11. $\frac{DA}{BD} = \frac{CA}{BC}$ $\frac{AD}{DB} = \frac{AC}{BC}$	**11.** Multiplication Property for Equality. Q E D (Both sides by $\frac{DA \cdot CA}{BD \cdot BC}$)

2.
$\frac{JH}{JK} = \frac{HL}{LK}$

$\frac{9}{10} = \frac{6}{LK}$

$10 \cdot 6 = 9 \cdot LK$

$\frac{10 \cdot 6}{9} = LK$

$\frac{2 \cdot 5 \cdot 2 \cdot \cancel{3}}{3 \cdot \cancel{3}} = LK$

$\frac{20}{3} = LK$

3.
$\frac{JK}{JH} = \frac{KL}{HL}$

$\frac{8}{14} = \frac{6}{HL}$

$14 \cdot 6 = 8 \cdot HL$

$\frac{14 \cdot 6}{8} = HL$

$\frac{\cancel{2} \cdot 7 \cdot \cancel{2} \cdot 3}{\cancel{2} \cdot \cancel{2} \cdot 2} = HL$

$\frac{21}{2} = HL$

4.
$\frac{HJ}{KJ} = \frac{HL}{KL}$

$\frac{25}{15} = \frac{10}{KL}$

KL + LH = KH

KL + 10 = KH

$15 \cdot 10 = 25 \cdot KL$

$\frac{15 \cdot 10}{25} = KL$

$\frac{3 \cdot \cancel{5} \cdot 2 \cdot \cancel{5}}{\cancel{5} \cdot \cancel{5}} = KL$

6 = KL

6 + 10 = KH

16 = KH

5. $\frac{JH}{JK} = \frac{HL}{KL}$

$HK = HL + LK$
$HK = HL + 6$
$HK - 6 = HL$

$\frac{20}{8} = \frac{HK - 6}{6}$

$8\,(HK - 6) = 20 \cdot 6$
$8 \cdot HK - 48 = 120$
$8\,HK = 168$
$HK = 21$

6. $\frac{KJ}{HJ} = \frac{KL}{HL}$

$KL + LH = KH$
$KL + LH = 9$
$LH = 9 - KL$

$\frac{10}{8} = \frac{KL}{9 - KL}$

$8 \cdot KL = 10\,(9 - KL)$
$8\,KL = 90 - 10\,KL$
$18\,KL = 90$
$KL = 5$
$HL = 9 - 5 = 4$

7. $\frac{HJ}{KJ} = \frac{LH}{LK}$

$KL + LH = KH$
$KL + LH = 18$
$LH = 18 - KL$

$\frac{12}{15} = \frac{18 - KL}{KL}$

$15\,(18 - KL) = 12 \cdot KL$
$270 - 15\,KL = 12\,KL$
$270 = 27\,KL$
$10 = KL$

8. $\frac{8}{z} = \frac{x}{y}$

9. $\frac{3}{5} = \frac{x}{y}$

10. $\frac{8}{z} = \frac{3}{5}$

$z \cdot 3 = 8 \cdot 5$
$z \cdot 3 = 40$
$z = \frac{40}{3}$

11. $\frac{DE}{\frac{40}{3}} = \frac{3}{8}$

$DE \cdot 8 = \frac{40}{3} \cdot 3$
$DE \cdot 8 = 40$
$DE = \frac{40}{8}$
$DE = 5$

12. $\frac{x}{y} = \frac{BA}{BC}$

$\frac{x}{7} = \frac{8}{\frac{40}{3}}$

$7 \cdot 8 = x \cdot \frac{40}{3}$

$\frac{7 \cdot 8}{1} \cdot \frac{3}{40} = x$

$\frac{7 \cdot \cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 5} = x$

$\frac{21}{5} = x$

13. $\frac{AD}{DH} = \frac{AF}{FH}$

Theorem 35 - If a ray bisects an angle of a triangle, then it divides the opposite side into segments that have the same ratio as the two other sides.

$\frac{12}{15} = \frac{AF}{10}$

$15 \cdot AF = 12 \cdot 10$
$15 \cdot AF = 120$
$AF = 8$

14. $\frac{AB}{BD} = \frac{AE}{EH}$

Theorem 28 - If a line is parallel to one side of a triangle and intersects the other two sides in different points, then it divides the two sides proportionally.

$\frac{2}{7} = \frac{AE}{14}$

$7 \cdot AE = 2 \cdot 14$
$7 \cdot AE = 28$
$AE = 4$

15. $\frac{AC}{CD} = \frac{AG}{GH}$

Theorem 28 - If a line is parallel to one side of a triangle and intersects the other two sides in different points, then it divides the two sides proportionally.

Let GH = x

AG = AH - GH

AG = 15 - x

$\frac{8}{3} = \frac{15-x}{x}$

3 (15 - x) = 8 · x

45 - 3x = 8x

45 = 11 x

$\frac{45}{11} = x = GH$

$AG = 15 - \frac{45}{11} = \frac{165}{11} - \frac{45}{11} = \frac{120}{11}$

16. $\frac{AF}{DH} = \frac{AD}{FH}$

Theorem 35 - If a ray bisects an angle of a triangle, then it divides the opposite side into segments that have the same ratio as the two other sides.

Let CD = x

AD = AC + CD

$\frac{6}{8} = \frac{AC + CD}{13}$

$\frac{6}{8} = \frac{7 + x}{13}$

8 (7 + x) = 6 · 13

56 + 8x = 78

8 x = 22

$x = \frac{22}{8} = \frac{\cancel{2} \cdot 11}{\cancel{2} \cdot 2 \cdot 2}$

$x = \frac{11}{4} = CD$

17. Using Theorem 35:

$\frac{AF}{FC} = \frac{BA}{BC}$

$\frac{6}{x + 6} = \frac{4 + 5}{12}$

$\frac{6}{x + 6} = \frac{9}{12}$

(x + 6) · 9 = 6 · 12

9 x + 54 = 72

9 x = 18

x = 2

Using Theorem 28:

$\frac{AD}{DB} = \frac{AE}{EC}$

$\frac{5}{4} = \frac{6 + x}{6}$

4 (6 + x) = 5 · 6

24 + 4 x = 30

4x = 6

4x = 6

$x = \frac{6}{4} = \frac{\cancel{2} \cdot 3}{\cancel{2} \cdot 2} = \frac{3}{2}$

The two theorems (and two equations) give different values for x. This is not possible (i.e., a contradiction).

p. 431 – Lesson 7 — Theorem 36

1. **4)** Definition of right angle
5) Definition of right triangle
7) Theorem 31 - If you have a right triangle, then the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the two legs.
8) Substitution
10) Substitution
11) Definition of Congruent Line Segments
12) S.S.S. Congruence Postulate
13) C.P.C.T.C.
14) Definition of Congruent Angles
15) Substitution
16) Definition of right angle
17) Definition of Right Triangle

2.

Statement	Reason
1. $\triangle ABC$ with altitude $\overline{CD}$	**1.** Given
2. $\overline{CD} \perp \overline{AB}$	**2.** Definiton of Altitude
3. $\angle CDB$ is a right angle $\angle CDA$ is a right angle	**3.** Definiton of Perpendicular
4. $\triangle CDB$ is a right triangle $\triangle CDA$ is a right triangle	**4.** Definition of Right Triangle
5. $a^2 = y^2 + (CD)^2$ $b^2 = x^2 + (CD)^2$	**5.** Theorem 31 - If you have a right triangle, then the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the two legs.
6. $a^2 - y^2 = (CD)^2$ $b^2 - x^2 = (CD)^2$	**6.** Addition Property (Subtraction) for equality.
7. $a^2 - y^2 = b^2 - x^2$	**7.** Substitution

3.

Statement	Reason
1. $\angle ACB$ is a right angle	**1.** Given
2. $\triangle ACB$ is a right triangle $\triangle DCB$ is a right triangle	**2.** Definiton of Right Triangle
3. $BD^2 = DC^2 + BC^2$ $AB^2 = AC^2 + BC^2$	**3.** Theorem 31 - If you have a right triangle, then the square of the measure of the hypotenuse is equal to the sum of the squares of the measures of the two legs.
4. $BD^2 - DC^2 = BC^2$ $AB^2 - AC^2 = BC^2$	**4.** Addition Property (Subtraction) for equality
5. $BD^2 - DC^2 = AB^2 - AC^2$	**5.** Substitution
6. $BD^2 - DC^2 + AC^2 = AB^2$	**6.** Addition Property for equality
7. $BD^2 + AC^2 = AB^2 + DC^2$	**7.** Addition Property for equality

4.

$$AC^2 + CB^2 = AB^2$$
$$4^2 + 4^2 = BD^2$$
$$16+16=BD^2$$
$$32=BD^2$$
$$\sqrt{32}=BD$$
$$\sqrt{16 \bullet 2}=BD$$
$$4\sqrt{2} = BD$$

$$CB + BD = CD$$
$$4 + 4\sqrt{2} = CD$$
$$AC^2 + CD^2 = AD^2$$
$$4^2 + (4 + 4\sqrt{2})^2 = AD^2$$
$$16 + (4 + 4\sqrt{2})(4 + 4\sqrt{2}) = AD^2$$
$$16 + 16 + 32\sqrt{2} + 16 \bullet 2 = AD^2$$
$$16 + 16 + 32\sqrt{2} + 32 = AD^2$$
$$64 + 32\sqrt{2} = AD^2$$
$$32(2 + \sqrt{2}) = AD^2$$
$$\sqrt{32(2+\sqrt{2})} = AD$$
$$\sqrt{32} \bullet \sqrt{2+\sqrt{2}} = AD$$
$$\sqrt{16 \bullet 2} \bullet \sqrt{2+\sqrt{2}} = AD$$
$$4\sqrt{2} \bullet \sqrt{2+\sqrt{2}} = AD$$
$$4 \bullet \sqrt{4 + 2\sqrt{2}} = AD$$

5.

a) $a^2 + b^2 = c^2$

$3^2 + 4^2 = 5^2$

$9 + 16 = 25$

$25 = 25$

Yes

$\angle B$

b) $a^2 + b^2 = c^2$

$2^2 + 3^2 = 4^2$

$4 + 9 = 16$

$13 \neq 16$

No

c) $a^2 + b^2 = c^2$

$(2.5)^2 + 3^2 = 4^2$

$6.25 + 9 = 16$

$15.25 \neq 16$

No

5.

d) $a^2 + b^2 = c^2$

$(7.5)^2 + 18^2 = (19.5)^2$

$56.25 + 324 = 380.25$

$380.25 = 380.25$

Yes

$\angle C$

e) $a^2 + b^2 = c^2$

$(\sqrt{5})^2 + (\sqrt{5})^2 = 5^2$

$5 + 5 = 25$

$10 \neq 25$

No

f) $a^2 + b^2 = c^2$

$(\sqrt{3})^2 + (\sqrt{8})^2 = (\sqrt{11})^2$

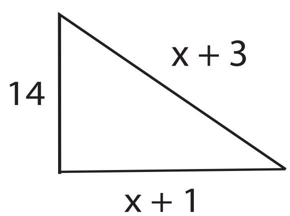
$3 + 8 = 11$

$11 = 11$

Yes

$\angle 8$

6.



$(x+3)^2 = 14^2 + (x+1)^2$

$(x+3)(x+3) = 196 + (x+1)(x+1)$

$x^2 + 6x + 9 = 19 + x^2 + 2x + 1$

$4x = 188$

$x = 47$

$x + 1 = 48$

$x + 3 = 50$

7.

$a^2 + b^2 = c^2$

$9^2 + 40^2 = 41^2$

$81 + 1600 = 1681$

$1681 = 1681$

Yes

$a^2 + b^2 = c^2$

$18^2 + 80^2 = 82^2$

$324 + 6400 = 6724$

$6724 = 6724$

Yes

Doubling the measure of each side of the triangle gives measures which fit the pythagorean theorem.

8.

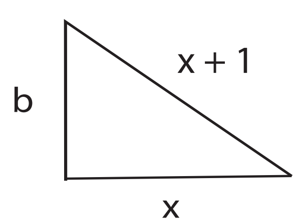
$x^2 + 24^2 = 25^2$

$x^2 + 576 = 625$

$x^2 = 49$

$x = 7$

9.



$b^2 + x^2 = (x+1)^2$

$b^2 + x^2 = (x+1)(x+1)$

$b^2 + x^2 = x^2 + 2x + 1$

$b^2 = 2x + 1$

$b = \sqrt{2x+1} = \sqrt{x + (x+1)}$

(Square Root of length of leg plus length of hypotenuse)

17.

$a^2 + b^2 \ ? \ c^2$

$9^2 + 10^2 \ ? \ 12^2$

$81 + 100 \ ? \ 144$

$181 > 144$

$\therefore a^2 + b^2 > c^2$

Acute Triangle

18.

$a^2 + b^2 \ ? \ c^2$

$(0.9)^2 + (4.0)^2 \ ? \ (4.1)^2$

$.81 + 16 \ ? \ 16.81$

$16.81 = 16.81$

$\therefore a^2 + b^2 = c^2$

Right Triangle

19.

$a^2 + b^2 \ ? \ c^2$

$(1/5)^2 + (4/5)^2 \ ? \ 1$

$\frac{1}{25} + \frac{16}{25} \ ? \ 1$

$\frac{17}{25} < 1$

$\therefore a^2 + b^2 < c^2$

Obtuse Triangle

20.

$(AD)^2 + (DC)^2 = (AC)^2$

$6^2 + 15^2 = (AC)^2$

$36 + 225 = (AC)^2$

$261 = (AC)^2$

$\sqrt{261} = (AC)^2$

$(AB)^2 + (BC)^2 = (AC)^2$

$10^2 + 17^2 = (\sqrt{261})^2$

$100 + 289 = 261$

$389 > 261$

$\therefore a^2 + b^2 > c^2$

Acute Triangle

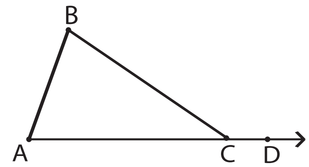
Unit IV — Triangles

Part G - Congruence - Part 3 (Triangle Inequalities)

p. 437 – Lesson 1 — Theorem 37

1. a.) Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle.

b.)



c.) Given: △ ABC with exterior ∠ BCD

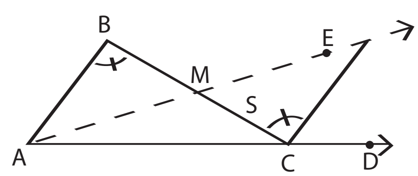
d.) Prove: m ∠ BCD > m ∠ B
m ∠ BCD > m ∠ A

e.) Proof

Statement	Reason
1. △ ABC with exterior ∠ BCD	**1.** Given
2. m ∠ BCD = m ∠ A + m ∠ B	**2.** Theorem 26 - If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
3. m ∠ BCD > m ∠ B	**3.** Definition of " is greater than " - For any numbers a and b, a > b if and only if there is a positive number c such that a = b + c.
4. m ∠ BCD > m ∠ A	**4.** Definition of " is greater than"

2. a.) Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle.

b.)



c.) Given: △ ABC with exterior ∠ BCD

d.) Prove: m ∠ BCD > m ∠ B
m ∠ BCD > m ∠ BAC

e.) Proof

Statement	Reason
1. △ ABC with exterior ∠ BCD	**1.** Given
2. Choose point M on $\overline{BC}$ so that point M is the midpoint of $\overline{BC}$.	**2.** Theorem 4 - If you have a given line segment, BC, then that segment has exactly one midpoint.
3. Draw $\overrightarrow{AE}$ through point M	**3.** Postulate 2 - Uniquenesss of lines, planes, and spaces - For any two different points, A and M, there is exactly one line, $\overrightarrow{AE}$, containing them.
4. Choose point E on $\overrightarrow{AE}$ so that AM = EM	**4.** Postulate 6 - Ruler - Second Assumption - To Every pair of points on a line, there corresponds exactly one real number, called the unique distance between the points.

2. (continued)

Statement	Reason
5. Draw $\overline{CE}$	**5.** Postulate 2 - Uniqueness of lines, planes, and spaces.
6. BM = MC	**6.** Definition of midpoint
7. $\overline{BM} \cong \overline{MC}$	**7.** Definition of congruent line segments.
8. ∠ AMB ≅ ∠ EMC	**8.** Theorem 15 - If two lines intersect, then the vertical angles formed are congruent.
9. $\overline{AM} \cong \overline{EM}$	**9.** Definition of congruent line segments
10. △ AMB ≅ △ EMC	**10.** Postulate 13 - SAS Congruence Assumption - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the two triangles are congruent.
11. ∠ ABM ≅ ∠ ECM	**11.** C.P.C.T.C.
12. m ∠ BCD = m ∠ DCE + m ∠ ECM	**12.** Postulate 7 - Protractor - Fourth Assumption - Angle Addition Assumption.
13. m ∠ BCD = m ∠ DCE + m ∠ ABM	**13.** Substitution
14. m ∠ BCD > m ∠ B (m ∠ ABM)	**14.** Definition of " is greater than" - For any numbers a and b, a > b is and only if there is a positive number c such a that a = b + c.
15. $\overline{AB} \parallel \overline{EC}$	**15.** Theorem 20 - If two lines are cut by a transversal so that alternate interior angles are congruent, then the two lines are parallel.
16. ∠ BAC ≅ ∠ DCE	**16.** Postulate 11 - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
17. m ∠ BAC = m ∠ DCE	**17.** Definition of congruent angles
18. m ∠ BCD = m ∠ DCE + m ∠ ECM	**18.** Postulate 7 - Protractor - Fourth Assumption - Angle Addition Assumption.
19. m ∠ BCD = m ∠ BAC + m ∠ ECM	**19.** Substitution
20. m ∠ BCD > m ∠ BAC	**20.** Definition of " is greater than". For any numbers a and b, a > b if and only if there is a positive number c such that a = b + c.

3. **a.)** ∠ 6 **b)** cannot be determined **c)** ∠ 2 **d)** ∠ 2

4. y > x **5.** x > y

6. x > y **7.** x < y

8. m ∠ 1 > m ∠ 2 **9.** m ∠ 1 < m ∠ 2

10. m ∠ 1 < m ∠ 2 **11.** m ∠ 1 < m ∠ 2

12.

Statement	Reason
1. b = x + c	**1.** Given
2. y > b	**2.** Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle.
3. y > x + c	**3.** Substitution
4. y > x	**4.** For any numbers a and b, if there is a positive number c such that a = b + c, then a > b.

13.

Statement	Reason
1. m ∠ 1 > m ∠ 3	**1.** Given
2. m ∠ 3 > m ∠ 2	**2.** Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote angle.
3. m ∠ 1 > m ∠ 2	**3.** Transitive property for the relation " is greater than. If a > b and b > c, then a > c.

14. **a.)** x, t, s
b.) x, t, r
c.) t, s, u
d.) x, t, v

15. **a.)** y, x, 40
b.) w, 40, 30
c.) w + x, 180, y
d.) w + x + y, 330, 2x

16. **a.)** ∠ ABC - Right
∠ BCA - Acute
∠ CAB - Acute

b) ∠ ABC - Obtuse
∠ BCA - Acute
∠ CAB - Acute

c.) ∠ ABC - Acute
∠ BCA - Obtuse, Acute, or Right
∠ CAB - Obtuse, Acute, or Right

17. False - The key word is "any". If an exterior angle is an acute angle, then the interior angle of the triangle adjacent to it would be obtuse.

18. True - If the exterior angle is a right angle, then the triangle itself is a right triangle.

19. False - If the angle of the triangle is a right angle then the exterior angle is a right angle. If the angle is obtuse, the exterior angle is acute.

20. True - The acute exterior angle would have an obtuse adjacent interior angle.

21. False - The exterior angle of the right angle would be a right angle. The other two angles of the triangle are acute with obtuse exterior angles.

22. True - If the triangle is equilateral, all of its interior angles are congruent.

23. True - Two adjacent angles are supplementary. Each must measure 90.

24. False - △ XYZ could have one right angle.

25. True - The base angles of an isosceles triangle must be acute angles. The exterior angles would therefore be obtuse.

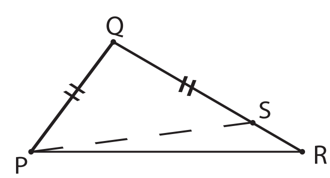
26.

Statement	Reason
1. E is in the interior of ∠ ABC	**1.** Given
2. m ∠ 1 > m ∠ EDC	**2.** Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle.
3. m ∠ EDC > m ∠ 2	**3.** Theorem 37
4. m ∠ 1 > m ∠ 2	**4.** Transitive Property for inequalities (for >)

27.

Statement	Reason
1. AB = BC	**1.** Given
2. $\overline{AB} \cong \overline{BC}$	**2.** Definition of congruent line segments.
3. ∠ A ≅ ∠ C	**3.** Theorem 33 - If two ides of a triangle are congruent, then the angles opposite them are congruent.
4. m ∠ A = m ∠ C	**4.** Definition of congruent angles
5. m ∠ 1 > m ∠ A	**5.** Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle.
6. m ∠ 1 > m ∠ C	**6.** Subsitution

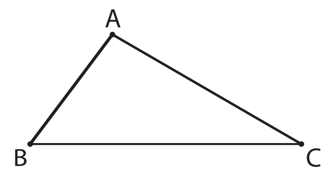
28. **a.)** Given: QR > PQ
b.) Prove: m ∠ QPR > m ∠ R



Statement	Reason
1. QR > PQ	**1.** Given
2.Choose a point S on $\overline{QR}$ such that QS = QP	**2.** Postulate 6 - Ruler - Second Assumption - To every pair of points on a line, there corresponds exactly one real number, called the unique distance between the points.
3. Draw $\overline{PS}$	**3.** Postulate 2 - First Assumption - For any two different points, there is exactly one line containing them.
4. m ∠ QPR = m ∠ QPS + m ∠ SPR	**4.** Postulate 7 - Protractor - Fourth Assumption - Angle Addition.
5. m ∠ QPR > m ∠ QPS	**5.** Definition of " is greater than " - For any numbers a and b, a > b if and only if there is a positive number c such that a = a + b
6. $\overline{QS} \cong \overline{QP}$	**6.** Definition of congruent line segments.
7. ∠ QPS ≅ ∠ QSP	**7.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite the are congruent.
8. m ∠ QPS = m ∠ QSP	**8.** Definition of congruent angles.
9. m ∠ QPR > m ∠ QSP	**9.** Substitution
10. m ∠ QSP > m ∠ R	**10.** Theorem 37 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle.
11. m ∠ QPR > m ∠ R	**11.** Transitive Property for Inequality

p. 443 – Lesson 2 - Theorem 38

1. **a.)** Theorem 38 - If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent.
   b.)

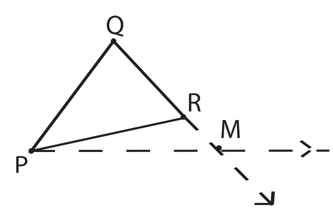


c.) Given: $\overline{AC} \ncong \overline{AB}$
d.) Prove: ∠ B ≇ ∠ C

e.) Proof

Statement	Reason
1. Assume ∠ B ≅ ∠ C	**1.** Indirect Proof Assumption
2. $\overline{AC} \cong \overline{AB}$	**2.** Theorem 34 - If two angles of a triangle are congruent, then the sides opposite them are congruent.
3. However, $\overline{AC} \ncong \overline{AB}$	**3.** Given
4. Our assumption is false, and ∠ B ≇ ∠ C	**4.** Reductio Ad Absurdum

2. **a.)** If the measure of one side of a triangle is greater than the measure of a second side, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.
   b.)



c.) Given: △ PQR
PQ > RQ
d.) Prove: m ∠ QRP > m ∠ QPR

e.)

Statement	Reason
1. △ PQR with PQ > RQ	**1.** Given
2. Extend $\overline{QR}$ to point M so that QM = QP.	**2.** Postulate 6 - Ruler - Second Assumption - To every pair of points on a line, there corresponds exactly one real number, called the unique distance between the points.
3. Draw $\overrightarrow{PM}$ through point M forming △ QPM	**3.** Postulate 2 - Uniquenesss of lines, planes, and spaces - For any two different points, P and M, the is exactly one line, (PM), containing them.
4. $\overline{QM} \cong \overline{QP}$	**4.** Definition of congruent line segments.
5. ∠ QPM ≅ ∠ QMP	**5.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.

2. (continued) Statement	Reason
6. m ∠ QPM = m ∠ QMP	**6.** Definition of congruent angles
7. m ∠ QPM = m ∠ QPR + m ∠ RPM	**7.** Postulate 7 - Protractor - Fourth Assumption - Angle Addition.
8. m ∠ QPR + m ∠ RPM = m ∠ QMP	**8.** Substitution
9. m ∠ QRP = m ∠ RPM + m ∠ QMP	**9.** Theorem 26 - If you have a given exterior angle of a triangle, then the measure of that angle is equal to the sum of the measures of the two remote interior angles.
10. m ∠ QRP = m ∠ RPM + m ∠ QPR + m ∠ RPM	**10.** Substitution
11. m ∠ QRP = m ∠ QPR + 2 m ∠ RPM	**11.** Collect like terms
12. m ∠ QRP > m ∠ QPR	**12.** Definition of " is greater than " - For any numbers a and b, a > b, if and only if, there is a positive number c such that a = b + c

3. **a.)** m ∠ C < m ∠ B < m ∠ A
b.) m ∠ C < m ∠ B < m ∠ A
c.) m ∠ C < m ∠ A < m ∠ B
d.) m ∠ B < m ∠ A < m ∠ C

4. LARGEST	SMALLEST
a.) ∠ C	∠ B
b.) ∠ N	∠ M
c.) ∠ W	∠ U
d.) ∠ H	∠ I

5. ∠ R ; Theorem 38 - If two sides of a triangle are not congruent, then the angles opposite those sides are not congruent.

6. m ∠ A
> m ∠ R

Corollary 38 a - If the measure of one side of a triangle is greater than the measure of a second side of the triangle, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.

7. ∠ 2 **8.** ∠ 1
9. ∠ 1 **10.** ∠ 2
11. ∠ 2 **12.** <
13. = , = , <

14. Never True

15. Always True

16. Sometimes True

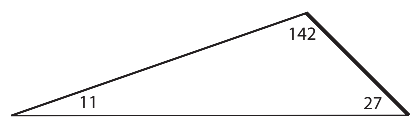
17. Sometimes True

18. Always True

19.

Statement	Reason
1. EB > AE ; CD > BC	**1.** Given
2. m ∠ A > m ∠ EBA	**2.** Corollary 38 a - If the measure of one side of a triangle is greater than the measure of a second side, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.
3. ∠ EBA ≅ ∠ CBD	**3.** Theorem 15 - If two lines intersect, then the vertical angles formed are congruent.
4. m ∠ EBA = m ∠ CBD	**4.** Definition of congruent angles
5. m ∠ CBD > m ∠ D	**5.** Corollary 38 a - If the measure of one side of a triangle is greater than the measure of a second side, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.
6. m ∠ A > m ∠ CBD	**6.** Substitution
7. m ∠ A > m ∠ D	**7.** Transitive Property for Inequality

20. Counter Example : 11 + 27 $\ngtr$ 142



21.

Statement	Reason
1. ABCD is a trapezoid with $\overline{AB} \parallel \overline{CD}$	**1.** Given
2. Assume ∠ C and ∠ D are right angles	**2.** Indirect Proof Assumption
3. $\overline{BC} \perp \overline{CD}$; $\overline{AD} \perp \overline{DC}$	**3.** Definitions of Perpendicular
4. $\overline{AD} \parallel \overline{BC}$	**4.** Theorem 22 - If two lines are perpendicular to a third line, then the two lines are parallel.
5. ABCD is a parallelogram	**5.** Definition of Parallelogram - A quadrilateral with both pairs of opposite sides parallel.
6. However, this contradicts the given.	**6.** A Trapezoid is a Quadrilateral with exactly one pair of parallel sides.
7. Our assumption is false, and ∠C and ∠D are not both right angles.	**7.** Reductio ad absurdum

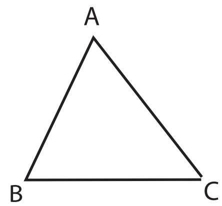
Unit IV — Triangles

Part G - Congruence - Part 3 (Triangle Inequalities)

p. 447 – Lesson 3 - Theorem 39

1. a.) Theorem 39 - If two angles of a triangle are not congruent, then the sides opposite those angles are not congruent.

b.)



c.) Given: $\angle B \ncong \angle C$

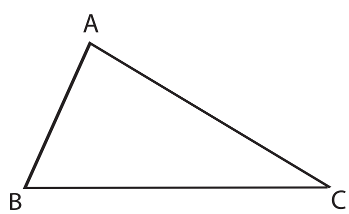
d.) Prove: $\overline{AC} \ncong \overline{AB}$

e.) Proof

Statement	Reason
1. Assume $\overline{AC} \cong \overline{AB}$	**1.** Indirect Proof Assumption
2. $\angle B \cong \angle C$	**2.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
3. However, $\angle B \ncong \angle C$	**3.** Given
4. Our assumption is false, and $\overline{AC} \ncong \overline{AB}$	**4.** Reductio Ad Absurdum

2. a.) If the measure of one angle of a triangle is greater than the measure of a second angle of the triangle, then the measure of the side opposite the larger angle is greater than the measure of the side opposite the smaller angle.

b.)



c.) Given: $m\angle A > m\angle C$

d.) Prove: BC > AB

e.)

Statement	Reason
1. Assume BC = AB	**1.** Indirect Proof Assumption
2. $\overline{BC} \cong \overline{AB}$	**2.** Definition of congruent line segments
3. $\angle A \cong \angle C$	**3.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
4. $m\angle A = m\angle C$	**4.** Definition of congruent angles
5. However, $m\angle A > m\angle C$	**5.** Given
6. Our Assumption is False	**6.** Reductio Ad Absurdum
7. Assume BC < AB or, written another way, AB > BC.	**7.** Indirect Proof Assumption
8. $m\angle C > m\angle A$	**8.** Corollary 38 - a - If the measure of one side of a triangle is greater than the measure of a second side, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.
9. However, $m\angle A > m\angle C$	**9.** Given
10. This contradicts the given information. Our assumption is false and BC > AB.	**10.** Reductio Ad Absurdum

3. **a.)** $\overline{AD}$, $\overline{AC}$, $\overline{DC}$
b.) $\overline{CB}$, $\overline{BE}$, $\overline{EC}$
c.) $\overline{DB}$, $\overline{AD}$, $\overline{AB}$
d.) $\overline{DE}$, $\overline{DC}$. $\overline{EC}$
e.) $\overline{BE}$, $\overline{AB}$, $\overline{AE}$

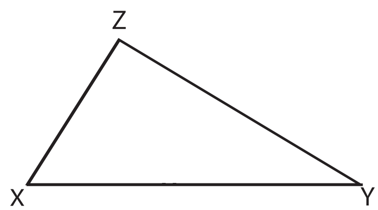
4. **a.)** $\overline{AB}$, $\overline{BC}$, $\overline{AC}$
b.) $\overline{AB}$, $\overline{AC}$, $\overline{BC}$
c.) $\overline{AB}$, $\overline{BC}$, $\overline{AC}$
d.) $\overline{AB}$, $\overline{AC}$, $\overline{BC}$

5. RM ; Theorem 39 - If two angles of a triangle are not congruent, then the sides opposite those angles are not congruent.

6. BC > AB
AB < AC

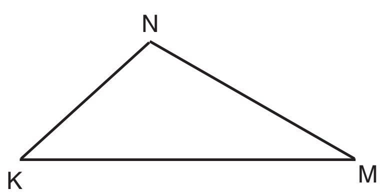
Corollary 39 a - If the measure of one angle of a triangle is greater than the measure of a second angle of the triangle, then the measure of the side opposite the larger angle is greater than the measure of the side opposite the smaller angle.

7.



a) XY = 1 5/16 inches or 33 millimeters
XZ = 13/16 inches or 21 millimeters

b)



c) ZY = 1 13/16 inches or 31 millimeters
NM = (answers will vary)
$\overline{ZY}$ is longer than $\overline{NM}$

d) m ∠ X is greater than m∠ K
$\overline{ZY}$ is greater than $\overline{NM}$
The longest side is opposite the larger angle.

e) the measure of the side opposite the larger angle is greater than the measure of the side opposite the smaller angle.

f) The Greater the opening of a door (on a hinge), or the greater the angle a door makes with the door opening, the greater the distance between the door edge and the door stop.

8. >
9. <
10. <
11. >
12. <
13. <
14. <

17.

Statement	Reason
1. ST > RS	**1.** Given
2. m ∠ TRS > m ∠ RTS	**2.** Corollary 38 a - If the measure of one side of a triangle is greater than the measure of a second side of the triangle, then the measure of the angle opposite the longer side is greater than the measure of the angle opposite the shorter side.
3. m ∠ TRS = m ∠ 3 + m ∠ 4 m ∠ STR = m ∠ 1 + m ∠ 2	**3.** Postulate 7 - Protractor - Fourth Assumption - Angle Addition Property.
4. ∠ 1 ≅ ∠ 2 ∠ 3 ≅ ∠ 4	**4.** Given
5. m ∠ 1 = m ∠ 2 m ∠ 3 = m ∠ 4	**5.** Definition of Congruent Angle
6. m ∠ 3 + m ∠ 4 > m ∠ 1 + m ∠ 2	**6.** Substitution
7. m ∠ 3 + m ∠ 3 > m ∠ 2 + m ∠ 2	**7.** Substitution
8. 2 m ∠ 3 > 2 m ∠ 2	**8.** Princliples of Arthmetic
9. m ∠ 3 > m ∠ 2	**9.** Multiplication Property for Inequalities
10. TU > UR	**10.** Corollary 39 a - If the measure of one angle of a triangle is larger than the measure of a second angle of the triangle, then the measure of the side opposite th larger angle is greater than the measure of the side opposite the smaller angle.

18.

Statement	Reason
1. C is the intersection of $\overline{AD}$ and $\overline{BE}$.	**1.** Given
2. m ∠ B > m ∠ A	**2.** Given
3. AC > BC	**3.** Corollary 39 a - If the measure of one angle of a triangle is greater than the measure of a second angle of the triangle, then the measure of the side opposite the larger angle is greater than the measure of the side opposite the smaller angle.
4. m ∠ E > m ∠ D	**4.** Given
5. CD > CE	**5.** Corollary 39 - a
6. AC + CD > BC + CE	**6.** Addition property for Inequalities
7. AC + CD = AD BC + CE = BE	**7.** Postulate 6 - Ruler - Fourth Assumption - Segment Addition
8. AD > BE	**8.** Substitution

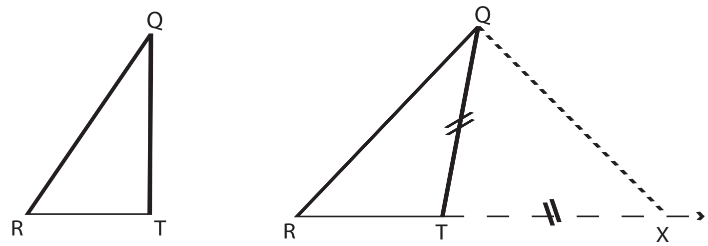
Unit IV — Triangles

Part G - Congruence - Part 3 (Triangle Inequalities)

p. 452 – Lesson 4 - Theorem 40

1. a.) Theorem 40 - If you have the sum of the measures of two sides of a triangle, then that sum is greater than the measure of the third side of the triangle

b.)



c.) Given: △ RTQ is any triangle

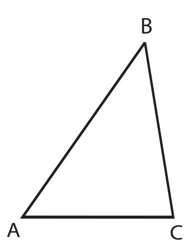
d.) Prove: RT + TQ > RQ

e.) Proof

Statement	Reason
1. △ RTQ is any triangle	**1.** Given
2. Extend $\overline{RT}$ to point X so that TX = TQ	**2.** Postulate 6 - Ruler - 2nd Assumption - To every pair of points on a line, there corresponds exactly one real number, called the unique distance between the points.
3. Draw $\overline{QX}$	**3.** Postulate 2 - For any two different points, there is exactly one line containing them.
4. $\overline{TX} \cong \overline{TQ}$	**4.** Definition of Congruent Segments
5. ∠ TQX ≅ ∠ TXQ	**5.** Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
6. m ∠ TQX = m ∠ TXQ	**6.** Definition of Congruent Angles
7. m ∠ RQX = m ∠ RQT + m ∠ TQX	**7.** Postulate 7 - Protractor - Fourth Assumption - Angle Addition
8. m ∠ RQX > m ∠ TQX	**8.** Inequality Theorem - For any numbers a and b, if there is a positive number c such that a = b + c, then a > b.
9. m ∠ RQX > m ∠ TXQ	**9.** Substitution
10. RX > RQ	**10.** Corollary 39a - If the measure of one angle of a triangle is greater than the measure of a second angle of the triangle, then the measure of the side opposite the larger angle is greater than the measure of the side opposite the smaller angle.
11. RX = RT + TX	**11.** Postulate 6 - Ruler - Fourth Assumption - Segment Addition.
12. RT + TX > RQ	**12** Substitution
13. RT + TQ > RQ	**13.** Substitution

2. No ; For this to be true, the three points would have to be colliner and there would be no triangle.

3.



BC + AC > AB
AB + AC > BC

4. Yes
9 + 7 > 14
7 + 14 > 9
9 + 14 > 7

5. No
7 + 3 > 4
7 + 4 > 3
But $3 + 4 \not> 7$

6. Yes
3 + 4 > 5
5 + 4 > 3
3 + 5 > 4

7. No
18 + 10 > 7
18 + 7 > 10
But 7 + 10 $\ngtr$ 18

8. No
x + 2 > x - 2
But x $\ngtr$ 2 + x - 2
And 2 $\ngtr$ x + x -2
Note: if x is 1 or 2, the problem is actually meaningless

9. Yes
2x + x > x + 1
x + x + 1 > 2x
2x + x + 1 > x

10. x + 5 > 7 AND 5 + 7 > x
x + 5 + -5 > 7 + -5   12 > x
x > 2   or
or   x < 12
2 < x

2 < x AND x < 12
2 < x < 12
Length of third side is between 2 and 12

11. x + 80 > 100 AND 80 + 100 > x
x + 80 + -80 > 100 + -80   180 > x
x > 20   or
or   x < 180
20 < x

20 < x AND x < 180
20 < x < 180
Length of third side is between 20 and 180

12. x + 3 1/2 > 7 AND 3 1/2 + 7 > x
x + 3 1/2 + -3 1/2 > 7 + -3 1/2 AND 10 1/2 > x
x > 3 1/2   or
or   x < 10 1/2
3 1/2 < x

3 1/2 < x AND x < 10 1/2
3 1/2 < x < 10 1/2
Length of third side is between 3 1/2 and 10 1/2

13. x + 8 > 8 AND 8 + 8 > x
x + 8 + -8 > 8 + -8 AND 16 > x
x > 0 AND 16 > x
0 < x AND x < 16

0 < x < 16
Length of third side is between 0 and 16

14. x + 2.5 > 7.5 AND 2.5 + 7.5 > x
x + 2.5 + -2.5 > 7.5 + -2.5 AND 10 > x
x > 5 AND 10 > x
5 < x AND x < 10

5 < x < 10
Length of third side is between 5 and 10

15. x + 2 1/4 > 5 AND 2 1/4 + 5 > x

x + 2 1/4 + -2 1/4 > 5 + -2 1/4 AND 7 1/4 > x

x > 2 3/4 AND 7 1/4 > x

2 3/4 < x AND x < 7 1/4

2 3/4 < x < 7 1/4

Length of third side is between 2 3/4 and 7 1/4

16.

Statement	Reason
1. Quadrilateral ABCD	**1.** Given
2. AB + BC > AC CD + DA > AC	**2.** Theorem 40 - If you have the sum of the measures of two sides of triangle, then that sum is greater than the measure of the third side of the triangle.
3. AB + BC + CD + DA > AC + AC	**3.** Addition Property for Inequalities
4. AB + BC + CD + DA > 2 AC	**4.** Collect like terms