Unit I — The Structure of Geometry

Part A — What is Geometry?

p. 2 – Lesson 1 — Origin and Structure

	Part of Speech	Meaning
1.	Number symbol	Represents the ratio of the circumference of a circle to the diameter of the circle. It has a value of approximately $\frac{22}{7}$ or 3.146…. It is an irrational, real number.
2.	Placeholder symbol	Holds the place of a number until it is identified.
3.	Number symbol	Fifteen and three hundred eighteen thousandths.
4.	Number symbol	Read "the square root of eleven". It is approximately equal to 3.317…. It is an irrational, real number.
5.	Operation symbol	Indicates division.
6.	Relation symbol	Read "is not greater than or equal to."
7.	Number symbol	Indicates the third root of -27
8.	Grouping symbol	Parentheses.
9.	Number symbol	Seventeen and one-third.
10.	Relation symbol	Symbol used in geometry to indicate that two figures have the same size and shape. The symbol is read "is congruent to."
11.	Number symbol	Symbol for an irrational, real number equal to approximately 2.7182818….
12.	Number symbol	Seven and repetend forty-five hundredths. It is a rational, real number.
13.	Grouping symbol	Braces, usually used in set-notation.
14.	Number symbol	Ten sevenths.
15.	Number symbol	Six to the power of three; indicates to use 6 as a factor three times as in 6 • 6 • 6.
16.	Operation symbol	Indicates addition.
17.	Relation symbol	Indicates "is perpendicular to" even though the "little square box" is not present.
18.	Number symbol	Indicates the second root of 16
19.	Grouping symbol	Brackets
20.	Operation symbol	Indicates multiplication
21.	Relation symbol	Read "is not equal to."

Unit I — The Structure of Geometry

Part A — What is Geometry?

p. 5 – Lesson 2 — More on Things

1. Line

2. Plane

3. Point

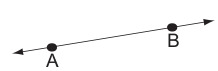
4. (Answers may vary); dot on a map; tip of a pencil; star in the sky.

5. (Answers may vary); laser beam; telephone wires strung from pole to pole; guitar string.

6. (Answers may vary); screen on a plasma television; floor in a gymnasium; table top.

7. Line , $\overleftrightarrow{XY}$.

8. $\overleftrightarrow{AR}$ or $\overleftrightarrow{RA}$; $\overleftrightarrow{RT}$ or $\overleftrightarrow{TR}$; $\overleftrightarrow{AT}$ or $\overleftrightarrow{TA}$;

9.


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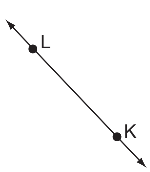

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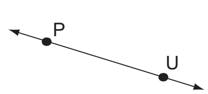
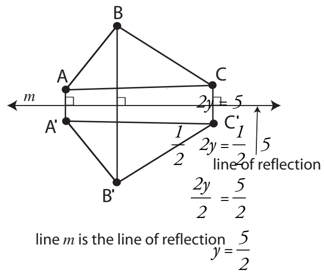
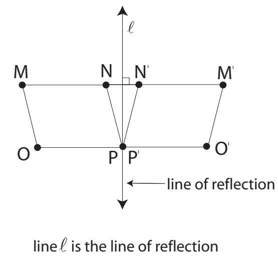


6. **a)** Trace the triangle ABC and triangle A'B'C'.
b) Draw a line segment connecting A to A' (or B to B' or C to C')
c) Draw the perpendicular bisector (line m) of the line segment(s).



7. **a)** Trace the quadrilateral MNOP and the quadrilateral M'N'O'P'.
b) Draw a line segment connecting M to M' (or N to N' or O to O')
c) Draw the perpendicular bisector (line) of the line segment(s).



8. In a reflection, size of segments and angles is maintained.

$2y = 5$

$\frac{1}{2} \cdot 2y = \frac{1}{2} \cdot 5$

$\frac{2y}{2} = \frac{5}{2}$

$y = \frac{5}{2}$

$x = 3$

$3^2 + c^2 = 5^2$

$9 + c^2 = 25$

$9 + -9 + c^2 = 25 + -9$

$c^2 = 16$

$c = 4$

$a° = 90°$

9. In a reflection, size of segments and angles is maintained.

$5 = y$

$3x = x + 6$

$3x + -x = x + 6 + -x$

$2x = 6$

$\frac{1}{2} \cdot 2x = \frac{1}{2} \cdot 6$

$\frac{2x}{2} = \frac{6}{2}$

$x = 3$

10. In a reflection, size of segments and angles is maintained.

$y° = 100°$ *The sum of the angles of a triangle is 180°.*

$50 + y° + x° = 180$

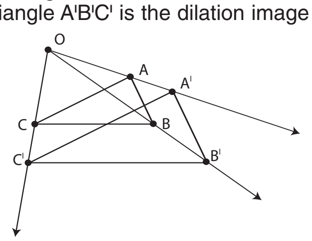
$50 + 100 + x° = 180$

$150 + x° = 180$

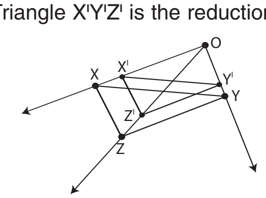
$150 + -150 + x° = 180 + -150$

$x° = 30°$

11. **a)** Trace triangle ABC with point O in the position shown.
b) Draw rays OA, OB, and OC (the rays will extend through the points A, B, C).
c) Measure line segments OA, OB, and OC.
d) Then draw line segments OA', OB', and OC' to lengths 1.5 times the lengths of OA, OB, and OC respectively.
e) Triangle A'B'C' is the dilation image of triangle ABC.



12. **a)** Trace triangle XYZ with point O in the position shown.
b) Draw rays OX, OY, and OZ.
c) Measure line segments OX', OY', and OZ' to lengths that are 0.75 times the length of line segments OX, OY, and OZ respectively.
d) Triangle X'Y'Z' is the reduction image of triangle XYZ.



Unit I — The Structure of Geometry

Part A — What is Geometry?

p. 16 – Lesson 4 — More on Relations

1. True	**2.** False	**3.** False	**4.** True	**5.** True	**6.** False	**7.** True
8. F	**9.** H	**10.** D	**11.** G	**12.** A	**13.** G	**14.** C
15. B	**16.** F	**17.** True	**18.** True	**19.** True	**20.** True	**21.** False
22. True	**23.** True	**24.** False	**25.** {K, Q, R} {K, L, M} {N, M, P} (other answers possible)			

26. {N, M, L} {Q, R, M} (other answers possible)

27. $\overleftrightarrow{KM}$ and $\overleftrightarrow{NQ}$ intersect at point L
$\overleftrightarrow{KP}$ and $\overleftrightarrow{NQ}$ intersect at point Q (other answers possible)
$\overleftrightarrow{NP}$ and $\overleftrightarrow{KL}$ intersect at point M

28. Point K; Point R

29. Yes; Yes; The given point would need to be the intersection of the two line or the intersection of the ten lines (through that one point.)

30. Yes; Yes; The given line would need to be the intersection of the two planes or the intersection of the ten planes (through that one line.)

Unit I — The Structure of Geometry

Part A — What is Geometry?

p. 23 – Lesson 5 — More on Groupings

1. **a)** {16, 17, 18, …} **b)** {G, E, O, M, T, R, Y} **c)** { } **d)** {s | s is a student enrolled in classes at Harvard}

2. **a)** Not well defined. We do not have agreement as to the meaning of "wealth."
b) Not well defined. A "great book" to one reader might not be considered a "great book" to another reader. We do not have agreement on what is a "great book."
c) Well defined. We can list each number and decide if the number is a natural number and if it is less than 100. We have accepted definitions to help us decide.
d) Well defined. We know how to form all subsets of the given set.
e) Not well defined. We can identify an employee. A "good" employee would not be so easy to identify unless an agreement could be made as to the meaning of "good."

3. **a)** P = {u, v, w, x, y, z} **b)** {1, 2} ⊂ {1, 2, 3, 4} **c)** {0, 1} ⊄ {1, 2, 3, 4} **d)** 0 ∉ { } **e)** {0} ≠ { }

4. A = C, E = I, F = H

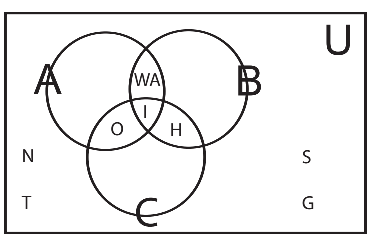
5. **a)** ⊆ **b)** ∉ **c)** ⊂ or ⊆ **d)** ∈ **e)** ⊄ **f)** ⊆ **g)** ⊄

6. A ∩ B = {3, 4}
A U B = {1, 2, 3, 4, 5, 6}

7. A ∩ B = { }
A U B = {0, 1, 2, 3, 4, 5, 6, …} or
A U B = {whole numbers}

8. A ∩ B = {2, 4, 6, 8, … }
A U B = {1, 2, 3, 4, 5, 6, 7, …} or
A U B = {counting numbers}

9.



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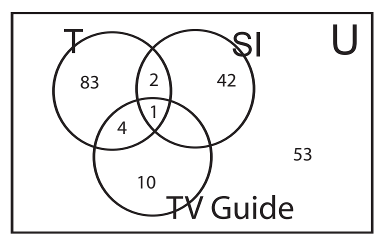


Fifty-three of those surveyed read none of the three magazines.

Unit I — The Structure of Geometry
Part B — The Scope of Our Geometry

p. 28 – Lesson 1 — Undefined Terms

1. **a)** coordinate geometry (Note: you might argue that it is synthetic as well, although these points represent subjects, not points in space.)
b) network geometry (or synthetic geometry)
c) discrete geometry (or coordinate geometry)
d) network geometry
e) synthetic geometry
f) discrete geometry

Note: Traversable Network **1)** only even vertices
2) two odd vertices with all others even

2. **a)** Traversable – Start at an odd node as in the upper middle of the box or lower middle of the box. Answers (or tracings) could vary.
b) Traversable – There are two odd nodes. Start at one of them. Traceable patterns are different.
c) Not Traversable – The network has four odd nodes. More than two odd nodes are not traversable.
d) Not Traversable – Actually same figure as "C".
e) Not Traversable – The figure has all odd nodes.
f) Traversable – Start at any node.
g) Traversable – Start at any node.
h) Traversable – The network has two odd nodes. Start at either odd node.
i) Traversable – Start at an odd node, either upper left or lower right. Answers may vary.
j) Not Traversable – The network has four odd nodes. More than two odd nodes are not traversable.

Unit I — The Structure of Geometry
Part B — The Scope of Our Geometry

p. 32 – Lesson 2 — Simple Plane Closed Curves

Geometric Figure	Curve	Plane Curve	Closed Plane Curve	Simple Closed Plane Curve	Simple Closed Plane Curve Made Up Of Only Straight Line Segments
					1
		2			
				3	
		4			
					5
		6			
			7		
8					
			9		
10					
11					
			12		

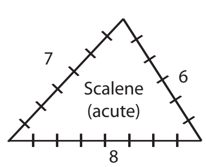
Unit I — The Structure of Geometry
Part B — The Scope of Our Geometry

p. 36 – Lesson 3 — Undefined Terms

1. A, E, H

2. **a)** Pentagon (5 sides)
b) Kite (quadrilateral - 4 sides)
c) Heptagon (7 sides)
d) Hexagon (6 sides)
e) Octagon (8 sides)
f) Quadrilateral (4 sides)

3.



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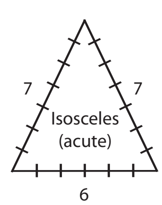

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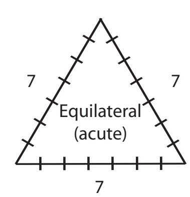
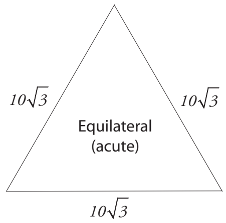


6. $10\sqrt{3}$

$5\sqrt{12}$
$5\sqrt{4} \cdot \sqrt{3}$
$5 \cdot 2 \cdot \sqrt{3}$
$10\sqrt{3}$

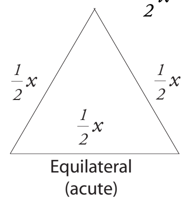
$\sqrt{300}$
$\sqrt{100} \cdot \sqrt{3}$
$10\sqrt{3}$



7. $\frac{1}{2}x$

$x - \frac{1}{2}x$
$1x - \frac{1}{2}$
$(1 - \frac{1}{2})\,x$
$\frac{1}{2}x$

$(\frac{1}{4})(2x)$
$(\frac{1}{4} \cdot 2) \cdot x$
$\frac{1 \cdot 2}{2 \cdot 2 \cdot 1} \cdot x$
$\frac{1}{2}x$



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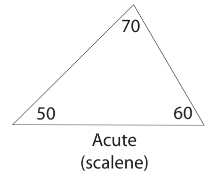

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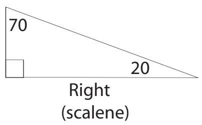

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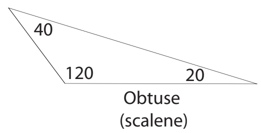

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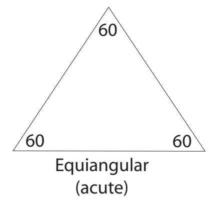
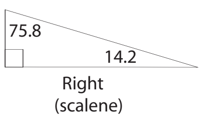


12.



$x = 90 - 14.2$
$= 75.8$

$\sqrt{200}$
$\sqrt{100} \cdot \sqrt{2}$
$10\sqrt{2}$
14.2

13. 0 **14.** 1 **15**. 2 **16.** 2

17. 1 **18**. 1 **19.** 3

Unit I — The Structure of Geometry

Part B — The Scope of Our Geometry

p. 41 – Lesson 4 — Solids

1. Triangular Prism
2. Pentagonal Pyramid
3. Cylinder (Right Circular)
4. Quadrilateral Prism (Trapezoid)
5. Cone (Right)
6. Triangular Prism
7. Sphere
8. Cylinder
9. Triangular Prism
10. Pentagonal Prism (Oblique)
11. Octagonal Pyramid
12. Square Prism
13. Hexagonal Prism
14. Rectangular Pyramid
15. Trapezoidal Prism

13.

$P = 4 \cdot s$
$= 4 \cdot 1$
$= 4$

Answer : 4 units

$A = s^2$
$= 1^2$
$= 1$

Answer : 1 square unit or 1 unit^2

14.

$P = 4 \cdot s$
$= 4 \cdot 2$
$= 8$

Answer : 8 units

$A = s^2$
$= 2^2$
$= 4$

Answer : 4 square units or 4 units^2

15.

$P = 4 \cdot s$
$= 4 \cdot 30$
$= 120$

Answer : 120 units

$A = s^2$
$= 30^2$
$= 900$

Answer : 900 square units or 900 units^2

16.

$P = 4 \cdot s$
$= 4 \cdot 4$
$= 16$

Answer : 16 units

$A = s^2$
$= 4^2$
$= 16$

Answer : 16 square units or 16 units^2

17.

$P = 4 \cdot s$
$= 4 \cdot 17$
$= 68$

Answer : 68 units

$A = s^2$
$= 17^2$
$= 289$

Answer : 289 square units or 289 units^2

18.

$P = 4 \cdot s$
$= 4i$

Answer : 4i kilometers

$A = s^2$
$= i^2$

Answer : i^2 square kilometers or i^2 km^2

19.

Perimeter = 36 ft
$P = 4 \cdot s$
$36 = 4 \cdot s$
$\frac{1}{4} \cdot 36 = \frac{1}{4} \cdot 4 \cdot s$
$\frac{1 \cdot \cancel{4} \cdot 9}{\cancel{4} \cdot 1} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{1} \cdot s$
$9 \text{ ft} = s$

$A = s^2$
$= 9^2$
$= 81$ square ft or 81 ft^2

20.

Perimeter = 20 in.
$P = 4 \cdot s$
$20 = 4 \cdot s$
$\frac{1}{4} \cdot 20 = \frac{1}{4} \cdot 4 \cdot s$
$\frac{1 \cdot \cancel{4} \cdot 5}{\cancel{4} \cdot 1} = \frac{1}{\cancel{4}} \cdot \frac{\cancel{4}}{1} \cdot s$
$5 \text{ in} = s$

$A = s^2$
$= 5^2$
$= 25$ square in. or 25 in^2

21.

Area = 49 mm^2
$A = s^2$
$49 = s^2$

Perimeter $= 4 \cdot s$
$P = 4 \cdot 7$
$= 28$ mm

There are 3 methods that can be used to solve this equation.

1.)
$49 = s^2$
$(49)^{\frac{1}{2}} = (s^2)^{\frac{1}{2}}$
$\sqrt[2]{49^1} = s^{2 \cdot \frac{1}{2}}$
$\sqrt[2]{(\pm 7)^2} = s^1$
$\pm 7 = s$
$7 \text{ mm} = s$

2.)
$49 = s^2$
$\pm \sqrt{49} = \sqrt{s^2}$
$\pm 7 = s$
$7 \text{ mm} = s$

3.)
$49 = s^2$
$49 - 49 = s^2 - 49$
$0 = s^2 - 49$
$0 = (s - 7)(s + 7)$
$0 = s - 7$ or $s + 7$
$0 + 7 = s - 7 + 7$ or $0 - 7 = s + 7 - 7$
$7 = s + 0$ or $-7 = s + 0$
$7 = s$ or $-7 = s$

Since the length of a side cannot be negative, the only acceptable answer is 7 mm.

22.

Area = 64 in^2
$A = s^2$
$64 = s^2$
$(64)^{\frac{1}{2}} = (s^2)^{\frac{1}{2}}$
$\sqrt[2]{64^1} = s^{2 \cdot \frac{1}{2}}$
$\sqrt[2]{(\pm 8)^2} = s^1$
$\pm 8 = s$
$8 \text{ in.} = s$

Perimeter $= 4 \cdot s$
$P = 4 \cdot 8$
$= 32$ inches

23. Side "S" = 2k units

$P = 4 \cdot s$
$= 4 \cdot 2k$
$= 8k$ units

$A = s^2$
$= (2k)(2k)$
$= 4k^2$ square units or $4k^2$ units^2

24. Side "S" = (k + 3) units

$P = 4 \cdot s$
$= 4 \cdot (k + 3)$
$= 4 \cdot k + 4 \cdot 3$
$= 4k + 12$
$= (4k + 12)$ units

$A = s^2$
$= (k + 3)(k + 3)$
$= (k + 3)(k) + (k + 3)(3)$
$= k \cdot k + 3k + k \cdot 3 + 3 \cdot 3$
$= k^2 + 3k + 3k + 9$
$= k^2 + 6k + 9$
$= (k^2 + 6k + 9)$ square units or
$= (k^2 + 6k + 9)$ units^2

Unit I — The Structure of Geometry

Part C — Measurement

p. 49 – Lesson 2 — Parallelograms

1. $A = b \cdot h$
$= 9 \text{ cm} \cdot 5 \text{ cm}$
$= 45$ square cm or 45 cm^2

2. $A = b \cdot h$
$= 6 \text{ in.} \cdot 12 \text{ in.}$
$= 72$ square in. or 72 in^2

3. $A = b \cdot h$
$= \frac{1}{2} \text{ in.} \cdot \frac{3}{8} \text{ in.}$
$= \frac{1 \cdot 3}{2 \cdot 8} \text{ in.}$
$= \frac{3}{16}$ square in. or $\frac{3}{16} \text{ in}^2$

4. $A = b \cdot h$
$= 7 \text{ ft} \cdot 4 \text{ ft}$
$= 28$ square ft or 28 ft^2

5. $A = b \cdot h$
$= 8 \text{ cm} \cdot 7 \text{ cm}$
$= 56$ square cm or 56 cm^2

6. $A = b \cdot h$
$= 11.5 \text{ ft} \cdot 6 \text{ ft}$
$= 69$ square ft or 69 ft^2

7. $A = b \cdot h$
$= 22 \text{ ft} \cdot 13 \text{ ft}$
$= 286$ square ft or 286 ft^2

8. $A = b \cdot h$
$= 7.6 \text{ ft} \cdot 4 \text{ ft}$
$= 30.4$ square ft or 30.4 ft^2

9. $A = b \cdot h$
$= 4 \text{ in.} \cdot 3 \text{ in.}$
$= 12$ square in. or 12 in^2

10. 8 ; 120
10 ; 15 ; 50

11. 10 ; 120
15 ; 10 ; 50

12. Perimeter = Sum of the lengths of the sides
$= 2k + (k + 1) + 2k + (k + 1)$
$= 2k + k + 1 + 2k + k + 1$
$= 2k + k + 2k + k + 1 + 1$
$= (2 + 1 + 2 + 1)k + 2$
$= (6k + 2)$ units

$A = b \cdot h$
$= (2k) \cdot (k)$
$= 2k^2$ square units

13. Perimeter = Sum of the lengths of the sides
$= (k + 2) + (k + 5) + (k + 2) + (k + 5)$
$= k + 2 + k + 5 + k + 2 + k + 5$
$= k + k + k + k + 2 + 5 + 2 + 5$
$= (1 + 1 + 1 + 1)k + 14$
$= (4k + 14)$ units

$A = b \quad h$
$= (k + 5)(k)$
$= (k)(k) + 5(k)$
$= (k^2 + 5k)$ square units

14. Perimeter = Sum of the lengths of the sides

$$= \frac{1}{3}k + \frac{1}{5}k + \frac{1}{3}k + \frac{1}{5}k$$
$$= \frac{1}{3}k + \frac{1}{3}k + \frac{1}{5}k + \frac{1}{5}k$$
$$= (\frac{1}{3} + \frac{1}{3}) \cdot k + (\frac{1}{5} + \frac{1}{5}) \cdot k$$
$$= (\frac{1+1}{3}) \cdot k + (\frac{1+1}{5}) \cdot k$$
$$= \frac{2}{3} \cdot k + \frac{2}{5} \cdot k$$
$$= (\frac{2}{3} + \frac{2}{5}) \cdot k$$
$$= (\frac{2 \cdot 5}{3 \cdot 5} + \frac{2 \cdot 3}{5 \cdot 3}) \cdot k$$
$$= (\frac{10}{15} + \frac{6}{15}) \cdot k$$
$$= \frac{10+6}{15} \cdot k$$
$$= \frac{16}{15} k \text{ units}$$

$A = b \cdot h$

$$= (\frac{1}{3}k) \cdot (\frac{1}{5}k)$$
$$= \frac{1}{3} \cdot k \cdot \frac{1}{5} \cdot k$$
$$= \frac{1}{3} \cdot \frac{1}{5} \cdot k \cdot k$$
$$= \frac{1 \cdot 1}{3 \cdot 5} \cdot k \cdot k$$
$$= \frac{1}{15} k^2 \text{ square units}$$

15. Perimeter = Sum of the lengths of the sides

$$= 5k + 5k + 5k + 5k$$
$$= (5 + 5 + 5 + 5) \cdot k$$
$$= 20k \text{ units}$$

$A = b \cdot h$

$$= 5k \cdot (k + 4)$$
$$= 5k \cdot k + 5k \cdot 4$$
$$= 5k^2 + 5 \cdot 4 \cdot k$$
$$= 5k^2 + 20k$$
$$= (5k^2 + 20k) \text{ units}^2$$

Unit I — The Structure of Geometry

Part C — Measurement

p. 53 – Lesson 3 — Triangles

1. Area $= \frac{1}{2} \cdot b \cdot h$

$$= \frac{1}{2} \cdot 7 \text{ ft} \cdot 4 \text{ ft}$$
$$= \frac{1 \cdot 7 \cdot 2 \cdot 2}{2 \cdot 1}$$
$$= 14 \text{ sq. ft or } 14 \text{ ft}^2$$

Perimeter = Sum of the lengths of the sides

$$= 3 \text{ ft} + 7 \text{ ft} + 6 \text{ ft}$$
$$= 10 \text{ ft} + 6 \text{ ft}$$
$$= 16 \text{ ft}$$

2. Area $= \frac{1}{2} \cdot b \cdot h$

$$= \frac{1}{2} \cdot 8 \text{ in.} \cdot 6 \text{ in.}$$
$$= \frac{1 \cdot 2 \cdot 4 \cdot 6}{2 \cdot 1}$$
$$= 24 \text{ sq. in. or } 24 \text{ in}^2$$

Perimeter = Sum of the lengths of the sides

$$= 6 \text{ in.} + 8 \text{ in.} + 10 \text{ in.}$$
$$= 14 \text{ in.} + 10 \text{ in.}$$
$$= 24 \text{ in.}$$

3. Area $= \frac{1}{2} \cdot b \cdot h$

$$= \frac{1}{2} \cdot 8 \text{ m} \cdot 6 \text{ m}$$
$$= \frac{1 \cdot 2 \cdot 4 \cdot 6}{2 \cdot 1}$$
$$= 24 \text{ sq. m or } 24 \text{ m}^2$$

Perimeter = Sum of the lengths of the sides

$$= 8 \text{ m} + 14 \text{ m} + 11 \text{ m}$$
$$= 22 \text{ m} + 11 \text{ m}$$
$$= 33 \text{ meters}$$

4. Area $= \frac{1}{2} \cdot b \cdot h$

$$= \frac{1}{2} \cdot 15 \text{ cm} \cdot \frac{15\sqrt{3}}{2} \text{ cm}$$
$$= \frac{1 \cdot 15 \cdot 15 \cdot \sqrt{3}}{2 \cdot 1 \cdot 2}$$
$$= \frac{225\sqrt{3}}{4} \text{ cm}^2$$

Perimeter = Sum of the lengths of the sides

$$= 15 \text{ cm} + 15 \text{ cm} + 15 \text{ cm}$$
$$= 30 \text{ cm} + 15 \text{ cm}$$
$$= 45 \text{ cm}$$

5. Area $= \frac{1}{2} \cdot b \cdot h$

$$= \frac{1}{2} \cdot (6 \text{ in.} + 8 \text{ in.}) \cdot 8 \text{ in.}$$
$$= \frac{1 \cdot 14 \cdot 8}{2 \cdot 1 \cdot 1}$$
$$= \frac{1 \cdot 2 \cdot 7 \cdot 8}{2 \cdot 1 \cdot 1}$$
$$= 56 \text{ sq. in. or } 56 \text{ in}^2$$

Perimeter = Sum of the lengths of the sides

$$= 10 \text{ in.} + 6 \text{ in.} + 8 \text{ in.} + 11 \text{ in.}$$
$$= 16 \text{ in.} + 8 \text{ in.} + 11 \text{ in.}$$
$$= 24 \text{ in.} + 11 \text{ in.}$$
$$= 35 \text{ inches}$$

6. Area $= \frac{1}{2} \cdot b \cdot h$

$$= \frac{1}{2} \cdot (10 \text{ ft} + 2 \text{ ft}) \cdot 5 \text{ ft}$$
$$= \frac{1 \cdot 12 \cdot 5}{2 \cdot 1 \cdot 1}$$
$$= \frac{1 \cdot 2 \cdot 6 \cdot 5}{2 \cdot 1 \cdot 1}$$
$$= 30 \text{ sq. ft or } 30 \text{ ft}^2$$

Perimeter = Sum of the lengths of the sides

$$= 11 \text{ ft} + 10 \text{ ft} + 2 \text{ ft} + 7 \text{ ft}$$
$$= 21 \text{ ft} + 9 \text{ ft}$$
$$= 30 \text{ feet}$$

Unit I — The Structure of Geometry

Part C — Measurement

p. 56 – Lesson 4 — Trapezoids

1.

$$
\begin{aligned}
Area &= \frac{1}{2} \cdot h \cdot (b_1 + b_2) \\
&= \frac{1}{2} \cdot 8 \cdot (14 + 10) \\
&= \frac{1 \cdot 8}{2} \cdot (24) \\
&= 4 \cdot 24 \\
&= 96 \text{ sq. cm or } 96 \text{ cm}^2
\end{aligned}
$$

$$
\begin{aligned}
Perimeter &= \text{Sum of the lengths of the sides} \\
&= 9 + 14 + 8\frac{1}{2} + 10 \\
&= 23 + 18\frac{1}{2} \\
&= 41\frac{1}{2} \text{ cm}
\end{aligned}
$$

2.

$$
\begin{aligned}
Area &= \frac{1}{2} \cdot h \cdot (b_1 + b_2) \\
&= \frac{1}{2} \cdot 6 \cdot (11 + 7) \\
&= \frac{1 \cdot 6}{2} \cdot (18) \\
&= 3 \cdot 18 \\
&= 54 \text{ sq. cm or } 54 \text{ cm}^2
\end{aligned}
$$

$$
\begin{aligned}
Perimeter &= \text{Sum of the lengths of the sides} \\
&= 6 + 11 + 6\frac{1}{4} + 7 \\
&= 17 + 13\frac{1}{4} \\
&= 30\frac{1}{4} \text{ cm}
\end{aligned}
$$

3.

$$
\begin{aligned}
Area &= \frac{1}{2} \cdot h \cdot (b_1 + b_2) \\
&= \frac{1}{2} \cdot 4 \cdot (4.3 + 7.7) \\
&= \frac{1 \cdot 4}{2} \cdot (12) \\
&= 2 \cdot 12 \\
&= 24 \text{ sq. cm or } 24 \text{ cm}^2
\end{aligned}
$$

$$
\begin{aligned}
Perimeter &= \text{Sum of the lengths of the sides} \\
&= 4.1 + 7.7 + 4 + 4.3 \\
&= 4.1 + 4 + 7.7 + 4.3 \\
&= 8.1 + 12 \\
&= 20.1 \text{ cm}
\end{aligned}
$$

4.

$$
\begin{aligned}
Area &= \frac{1}{2} \cdot h \cdot (b_1 + b_2) \\
&= \frac{1}{2} \cdot 7 \cdot (19 + 12) \\
&= \frac{1 \cdot 7}{2} \cdot (31) \\
&= \frac{7}{2} \cdot 31 \\
&= \frac{7 \cdot 31}{2 \cdot 1} \\
&= \frac{217}{2} \\
&= 108\frac{1}{2} \text{ sq. cm or } 108\frac{1}{2} \text{ cm}^2
\end{aligned}
$$

$$
\begin{aligned}
Perimeter &= \text{Sum of the lengths of the sides} \\
&= 8 + 19 + 9 + 12 \\
&= 8 + 28 + 12 \\
&= 8 + 40 \\
&= 48 \text{ cm}
\end{aligned}
$$

5.

$$
\begin{aligned}
Area &= \frac{1}{2} \cdot h \cdot (b_1 + b_2) \\
&= \frac{1}{2} \cdot 2 \cdot (3\frac{2}{3} + 2\frac{1}{2}) \\
&= \frac{1}{2} \cdot 2 \cdot (\frac{11}{3} + \frac{5}{2}) \\
&= \frac{1 \cdot 2}{2} \cdot (\frac{11 \cdot 2}{3 \cdot 2} + \frac{5 \cdot 3}{2 \cdot 3}) \\
&= 1 \cdot (\frac{22}{6} + \frac{15}{6}) \\
&= \frac{22 + 15}{6} \\
&= \frac{37}{6} \\
&= 6\frac{1}{6} \text{ sq. cm or } 6\frac{1}{6} \text{ cm}^2
\end{aligned}
$$

$$
\begin{aligned}
Perimeter &= \text{Sum of the lengths of the sides} \\
&= 3\frac{1}{3} + 3\frac{2}{3} + 2 + 2\frac{1}{2} \\
&= \frac{10}{3} + \frac{11}{3} + 4\frac{1}{2} \\
&= \frac{10 + 11}{3} + 4\frac{1}{2} \\
&= \frac{21}{3} + 4\frac{1}{2} \\
&= \frac{3 \cdot 7}{3} + 4\frac{1}{2} \\
&= 7 + 4\frac{1}{2} \\
&= 11\frac{1}{2} \text{ cm}
\end{aligned}
$$

6.

$$
\begin{aligned}
Area &= \frac{1}{2} \cdot h \cdot (b_1 + b_2) \\
&= \frac{1}{2} \cdot \frac{9}{2} \cdot (4 + 1) \\
&= \frac{1 \cdot 9}{2 \cdot 2} \cdot (5) \\
&= \frac{9}{4} \cdot \frac{5}{1} \\
&= \frac{9 \cdot 5}{4 \cdot 1} \\
&= \frac{45}{4} \\
&= 11\frac{1}{4} \text{ sq. cm or } 11\frac{1}{4} \text{ cm}^2
\end{aligned}
$$

$$
\begin{aligned}
Perimeter &= \text{Sum of the lengths of the sides} \\
&= 5 + 4 + 4.2 + 1 \\
&= 9 + 5.2 \\
&= 14.2 \text{ cm}
\end{aligned}
$$

p. 60 – Lesson 5 — Regular Polygons

1. **a)** Inscribed **b)** Circumscribed **c)** 12 • 8 or 96 **d)** n • s **e)** Octagon has 8 sides

$$\begin{aligned} Perimeter &= n \cdot s \\ 40 &= 8 \cdot s \\ 5 &= s \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2} \cdot a \cdot s \cdot n \\ &= \frac{1}{2} \cdot 6 \cdot 5 \cdot 8 \\ &= 3 \cdot 5 \cdot 8 \\ &= 120 \text{ sq. units or } 120 \text{ units}^2 \end{aligned}$$

2. Regular Triangle (Equilateral Triangle) 3 sides

$$\begin{aligned} Perimeter &= n \cdot s \\ &= 3 \cdot 4\sqrt{3} \\ &= 12\sqrt{3} \text{ cm} \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2} \cdot a \cdot s \cdot n \\ &= \frac{1}{2} \cdot 2 \cdot 4\sqrt{3} \cdot 3 \\ &= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{1} \cdot 4\sqrt{3} \cdot 3 \\ &= 4 \cdot 3 \cdot \sqrt{3} \\ &= 12\sqrt{3} \text{ sq. cm or } 12\sqrt{3} \text{ cm}^2 \end{aligned}$$

3. Regular Hexagon 6 sides

$$\begin{aligned} Perimeter &= n \cdot s \\ &= 6 \cdot 11 \\ &= 66 \text{ cm} \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2} \cdot a \cdot s \cdot n \\ &= \frac{1}{2} \cdot 9.5 \cdot 11 \cdot 6 \\ &= \frac{1}{2} \cdot \frac{9.5 \cdot 11 \cdot 6}{1} \\ &= \frac{1 \cdot 9.5 \cdot 11 \cdot \cancel{2} \cdot 3}{\cancel{2}} \\ &= 9.5 \cdot 11 \cdot 3 \\ &= 104.5 \cdot 3 \\ &= 313.5 \text{ sq. cm or } 313.5 \text{ cm}^2 \end{aligned}$$

4. Regular Quadrilateral (Square) 4 sides

$$\begin{aligned} Perimeter &= n \cdot s \\ &= 4 \cdot 8\sqrt{2} \\ &= 32\sqrt{2} \text{ cm} \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2} \cdot a \cdot s \cdot n \\ &= \frac{1}{2} \cdot 4\sqrt{2} \cdot 8\sqrt{2} \cdot 4 \\ &= \frac{1}{2} \cdot \frac{4\sqrt{2}}{1} \cdot \frac{8\sqrt{2}}{1} \cdot \frac{4}{1} \\ &= \frac{1 \cdot 2 \cdot \cancel{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot 8 \cdot 4}{\cancel{2}} \\ &= 2 \cdot \sqrt{4} \cdot 32 \\ &= 2 \cdot 2 \cdot 32 \\ &= 128 \text{ sq. cm or } 128 \text{ cm}^2 \end{aligned}$$

Note : For a square, area is given by $A = s \cdot s$

$$\begin{aligned} &= 8\sqrt{2} \cdot 8\sqrt{2} \\ &= 64\sqrt{4} \\ &= 64 \cdot 2 \\ &= 128 \text{ cm}^2 \end{aligned}$$

5. Regular Octagon 8 sides

$$\begin{aligned} Perimeter &= n \cdot s \\ &= 8 \cdot 12 \\ &= 96 \text{ cm} \end{aligned}$$

$$\begin{aligned} Area &= \frac{1}{2} \cdot a \cdot s \cdot n \\ &= \frac{1}{2} \cdot 14.5 \cdot 12 \cdot 8 \\ &= \frac{1}{2} \cdot \frac{14.5 \cdot 12 \cdot 8}{1} \\ &= \frac{1 \cdot 14.5 \cdot 6 \cdot \cancel{2} \cdot 8}{\cancel{2} \cdot 1} \\ &= 14.5 \cdot 6 \cdot 8 \\ &= 87 \cdot 8 \\ &= 696 \text{ sq. cm or } 696 \text{ cm}^2 \end{aligned}$$

6. Regular Pentagon 5 sides

$$\begin{aligned}
Perimeter &= n \cdot s \\
&= 5 \cdot 30 \\
&= 150\ cm
\end{aligned}$$

$$\begin{aligned}
Area &= \frac{1}{2} \cdot a \cdot s \cdot n \\
&= \frac{1}{2} \cdot 20.6 \cdot 30 \cdot 5 \\
&= \frac{1}{2} \cdot \frac{20.6 \cdot 30 \cdot 5}{1} \\
&= \frac{20.6 \cdot \cancel{2} \cdot 15 \cdot 5}{\cancel{2} \cdot 1} \\
&= 20.6 \cdot 15 \cdot 5 \\
&= 20.6 \quad 75 \\
&= 1545\ sq.\ cm \ \text{ or } 1545\ cm^2
\end{aligned}$$

7. Regular Quadrilateral (Square) 4 sides

$$\begin{aligned}
Perimeter &= n \cdot s \\
&= 4 \cdot 12\sqrt{3} \\
&= 48\sqrt{3}\ cm
\end{aligned}$$

In a square, if the apothem is known, the length of a side will be 2 times the length of the apothem.

$$s = 12\sqrt{3}$$

$$\begin{aligned}
Area &= \frac{1}{2} \cdot a \cdot s \cdot n \\
&= \frac{1}{2} \cdot 6\sqrt{3} \cdot 12\sqrt{3} \cdot 4 \\
&= \frac{1 \cdot 6\sqrt{3} \cdot 12\sqrt{3} \cdot 2 \cdot \cancel{2}}{\cancel{2} \cdot 1} \\
&= 6\sqrt{3} \cdot 12\sqrt{3} \cdot 2 \\
&= 6 \cdot \sqrt{3} \cdot 12 \cdot \sqrt{3} \cdot 2 \\
&= 6 \cdot 12 \cdot \sqrt{3} \cdot \sqrt{3} \cdot 2 \\
&= 6 \cdot 12 \cdot \sqrt{9} \cdot 2 \\
&= 6 \cdot 12 \cdot 3 \cdot 2 \\
&= 72 \cdot 6 \\
&= 432\ sq.\ cm \ \text{ or } 432\ cm^2
\end{aligned}$$

Unit I — The Structure of Geometry

Part C — Measurement

p. 63 – Lesson 6 — Circles

The diameter of a circle is the length of the radius multiplied by two.

1. $Diameter = 2 \cdot 14\ m$
$= 28\ m$

2. $Diameter = 2 \cdot 5.7\ cm$
$= 11.4\ cm$

3. $Diameter = 2 \cdot 3x\ units$
$= 6x\ units$

The length of the radius of a circle is one-half of a diameter.

4.
$$\begin{aligned}
Radius &= \frac{1}{2} \cdot 41.6\ inches \\
&= \frac{1}{2} \cdot \frac{41.6}{1} \\
&= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\ \ 20.8}{1} \\
&= 20.8\ inches
\end{aligned}$$

5.
$$\begin{aligned}
Radius &= \frac{1}{2} \cdot 8y\ units \\
&= \frac{1}{2} \cdot \frac{8y}{1} \\
&= \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\ \ 4y}{1} \\
&= 4y\ units
\end{aligned}$$

6.
$$\begin{aligned}
Radius &= \frac{1}{2} \cdot \sqrt{45}\ meters \\
&= \frac{1}{2} \cdot \frac{\sqrt{45}}{1} \\
&= \frac{1}{2} \cdot \frac{\sqrt{9 \cdot 5}}{1} \\
&= \frac{1}{2} \cdot \frac{\sqrt{9} \cdot \sqrt{5}}{1} \\
&= \frac{3\sqrt{5}}{2}\ meters
\end{aligned}$$

7. $C = \pi \cdot d \qquad d = 2 \cdot r$

$= \pi \cdot 2 \cdot r$

$= \pi \cdot 2 \cdot 6$

$= \pi \cdot 12$

$= 12\pi$ units (exact answer) or

$= 12 \cdot 3.14$

$= 37.68$ units (approximate answer)

$A = \pi \cdot r^2$

$= \pi \cdot 6^2$

$= \pi \cdot 36$

$= 36\pi$ sq. units (exact answer) or

$= 36 \cdot 3.14$

$= 113.04$ sq. units (approximate answer)

8. $C = \pi \cdot d$

$= \pi \cdot 18.4$ km

$= 18.4\pi$ km (exact answer) or

$= 18.4 \cdot 3.14$

$= 57.776$ km (approximate answer)

$r = \frac{1}{2} \cdot d$

$= \frac{1}{2} \cdot 18.4$

$= \frac{1}{2} \cdot \frac{2 \cdot 9.2}{1}$

$= 9.2$ km

$A = \pi \cdot r^2$

$= \pi \cdot 9.2^2$

$= \pi \cdot 84.64$

$= 84.64\pi$ sq. km (exact answer) or

$= 84.64 \cdot 3.14$

$= 265.7696$ sq. km (approximate answer)

9. $C = \pi \cdot d$

$= \pi \cdot 3$ cm

$= 3\pi$ cm (exact answer) or

$= 3 \cdot 3.14$

$= 9.42$ cm (approximate answer)

$r = \frac{1}{2} \cdot d$

$= \frac{1}{2} \cdot 3$ cm

$= \frac{3}{2}$ cm

$A = \pi \cdot r^2$

$= \pi \cdot (\frac{3}{2})^2$

$= \pi \cdot \frac{9}{4}$

$= \frac{9}{4}\pi$ sq. cm or $\frac{9\pi}{4}$ (exact answer) or

$= \frac{9 \cdot 3.14}{4}$

$= 7.065$ sq. cm (approximate answer)

10. $C = \pi \cdot d$

$= \pi \cdot 28$ inches

$= 28\pi$ inches (exact answer) or

$= 28 \cdot 3.14$

$= 87.92$ inches (approximate answer)

$r = \frac{1}{2} \cdot d$

$= \frac{1}{2} \cdot 28$

$= 14$ inches

$A = \pi \cdot r^2$

$= \pi \cdot 14^2$

$= \pi \cdot 196$

$= 196\pi$ sq. inches (exact answer) or

$= 196 \cdot 3.14$

$= 615.44$ sq. inches (approximate answer)

11. $d = 2 \cdot r$

$= 2 \cdot 1$

$= 2$ cm

$C = \pi \cdot d$

$= \pi \cdot 2$ cm

$= 2\pi$ cm (exact answer) or

$= 2 \cdot 3.14$

$= 6.28$ cm (approximate answer)

$A = \pi \cdot r^2$

$= \pi \cdot 1^2$

$= \pi \cdot 1$

$= \pi$ sq. cm (exact answer) or

$= 3.14$ sq. cm (approximate answer)

12. $C = \pi \cdot d$

$= \pi \cdot 10\sqrt{2}$ units

$= 10\sqrt{2}\pi$ units (exact answer) or

$= 10 \cdot 3.14 \cdot \sqrt{2}$

$= 31.4\sqrt{2}$

$= 44.406$ units (approximate answer)

$r = \frac{1}{2} \cdot d$

$= \frac{1}{2} \cdot 10\sqrt{2}$

$= 5\sqrt{2}$

$A = \pi \cdot r^2$

$= \pi \cdot 5\sqrt{2}^2$

$= \pi \cdot 5\sqrt{2} \cdot 5\sqrt{2}$

$= \pi \cdot 5 \cdot 5 \cdot \sqrt{2} \cdot \sqrt{2}$

$= \pi \cdot 25 \cdot \sqrt{4}$

$= \pi \cdot 25 \cdot 2$

$= \pi \cdot 50$ sq units

$= 50\pi$ sq. units (exact answer) or

$= 50 \cdot 3.14$

$= 157$ sq. units (approximate answer)

13. $d = 2 \cdot r$

$= 2 \cdot 9.8\ cm$

$= 19.6\ cm$

$C = \pi \cdot d$

$= \pi \cdot 19.6\ cm$

$= 19.6\pi\ cm$ (exact answer) or

$= 19.6 \cdot 3.14$

$= 61.544\ cm$ (approximate answer)

$A = \pi \cdot r^2$

$= \pi \cdot 9.8\ cm \cdot 9.8\ cm$

$= \pi \cdot 96.04\ cm^2$

$= 96.04\pi$ sq. cm (exact answer) or

$= 96.04 \cdot 3.14$

$= 301.5656$ sq. cm (approximate answer)

14. $C = \pi \cdot d$

$= \pi \cdot 2\frac{3}{5}\ ft$

$= \frac{\pi}{1} \cdot \frac{13}{5}\ ft$

$= \frac{\pi \cdot 13}{1 \cdot 5}\ ft$

$= \frac{13\pi}{5}\ ft$ (exact answer) or

$= \frac{13 \cdot 3.14}{5}\ ft$

$= \frac{40.82}{5}\ ft$

$= 8.164\ ft$ (approximate answer)

$r = \frac{1}{2} \cdot d$

$= \frac{1}{2} \cdot 2\frac{3}{5}\ ft$

$= \frac{1}{2} \cdot \frac{13}{5}$

$= \frac{1 \cdot 13}{2 \cdot 5}$

$= \frac{13}{10}\ ft$

$A = \pi \cdot r^2$

$= \pi \cdot (\frac{13}{10})\ ft \cdot (\frac{13}{10})\ ft$

$= \pi \cdot \frac{13}{10} \cdot \frac{13}{10}\ ft^2$

$= \frac{\pi \cdot 13 \cdot 13}{10 \cdot 10}\ ft^2$

$= \frac{\pi \cdot 169}{100}\ ft^2$

$= \frac{169\pi}{100}\ ft^2$ (exact answer) or

$= \frac{169 \cdot 3.14}{100}\ ft^2$

$= 5.3066$ sq. ft (approximate answer)

15. $C = \pi \cdot d$

$= 61\pi$

$61\pi = \pi d$

$\frac{61\cancel{\pi}}{1} \cdot \frac{1}{\cancel{\pi}} = \frac{1}{\cancel{\pi}} \cdot \frac{\cancel{\pi}d}{1}$

$61 = d$

$d = 2 \cdot r$

$61 = 2 \cdot r$

$\frac{1}{2} \cdot \frac{61}{1} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \cdot r}{1}$

$\frac{61}{2}$ units $= r$

16. $C = \pi \cdot d$

$= 4\pi\sqrt{3}\ cm$

$4\pi\sqrt{3} = \pi d$

$\frac{1}{\cancel{\pi}} \cdot \frac{4\cancel{\pi}\sqrt{3}}{1} = \frac{1}{\cancel{\pi}} \cdot \frac{\cancel{\pi}d}{1}$

$4\sqrt{3} = d$

$d = 2 \cdot r$

$4\sqrt{3} = 2 \cdot r$

$\frac{1}{2} \cdot \frac{4\sqrt{3}}{1} = \frac{1}{2} \cdot \frac{2 \cdot r}{1}$

$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \cdot 2\sqrt{3}}{1} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \cdot r}{1}$

$2\sqrt{3}\ cm = r$

17. $C = \pi \cdot d$

$= \frac{\pi\sqrt{6}}{2}\ m$

$\frac{\pi\sqrt{6}}{2} = \pi d$

$\frac{1}{\cancel{\pi}} \cdot \frac{\cancel{\pi}\sqrt{6}}{2} = \frac{1}{\cancel{\pi}} \cdot \frac{\cancel{\pi}d}{1}$

$\frac{\sqrt{6}}{2} = d$

$d = 2 \cdot r$

$\frac{\sqrt{6}}{2} = 2 \cdot r$

$\frac{1}{2} \cdot \frac{\sqrt{6}}{2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2} \cdot r}{1}$

$\frac{1 \cdot \sqrt{6}}{2 \cdot 2} = r$

$\frac{\sqrt{6}}{4}\ m = r$

18. $A = \pi \cdot r^2$

$4 = \pi \cdot r^2$

$\frac{1}{\pi} \cdot 4 = r^2$

$\frac{4}{\pi} = r^2$

$\sqrt{\frac{4}{\pi}} = r$

$\frac{2}{\sqrt{\pi}} = r$

$\frac{2 \cdot \sqrt{\pi}}{\pi} = r$

19.

$$A = \pi \cdot r^2$$
$$A = 12\ in^2$$
$$\pi \cdot r^2 = 12$$
$$\frac{1}{\pi} \cdot \frac{\pi \cdot r^2}{1} = \frac{1}{\pi} \cdot \frac{12}{1}$$
$$r^2 = \frac{12}{\pi}$$
$$r = \pm\sqrt{\frac{12}{\pi}} \quad \text{(can't be negative)}$$
$$r = \frac{\sqrt{12}}{\sqrt{\pi}}$$
$$r = \frac{\sqrt{3} \cdot \sqrt{4} \cdot \sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{\pi}}$$
$$r = \frac{2\sqrt{3\pi}}{\pi} \text{ inches}$$

20.

$$A = \pi \cdot r^2$$
$$A = 8\ cm^2$$
$$8 = \pi \cdot r^2$$
$$\frac{1}{\pi} \cdot \frac{8}{1} = \frac{1}{\pi} \cdot \frac{\pi \cdot r^2}{1}$$
$$\frac{8}{\pi} = r^2$$
$$\pm\sqrt{\frac{8}{\pi}} = r \quad \text{(can't be negative)}$$
$$\sqrt{\frac{8}{\pi}} = r$$
$$\frac{\sqrt{8}}{\sqrt{\pi}} = r$$
$$\frac{\sqrt{4} \cdot \sqrt{2} \cdot \sqrt{\pi}}{\sqrt{\pi} \cdot \sqrt{\pi}} = r$$
$$\frac{2\sqrt{2\pi}}{\pi}\ cm = r$$

21. radius = 5 yds

$$d = 2 \cdot r$$
$$= 2 \cdot 5 \text{ yds}$$
$$= 10 \text{ yds}$$

$$C = \pi \cdot d$$
$$= \pi \cdot 10 \text{ yds}$$
$$= 10\pi \text{ yds}$$
$$\doteq 31.42 \text{ yds}$$

$$A = \pi \cdot r^2$$
$$= \pi \cdot (5 \text{ yds})^2$$
$$= \pi \cdot 5 \cdot 5 \text{ yds}^2$$
$$= 25\pi \text{ sq. yds}$$
$$= 25\pi \text{ yds}^2$$
$$\doteq 78.55 \text{ yds}^2$$

22. diameter = 8 m

$$\text{Radius} = \frac{1}{2} \cdot \text{diameter}$$
$$= \frac{1}{2} \cdot 8$$
$$= \frac{1}{2} \cdot \frac{2 \cdot 4}{1}$$
$$= 4\ m$$

$$C = \pi \cdot d$$
$$= \pi \cdot 8\ m$$
$$= 8\pi\ m$$
$$\doteq 25.14\ m$$

$$A = \pi \cdot r^2$$
$$= \pi \cdot (4\ m)^2$$
$$= \pi \cdot 4 \cdot 4\ m^2$$
$$= 16\pi \text{ sq. m}$$
$$= 16\pi\ m^2$$
$$\doteq 50.27\ m^2$$

23.

$$C = \pi \cdot d$$
$$= 28\pi \text{ in.}$$
$$28\pi = \pi d$$
$$\frac{1}{\pi} \cdot \frac{28\pi}{1} = \frac{1}{\pi} \cdot \frac{\pi d}{1}$$
$$28 \text{ in.} = d$$

$$\text{Radius} = \frac{1}{2} \cdot \text{diameter}$$
$$= \frac{1}{2} \cdot \frac{28}{1}$$
$$= \frac{1}{2} \cdot \frac{2 \cdot 14}{1}$$
$$= 14 \text{ in.}$$

$$A = \pi \cdot r^2$$
$$= \pi \cdot (14 \text{ in.})^2$$
$$= \pi \cdot 14 \cdot 14\ in^2$$
$$= 196\pi \text{ sq. in}$$
$$= 196\pi\ in^2$$
$$\doteq 615.83\ in^2$$

24.

$$A = \pi \cdot r^2$$
$$12\pi = \pi \cdot r^2$$
$$\frac{1}{\pi} \cdot \frac{12\pi}{1} = \frac{1}{\pi} \cdot \frac{\pi r^2}{1}$$
$$12 = r^2$$
$$\pm 12 = r \quad \text{(can't be negative)}$$
$$\sqrt{12} = r$$
$$\sqrt{4 \cdot 3} = r$$
$$\sqrt{4} \cdot \sqrt{3} = r$$
$$2\sqrt{3}\ m = r$$

$$\text{Diameter} = 2 \cdot r$$
$$= 2 \cdot 2\sqrt{3}$$
$$= 4\sqrt{3}\ m$$

$$C = \pi \cdot d$$
$$= \pi \cdot 4\sqrt{3}\ m$$
$$= 4\pi\sqrt{3}\ m$$
$$\doteq 12.57\sqrt{3}\ m$$

25.

Area of Square

$$A = s^2$$
$$= 16^2$$
$$= 16 \cdot 16$$
$$= 256\ cm^2$$

Area of Circle

$$A = \pi \cdot r^2$$
$$= \pi \cdot 8^2$$
$$= \pi \cdot 64$$
$$= 64\pi\ cm^2$$

Area of Shaded Region = Area of Square - Area of Circle

Area of Shaded Region $= (256 - 64\pi)\ cm^2$

$$= 64(4 - \pi)\ cm^2$$

p. 69 – Lesson 7 — Prisms

1. ***Lateral Area –***

The lateral faces of this prism are four identical squares.

$A = s \cdot s$ (area of one face)

$= 6 \text{ ft} \cdot 6 \text{ ft}$

$= 36$ sq. ft

Lateral area is $4 \cdot 36 = 144$ sq. ft

The bases of this prism are two identical squares.

$A = s \cdot s$

$= 6 \text{ ft} \cdot 6 \text{ ft}$

$= 36$ sq. ft

Area of bases is $2 \cdot 36 = 72$ sq. ft

Total area is the sum of the two bases of the prism and the area of all the lateral faces of the prism.

T.A. = B.A. + L.A.

$= 72 + 144$

$= 216$ sq. ft

Volume –

The volume of a prism is found by multiplying the area of one base by the altitude "h" of the prism.

Area of one base is 36 sq. ft

Altitude is the length of one of the lateral edges.

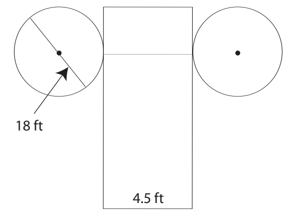
$V = B \cdot h$

$= 36 \cdot 6$

$= 216$ cubic ft or 216 ft^3

2. ***Lateral Area –***

Consider the cylinder, unfolded, and laid out, in a "net."



We can now see that the "lateral face" of the cylinder is a rectangle with length equal to the circumference of a circular base and width equal to the height of the cylinder.

$C = \pi \cdot d$

$= \pi \cdot 18$

$= 18\pi$ ft

Lateral Area = length • width of the rectangled or

= Circumference • height

$= 18\pi \text{ ft} \cdot 4.5 \text{ ft}$

$= 81\pi$ sq. ft

The bases of the cylinder are identical circles.

$A = \pi \cdot r^2$ | radius $= \frac{1}{2} \cdot$ diameter

$= \pi \cdot 9^2$ | $= \frac{1}{2} \cdot 18$ ft

$= \pi \cdot 81$ | $= 9$ ft

$= 81\pi$ sq. ft

Total Area is the sum of the area of the two bases of the cylinder and the area of the lateral face.

T.A. = B.A. + L.A.

$= [(81\pi + 81\pi) + 81\pi]$ sq. ft

$= (81 + 81 + 81)\pi$

$= 243\pi$ sq. ft

Volume –

The volume of a cylinder is found by multiplying the area of the base times the height.

$V = B \cdot h$

$= 81\pi \text{ sq. ft} \cdot 4.5 \text{ ft}$

$= 364.5\pi$ cubic ft or $364.5\pi \text{ ft}^3$

$\doteq 365\pi \text{ ft}^3$

5. ***Lateral Area –***

The lateral faces of this prism are four rectangles all of different dimensions.

$A = l \cdot w$ $\quad$ $A = l \cdot w$ $\quad$ $A = l \cdot w$ $\quad$ $A = l \cdot w$

$= 2\text{ in.} \cdot 3\text{ in.}$ $\quad$ $= 4\text{ in.} \cdot 3\text{ in.}$ $\quad$ $= 6\text{ in.} \cdot 3\text{ in.}$ $\quad$ $= 4\sqrt{2}\text{ in.} \cdot 3\text{ in.}$

$= 6\text{ sq. in.}$ $\quad$ $= 12\text{ sq. in.}$ $\quad$ $= 18\text{ sq. in.}$ $\quad$ $= 12\sqrt{2}\text{ sq. in.}$

The lateral area is $(6 + 12 + 18 + 12\sqrt{2})$ *sq in. or* $(36 + 12\sqrt{2})$ *sq in.*

The bases of this prism are identical trapezoids with bases of 6 in. and 2 in. and a height of 4 in.

$A = \frac{1}{2} \cdot h \cdot (b_1 + b_2)$

$= \frac{1}{2} \cdot 4\text{ in.} \cdot (6\text{ in.} + 2\text{ in.})$

$= 2\text{ in.} \cdot 8\text{ in.}$

$= 16\text{ sq. in.}$ *(area of one base)*

So, the area of the bases is $2 \cdot 16$ *or 32 sq. in.*

Total Area

$T.A. = B.A. + L.A.$

$= 32\text{ sq. in.} + (36 + 12\sqrt{2})\text{ sq. in.}$

$= (32 + 36 + 12\sqrt{2})\text{ sq. in.}$

$= (68 + 12\sqrt{2})\text{ sq. in.}$

Volume –

The volume of a prism is found by multiplying the area of one base by the altitude "h" of the prism.

$V = B \cdot h$

$= 16\text{ sq. in.} \cdot 3\text{ in.}$

$= 48\text{ in}^3$ *or 48 cubic in.*

6. ***Lateral Area –***

The lateral faces of this prism are four rectangles all of different dimensions.

$A = b \cdot h$ $\quad$ $A = b \cdot h$

$= 3\frac{1}{4}\text{ in.} \cdot 5\frac{1}{3}\text{ in.}$ $\quad$ $= 10\text{ in.} \cdot 5\frac{1}{3}\text{ in.}$

$= \frac{13}{4} \cdot \frac{16}{3}$ $\quad$ $= \frac{10}{1} \cdot \frac{16}{3}$

$= \frac{52}{3}$ or $17\frac{1}{3}$ sq. in. $\quad$ $= \frac{160}{3}$ or $53\frac{1}{3}$ sq. in.

The lateral area is $17\frac{1}{3} + 17\frac{1}{3} + 53\frac{1}{3} + 53\frac{1}{3} = 141\frac{1}{3}\text{ in}^2$

The bases of this prism are two identical rectangles.

$A = b \cdot h$

$= 10\text{ in.} \cdot 3\frac{1}{4}\text{ in.}$

$= \frac{10}{1}\text{ in.} \cdot \frac{13}{4}\text{ in.}$

$= \frac{130}{4}$ or 32.5 sq. in.

Area of the bases is $2 \cdot 32.5$ *or 65 sq. in.*

Total Area is the sum of the two bases of the prism and the area of all the lateral faces of the prism.

$T.A. = B.A. + L.A.$

$= 65\text{ sq. in.} + 141\frac{1}{3}\text{ sq. in.}$

$= 206\frac{1}{3}\text{ sq. in.}$

Volume –

The volume of a prism is found by multiplying the area of one base by the altitude "h" of the prism.

Area of one base is 32.5 sq. in.

$V = B \cdot h$

$= 32.5\text{ sq. in.} \cdot 5\frac{1}{3}\text{ in.}$

$= 32\frac{1}{2} \cdot 5\frac{1}{3}$

$= \frac{65}{2} \cdot \frac{16}{3}$

$= \frac{65}{1} \cdot \frac{8}{3}$

$= \frac{65 \cdot 8}{3}$

$= \frac{520}{3}$

$= 173\frac{1}{3}\text{ in}^3$ or $173\frac{1}{3}$ cubic in.

11. *Find h (height of pyramid)*

Pythagorean Theorem : $a^2 + b^2 = c^2$

$$h^2 + (\frac{1}{2} \cdot e)^2 = \ell^2$$
$$h^2 + 4^2 = 5^2$$
$$h^2 + 16 = 25$$
$$h^2 + 16 - 16 = 25 - 16$$
$$h^2 = 9$$
$$h = 3$$

$L.A. = \frac{1}{2} \cdot P \cdot \ell$

$= \frac{1}{2} \cdot (8 + 8 + 8 + 8)$ *units* $\cdot$ 5 *units*

$= \frac{1}{2} \cdot 32 \cdot 5$ *sq units*

$= \frac{1 \cdot 2 \cdot 16 \cdot 5}{2 \cdot 1 \cdot 1 \cdot 1}$ *sq units*

= 80 *sq units*

$T.A. = B.A. + L.A.$

$= L.A. + B.A.$

= 80 *sq units + area of square base*

= 80 *sq units* + (8 *units* 8 *units*)

= 80 *sq units* + 64 *sq units*

= 144 *sq units*

$V = \frac{1}{3} \cdot B \cdot h$

$= \frac{1}{3} \cdot 64$ *sq units* $\cdot$ 3 *units*

$= \frac{1 \cdot 64 \cdot 3}{3 \cdot 1 \cdot 1}$ *cubic units*

= 64 *cubic units*

12. $L.A. = \frac{1}{2} \cdot P \cdot$

$= \frac{1}{2} \cdot (10x + 10x + 10x + 10x)$ *units* $(13x)$ *units*

$= \frac{1}{2} \cdot (40x) \cdot (13x)$ *sq units*

$= \frac{1 \cdot 2 \cdot 20x \cdot 13x}{2 \cdot 1 \cdot 1 \cdot 1}$ *sq units*

$= 260x^2$ *sq units*

$T.A. = B.A. + L.A.$

$= L.A. + B.A.$

$= 260x^2$ *sq units + area of square base*

$= 260x^2$ *sq units* + ($10x$ *units* $\cdot$ $10x$ *units*)

$= 260x^2$ *sq units* + $100x^2$ *sq units*

$= (260 + 100)x^2$ *sq units*

$= 360x^2$ *sq units*

$V = \frac{1}{3} \cdot B \cdot h$

$= \frac{1}{3} \cdot 100x^2$ *sq units* $\cdot$ $12x$ *units*

$= \frac{1 \cdot 100 \cdot x^2 \cdot 3 \cdot 4x}{3 \cdot 1 \cdot 1 \cdot 1 \cdot 1}$ *cubic units*

$= 400x^3$ *cubic units*

p. 78 – Lesson 9 — Spheres

1. $(\frac{2}{3})^3 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2 \cdot 2 \cdot 2}{3 \cdot 3 \cdot 3} = \frac{8}{27}$

2. $(\frac{4}{5})^3 = \frac{4}{5} \cdot \frac{4}{5} \cdot \frac{4}{5} = \frac{4 \cdot 4 \cdot 4}{5 \cdot 5 \cdot 5} = \frac{64}{125}$

3. $\sqrt[3]{64} = \sqrt[3]{4^3} = 4^{\frac{3}{3}} = 4^1 = 4$

4. $\sqrt[3]{\frac{27}{125}} = \frac{\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{\sqrt[3]{3^3}}{\sqrt[3]{5^3}} = \frac{3^{\frac{3}{3}}}{5^{\frac{3}{3}}} = \frac{3^1}{5^1} = \frac{3}{5}$

5. $(\sqrt{3})^3 = \sqrt{3} \cdot \sqrt{3} \cdot \sqrt{3} = \sqrt{3 \cdot 3} \cdot \sqrt{3} = \sqrt{9} \cdot \sqrt{3} = 3\sqrt{3}$

6 $(.1)^3 = (.1)(.1)(.1) = 0.001$

7. $(.01)^3 = (.01)(.01)(0.1) = 0.000001$

8. $(\frac{3\sqrt{2}}{4})^3 = \frac{3\sqrt{2}}{4} \cdot \frac{3\sqrt{2}}{4} \cdot \frac{3\sqrt{2}}{4} = \frac{3 \cdot 3 \cdot 3 \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2}}{4 \cdot 4 \cdot 4} = \frac{27 \cdot 2 \cdot \sqrt{2}}{4 \cdot 2 \cdot 2 \cdot 4} = \frac{27\sqrt{2}}{32}$

9. *Surface area of a sphere* $= 4 \cdot \pi \cdot r^2$ *(Surface area is Total Area)*

$T.A. = 4 \cdot \pi \cdot r^2$
$= 4 \cdot \pi \cdot 9\ cm \cdot 9\ cm$
$= 4 \cdot \pi \cdot 81\ cm^2$
$= 324\pi\ cm^2$ *(exact answer)*
$\doteq 324 \cdot 3.14\ cm^2$
$\doteq 1{,}017.36\ cm^2$
$\doteq 1{,}017.4\ cm^2$ *(rounded to tenths)*

Volume of Sphere $= \frac{4}{3} \cdot \pi \cdot r^3$
$= \frac{4}{3} \cdot \pi \cdot (9\ cm)^3$
$= \frac{4}{3} \cdot \pi \cdot (9\ cm)(9\ cm)(9\ cm)$
$= \frac{4}{3} \cdot \pi \cdot 729\ cm^3$
$= \frac{4}{3} \cdot \frac{\pi}{1} \cdot \frac{3 \cdot 243}{1}\ cm^3$
$= 972\pi\ cm^3$ *(exact answer)*
$\doteq 972 \cdot 3.14\ cm^3$
$\doteq 3052.08\ cm^3$
$\doteq 3052.1\ cm^3$ *(rounded answer)*

10. *Surface area of a sphere* $= 4 \cdot \pi \cdot r^2$

$T.A. = 4 \cdot \pi \cdot r^2$
$= 4 \cdot \pi \cdot (2.25\ m)^2$
$= 4 \cdot \pi \cdot 5.0625\ m^2$
$= 20.25\pi\ m^2$ *(exact answer)*
$\doteq 20.25 \cdot 3.14 m^2$
$\doteq 63.585 m^2$
$\doteq 63.6 m^2$ *(rounded answer)*

diameter $= 2 \cdot r$
$4.5\ m = 2 \cdot r$
$\frac{1}{2} \cdot \frac{4.5}{1} = \frac{1}{2} \cdot \frac{2}{2} \cdot r$
$\frac{4.5}{2} = r = \frac{2 \cdot 2.25}{2} = 2.25\ m = r$

Volume of Sphere $= \frac{4}{3} \cdot \pi \cdot r^3$
$= \frac{4}{3} \cdot \pi \cdot (2.25\ m)^3$
$= \frac{4}{3} \cdot \pi \cdot (11.390625) m^3$
$= \frac{4 \cdot 11.390625 \cdot \pi}{3}\ m^3$
$= \frac{45.5625 \cdot \pi}{3}\ m^3$
$= 15.1875\pi\ m^3$ *(exact answer)*
$\doteq 15.1875 \cdot 3.14\ m^3$
$\doteq 47.68875\ m^3$
$\doteq 47.7\ m^3$ *(rounded answer)*

11. *Surface area of a sphere* $= 4 \cdot \pi \cdot r^2$

$$
\begin{aligned}
T.A. &= 4 \cdot \pi \cdot r^2 \\
&= 4 \cdot \pi \cdot (5\tfrac{1}{3} \text{ in.})^2 \\
&= 4 \cdot \pi \cdot \left(\frac{16}{3} \text{ in}\right)^2 \\
&= \frac{4}{1} \cdot \frac{\pi}{1} \cdot \frac{256}{9} \text{ in}^2 \\
&= \frac{4 \cdot 256 \cdot \pi}{1 \cdot 1 \cdot 9} \text{ in}^2 \\
&= \frac{1024\pi}{9} \text{ in}^2 \text{ (exact answer)} \\
&\doteq \frac{1024 \cdot 3.14}{9} \text{ in}^2 \\
&\doteq \frac{3215.36}{9} \text{ in}^2 \\
&\doteq 357.26222 \text{ in}^2 \\
&\doteq 357.3 \text{ in}^2 \text{ (nearest tenth)}
\end{aligned}
$$

$$
\begin{aligned}
\text{Volume of Sphere} &= \frac{4}{3} \cdot \pi \cdot r^3 \\
&= \frac{4}{3} \cdot \pi \cdot (5\tfrac{1}{3} \text{ in.})^3 \\
&= \frac{4}{3} \cdot \pi \cdot (\frac{16}{3} \text{ in.})^3 \\
&= \frac{4}{3} \cdot \frac{\pi}{1} \cdot \frac{16}{3} \cdot \frac{16}{3} \cdot \frac{16}{3} \text{ in.}^3 \\
&= \frac{4 \cdot 16 \cdot 16 \cdot 16 \cdot \pi}{3 \cdot 3 \cdot 3 \cdot 3 \cdot 1} \text{in}^3 \\
&= \frac{16{,}384}{81} \pi \text{ in}^3 \text{ (exact answer)} \\
&\doteq \frac{16{,}384}{81} \cdot \frac{3.14}{1} \text{ in}^3 \\
&\doteq \frac{51{,}445.76}{81} \text{ in}^3 \\
&\doteq 635.13283 \text{ in}^3 \\
&\doteq 635.1 \text{ in}^3 \text{ (nearest tenth)}
\end{aligned}
$$

12. *Surface area of a sphere* $= 4 \cdot \pi \cdot r^2$

$$
\begin{aligned}
\text{diameter} &= 2 \cdot r \\
5.5 \text{ yd} &= 2 \cdot r \\
\frac{1}{2} \cdot 5.5 \text{ yd} &= \frac{1}{2} \cdot 2 \cdot r \\
\frac{5.5}{2} \text{ yd} &= r \\
2.75 \text{ yd} &= r \text{ or } 2\tfrac{3}{4} \text{ yd}
\end{aligned}
$$

$$
\begin{aligned}
T.A. &= 4 \cdot \pi \cdot r^2 \\
&= 4 \cdot \pi \cdot (2.75 \text{ yd})^2 \\
&= 4 \cdot \pi \cdot 7.5625 \text{ yd}^2 \\
&= 30.25\pi \text{ sq yd (exact answer)} \\
&\doteq 30.25 \cdot 3.14 \text{ sq yd} \\
&\doteq 94.985 \text{ sq yd} \\
&\doteq 95.0 \text{ sq yd (nearest tenth)}
\end{aligned}
$$

$$
\begin{aligned}
\text{Volume of Sphere} &= \frac{4}{3} \cdot \pi \cdot r^3 \\
&= \frac{4}{3} \cdot \pi \cdot (2\tfrac{3}{4} \text{ yd})^3 \\
&= \frac{4}{3} \cdot \pi \cdot \frac{11}{4} \cdot \frac{11}{4} \cdot \frac{11}{4} \text{ yd}^3 \\
&= \frac{1 \cdot \pi \cdot 11 \cdot 11 \cdot 11}{3 \cdot 1 \cdot 1 \cdot 4 \cdot 4} \text{ yd}^3 \\
&= \frac{1331\pi}{48} \text{ yd}^3 \text{ (exact answer)} \\
&\doteq \frac{1331 \cdot 3.14}{48} \text{ yd}^3 \\
&\doteq 87.069583 \text{ yd}^3 \\
&\doteq 87.1 \text{ yd}^3 \text{ (nearest tenth)}
\end{aligned}
$$

Unit I — The Structure of Geometry

Part D — Inductive Reasoning

p. 80 — Lesson 1 — General Nature

1. He may conclude his method of weed control will be successful in all his fields. Twelve fields is a good test. However, he must take other factors into consideration such as weather conditions from year to year. Based on this experience, though, he should continue to use it until he has a series of failures.

2. Charlene will probably conlude that she is allergic to marigolds.

3. Chauncy has good reason to conclude that broccoli will be on the lunch menu every Thursday.

4. Her conclusion cannot be justified. She has forgotten about scarlet flowers such as roses or tulips.

5. a) This attitude toward the store is not justified. One dress of poor quality over a period of two years when the store would have sold perhaps hundreds of dresses would not warrant a boycott of the store.
b) This action is not justified. Joe's market might not even be the responsible party for the underweight bag. If other instances of similar complaints by customers become known, then her attitude might be justified.
c) Mr. Allen does not "know it will not rain". He has good reason to feel he can expect good weather. Weather forecasters do, in fact, use weather patterns from a particular period in the past to help them make current and future weather predictions.

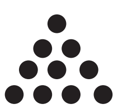
6. a) Students in a math class may come to feel that their teacher will not give a quiz on Monday since the first seven weeks of the school year have passed without a quiz on Monday.
b) During the past seventeen football seasons, the University of Indiana has lost the nine games it has played against the University of Illinois when playing at memorial stadium in Urbana, Illinois. The local sports writer is predicting the "jinx" will continue.
c) Although the posted speed limit is 55 mph on the local expressway, Jim has driven 70 mph without receiving a ticket for the past five weeks when driving to his new job across town. He surmises he can do this whenever he wishes without receiving a citation.
d) Over the past three years, Jonathan has observed three particular stocks in his portfolio gaining little or no value. He feels he should take the money from these three investments and reinvest it in some other area since the three stocks will never do well.
e) Jerry has hired the last five engineers for his company from the University near his home town. They have all been excellent empoyees. He will give future candidates from the university top priority when hiring in the future.
f) The local newspaper has predicted the winner of the upcoming election based on the opinions of 500 likely voters taken in a poll at the local shopping mall.

Unit I — The Structure of Geometry

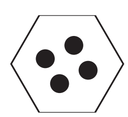
Part D — Inductive Reasoning

p. 83 — Lesson 2 — Applications in Mathematics

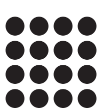
1. a)



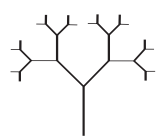
b)



c)



d)



make a right angle at each tip

2. a) 111,111,111

check: 12,345,678
x9
111,111,102
+9
111,111,111

b) 88,888,888

check: 9,876,543
x9
88,888,887
+1
88,888,888

3. Since an equilateral triangle is by definition a triangle with three sides of equal measure, Allan could find the perimeter by multiplying the length of a side by three.

4. Even; Negative 1

5. a) 256; 1024 — Each number can be found by multiplying the previous number by 4.

b) 6; 3 — Each number can be found by subtracting three from the previous number.

c) $\frac{1}{81}$; $\frac{1}{243}$ — Each number is found by multiplying the previous number by 1/3.

d) 25; 36 — Each number in the sequence can be found by adding consecutive odd integers, beginning with 3, to the previous number. (1 + 3 = 4, 4 + 5 = 9, 9 + 7 = 16, 16 + 9 = 25, 25 + 11 = 36, and so on.)

e) 17; 23 — Each number in the sequence can be found by adding consecutive integers, beginning with 1, to the previous number. (2 + 1 = 3, 3 + 2 = 5, 5 + 3 = 8, 8 + 4 = 12, and so on.)

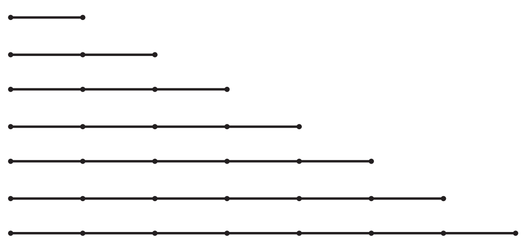
f) 40; 52 — Each number in the sequence can be found by adding consecutive even integers, beginning with 2, to the previous number. (10 + 2 = 12, 12 + 4 = 16, 16 + 6 = 22, and so on.)

g) 15; 4 — In this sequence of numbers, each number is found by subtracting odd integers, beginning with 1, from the previous number.

6. No. ... 23, 29, 35, 41, 47, 53, 59, 65, 71, 77, 83, 89, 95,...

35 is not prime
65 is not prime
77 is not prime
95 is not prime

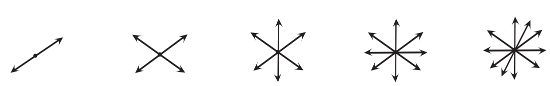
7.



2 points		1 line segment
3 points		3 line segments
4 points		6 line segments
5 points		10 line segments
6 points		15 line segments
7 points		21 line segments
8 points		28 line segments
9	28 + 8 =	36
10	36 + 9 =	45
11	45 + 10 =	55
12	55 + 11 =	66
13	66 + 12 =	78
14	78 + 13 =	91
15	91 + 14 =	105
16	105 + 15 =	120
17	120 + 16 =	136
18	136 + 17 =	153
19	153 + 18 =	171
20	171 + 19 =	190

190 Line segments

8.



2	4	6	8	10
1 x 2	2 x 2	3 x 2	4 x 2	5 x 2

Answer is 36. (18 x 2)

Unit I — The Structure of Geometry
Part E — Deductive Reasoning

p. 86 — Lesson 1 — General Nature

1. a) John is passing in three one-credit subjects.
   b) Mario's car was not manufactured in 1966.
   c) A Robin has feathers.

2. a) If Sue goes home from school, then she rides on the school bus. We cannot conclude with **certainty** that Sue is going home from school. The specific statement, "Sue is riding on the bus", does not satisfy the condition of the general statement that Sue is going home from school. The specific statement could mean Sue is going to a school event after school, or she might actually be going to school.
   b) The general statement written in another way would say, "If a stranger enters the yard, then Pal barks." The condition is that "A stranger enters the yard." The specific statement, "Pal is barking", could be caused by events other than a stranger coming into the yard. Since the specific statement does not meet the condition of the general statement, no valid conclusion can be made.
   c) The general statement written another way is "If a person is on Mr. Hoopster's basketball team, then that person is at least six feet tall." The specific statement, "Paul Bevars is on Mr. Hoopster's basketball team", satisfies the condition of the general statement. We can conclude, with certainty, that Paul Bevars is at least six feet tall.
   d) The general statement written another way is, "If it is Monday, Tuesday, Wednesday, Thursday, or Friday, then Cheryl is going to work." The specific statement, "Cheryl is not going to work today", does not satisfy the requirements of the general statement (what day is it?). We might be inclined to say that today is Saturday or Sunday based on the specific statement, but we cannot do so if we consider the formal structure of a valid argument.
   e) Adrienne is not in elementary school. The general statement restated is "If a student is old enough to vote, then the student is not in elementary school." The specific statement "Adrienne is old enough to vote" satisfies the condition of the general statement.

Unit I — The Structure of Geometry
Part E — Deductive Reasoning

p. 90 — Lesson 2 - Applications in Mathematics

1. a) Point P bisects line segment AB.
   b) Angle A and Angle B are equal in measure.
   c) Line AB and line CD will not intersect.
   d) Line segment DE is one-half of the third side WY.

2. a) An integer is a real number.
   b) A, B, C have no size.
   c) Here we cannot draw a valid conclusion. We can restate the given statement as, "If geometric figures are line segments, then the figures are set of points". Here the specific statements is about triangles, not line segments. So, the condition of the general statement is not met. While it is true that a triangle is a set of points, we cannot make this statement using the formal structure of a valid argument as the basis for our conclusion.
   d) Lines p and q have exactly one point in common.
   e) The phrase "3(7 – 5)" does not contain a relation symbol or a placeholder symbol.

Unit I — The Structure of Geometry

Part F — Logic

p. 96 — Lesson 1 — Simple Statements

1.

a) $x < 2$	This sentence is not a statement. If cannot be judged true or false until we know what "x" is. This is known in Algebra as an open sentence.
b) Paris is the capital city of France.	This sentence is a statement. It can be judged true or false.
c) The moon is made of green cheese.	This sentence is a statement. It can be judged true or false. Now that man has been to the moon, we think this statement would be false.
d) $x + y = y + 2x$	This sentence is not a statement. It cannot be judged true or false until we know what values are assigned to x and y.
e) Way to go!	This sentence is not a statement. It cannot be judged true or false. In English it would be called an exclamatory sentence.
f) This sentence is false	This sentence is not a statement. Until we know what "sentence" we are talking about, we cannot judge the original sentence to be true or false.

2. a) I like poetry and you like the classics.
b) I do not like poetry.
c) You do not like the classics.
d) I like poetry or you like the classics.
e) I like poetry and you do not like the classics.
f) It is not true that I like poetry and you like the classics.
g) I do not like poetry or you do not like the classics.
h) I do not like poetry and you like the classics.
i) It is not the case that I like poetry or you like the classics.
j) I do not like poetry and you do not like the classics.

3. a) $q \wedge p$
b) $\sim p$
c) $q \wedge \sim p$
d) $\sim q$
e) $\sim q \wedge \sim p$
f) $p \vee \sim p$
g) $\sim q \wedge p$
h) $\sim(\sim p)$

4. a) Yes b) No
c) No d) No
e) Yes f) No

5. a) No b) No
c) No d) Yes
e) No f) No

6. a) False b) False
c) True d) False
e) True f) True
g) False h) True
i) True

7. p is True q is false

a) $p \wedge q$	No, she is not telling the truth.
b) $p \vee q$	Yes, she is telling the truth.
c) $\sim p$	No, she is not telling the truth.
d) $\sim q$	Yes, she is telling the truth.

8. a) True b) False
c) True d) True
e) True f) True
g) False h) True
i) False j) False
k) True l) True
m) True n) True

9.

	$p \wedge q$	$(p \wedge q) \vee r$
a)	T	T
b)	T	T
c)	F	T
d)	F	F
e)	F	T
f)	F	F
g)	F	T
h)	F	F

10. a) $x = 3$ or $x = 2$ True
b) $2x = 10$
$x = 5$ $y = 1$ True

11. a) $F \wedge T$ False
b) $F \wedge F$ False
c) $T \wedge T$ True

12. a) $F \vee T$ True
b) $F \vee F$ False
c) $T \vee T$ True
d) $F \vee F$ False

13. a)
$$-2 + 1 = -1$$
$$-3(-2) - 1 = 5$$
$$\text{Therefore, } T \wedge T = T$$

b)
$$2(-2) - 1 = 5$$
$$3(-2) + 2(1) = 3$$
$$\text{Therefore, } F \wedge F = F$$

Unit I — The Structure of Geometry

Part F — Logic

p. 104 — Lesson 2 - Conditionals

1. David is out.

2. Georgio is a brave person.

3. A yorkshire has four legs.

4. Slokum is a gaggle.

5. Minor Premise: Samantha is a girl.

6. Major Premise: All gentlemen are scholars.

7. Major Premise: All tall men are Texans.

8. Major Premise: An irrational number is a non-terminating, non repeating decimal.

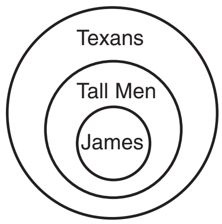
9. Valid

10. Not Valid — All boys are clever.
Josie is a boy.
Josie is clever.

11. Not Valid — All class presidents are bright.
Frankie is a class president.
Frankie is bright.

12. Valid

13.



Notice that this special type of Venn Diagram shows various sets as subsets of other sets. In other words, they are completely contained in other sets. This type of Venn Diagram was devised by a famous mathematician, Leonard Euler (pronounced "Oiler"), so the circles it uses are commonly called "Euler Circles".

14.

$A \subset F$	Set of "Americans" is a subset of set of "Free People."
$f \in A$	A "Floridian" is a member of the set of "Americans."
Therefore, $f \in F$	A "Floridian" is a member of the set of "Free People."

	Hypothesis	**Conclusion**
15.	Today is March 10.	Yesterday was March 9
16.	$x - 6 = 20$	$x = 26$
17.	You ride a bicycle.	You have strong legs.
18.	$x < 0$	$5x > 6x$

19. If you are a resident of Dallas, then you are a resident of Texas.

20. If a number is a rational number, then that number is a real number.

21. If two numbers are odd, then their sum is even. (Note: The conventional use of the expression "only if" in logic, implies that the phrase following the expression is the conclusion.)

22. If you are a resident of Chicago, then you are a resident of Illinois.

23.

$p \rightarrow q$	If Clayton is a 12[th] grader, then Clayton is a senior.
$q \rightarrow r$	If Clayton is a senior, then Clayton is in a government class.
$p \rightarrow r$	If Clayton is a 12[th] grader, then Clayton is in government class.

24.

$p \rightarrow q$	If it is raining, then the sky is cloudy.
$q \rightarrow r$	If the sky is cloudy, then you cannot see the sun.
$p \rightarrow r$	If it is raining, then you cannot see the sun.

25. Lines AB and CD intersect.

26. No conclusion may be reached, purely from the perspective of logic.

27. The law of detachment asserts that, if we know that "$p \rightarrow r$ and p" are true, we may detach "q" from "$p \rightarrow q$"and assert that "q" is true. We start with a conditional statement and the truth of the hypothesis (i.e., not the truth of the conclusion.) To write another true statement in the exercise, we need to assert "A cat is a mammal." then we could conclude "A cat is warm blooded."

28. According to the Law of Syllogism (or chain rule), if you accept "If p, then q" as true, and if you accept "If q, then r" as true, then you must logically accept "If p, then r" as true. We draw a conditional conclusion from two conditional statements in which the hypothesis of the second conditional is the conclusion of the first conditional. To write a true statement in this exercise, the second conditional should be "If your taxes are lower,..." We could then write the conditional conclusion, "If you vote for me, then I will save you money."

29. If a man lives in Indiana, then he lives in Indianapolis.	Sometimes True
30. If an animal has four legs, then it is a normal horse.	Sometimes True
31. If the grass is wet, then it is raining.	Sometimes True
32. If you were born in California, then you are a native Californian.	Always True
33. If a number is an odd number, then it is a multiple of 3.	Sometimes True
34. If 6 is a factor of the square of an integer, then 6 is a factor of the integer.	Always True

35. If a triangle is isosceles, then the triangle is equilateral. Sometimes True

36. If the sum of two numbers is odd, then the two numbers are both odd. Never True

37. If a triangle is equilateral, then the triangle is equilangular.

38. If a number is odd, then the number is evenly divisible by 2.

39. If a number is a multiple of 6, then the number is divisible by 3.

40. If you are a politician, then you are president of the United States.

41. x is an odd integer if and only if x^2 is an odd integer (or, x^2 is an odd integer if and only if x is an odd integer)

42. A geometric figure has four sides if and only if a geometric figure is a quadrilateral (or, A geometric figure is a quadrilateral if and only if a geometric figure has four sides.)

Unit I — The Structure of Geometry

Part F — Logic

p. 109 — Lesson 3 - Negations of Condtitionals

1. Inverse: If $x^2 \neq 4$, then $x \neq 2$ or $x \neq -2$ True
Converse: If $x = 2$, or $x = -2$, then $x^2 = 4$ True
Contrapositive: If $x \neq 2$, or $x \neq -2$, then $x^2 \neq 4$ True

2. Inverse: If two angles are not right angles, then they are not congruent. False
Converse: If two angles are congruent, then they are right angles. False
Contrapositive: If two angles are not congruent, then they are not right angles. True

3. Inverse: If you do not finish a marathon, then your physical condition is not good. False
Converse: If your physical condition is good, then you do finish a marathon. False
Contrapositive: If your physical condition is not good, then you do not finish a marathon. True

4. Inverse: If you are not a zebra, then you do know how to fly. False
Converse: If you do not know how to fly, then you are a zebra. False
Contrapositive: If you do know how to fly, then you are not a zebra. True

5. Inverse: If you do not have a cold, then you are not sick. False
Converse: If you are sick, then you have a cold. False
Contrapositive: If you are not sick, then you do not have a cold. True

6. Inverse: If an angle is not a right angle, then the angle does not measure 90°.
Converse: If an angle measures 90°, then an angle is a right angle.

Both conditionals are true.

7. Inverse: If $m \not> 0$, then $m^2 \not> 0$
Converse: If $m^2 > 0$, then $m > 0$

Both conditionals are false.

8. Inverse: If an animal is not a trout, then the animal is not a fish.
Converse: If an animal is a fish, then the animal is a trout.

Both conditionals are false.

9. Conditional: If a person studies Geometry, then that person is a mathematics student.
Contrapositive: If a person is not a mathematics student, then that person does not study Geometry.
Converse of the contrapositive: If a person does not study Geometry, then that person is not a mathematics student.
Inverse of the Converse of the Contrapositive: If a person studies Geometry, then that person is a mathematics student.

The result is the same statement as the original conditional statement. If you interchange the hypothesis and conclusion twice, and reverse the truth values of each twice, you will be brought back to the original statement.

10. Dana should use the contrapositive to try to convince the jury that Jim is innocent.

If Jim stole the watch, then he was in the jewelry store. The contrapositive of this statement is, " If Jim was not in the jewelry store, then Jim did not steal the watch." If she can show that Jim was not in the jewelry store, the jury will have to conclude Jim did not take the watch.

Using this strategy relies on the fact that a statement and its contrapositive have the same truth value.

11.	Conditional:	If a person has an engineering degree, then that person has a good chance of getting a high-salaried job.	True
	Contrapositive:	If a person does not have a good chance of getting a high-salaried job, then that person does not have an engineering degree.	True
12.	Conditional:	If a person is a lifelong resident of the West Indies, then that person is accustomed to hot weather.	True
	Contrapositive:	If a person is not accustomed to hot weather, then that person is not a lifelong resident of the West Indies.	True
13.	Conditional:	If there is an earthquake, then the ground trembles.	True
	Contrapositive:	If the ground does not tremble, then there is not an earthquake. (or then an earthquake is not occurring.)	True
14.	Conditional:	If you are funny, then you will make people laugh.	True
	Contrapositive:	If you do not make people laugh, then you are not funny.	True
15.	Conditional:	If points are collinear points, then they are points on the same line.	True
	Contrapositive:	If points are not on the same line, then the points are not collinear	True
16.	Conditional:	If Jim can vote, then he is at least 18 years old.	True
	Contrapositive:	If Jim is not at least 18 years old, then he cannot vote.	True
17.	Conditional:	If two angles have the same measure, then they are congruent.	True
	Contrapositive:	If two angles are not congruent, then they do not have the same measure.	True
18.	Conditional:	If the road is slippery, then there is ice on the road.	False
	Contrapositive:	If there is no ice on the road, then the road is not slippery.	False
19.	Conditional:	If two angles in a right triangle are acute, then the two angles are complementary. (VTI Algebra Page 129)	True
	Contrapositive:	If two angles in a right triangle are not complementary, then the two angles are not the acute angles of the right triangle.	True
20.	Conditional:	If a number is even, then the number ends in five.	False
	Contrapositive:	If a number does not end in five, then the number is not even.	False
21.	Conditional:	If a figure is a square, then the figure is not a triangle.	True
	Contrapositive:	If a figure is a triangle, then the figure is not a square.	True

22. a) Jamaal has used the converse of the original statement. He is saying, "If I wear shirts from Rachbach's, then I am fashionable." This is "If q, then p", the converse of "If p, then q."

b) No, Jamaal's reasoning is not correct. A true statement does not always have a true converse. In this case, wearing a shirt from a high fashion clothing store does not necessarily ensure that you are fashionable. Other factors enter into the definition of what is "fashionable" and what is "not fashionable".

Unit I — The Structure of Geometry

Part F — Logic

p. 112 — Lesson 4 - Fallacies

1. If you are shopping for the latest technology (plasma screens) in television sets, models made by the ACR Company would probably be the benchmark to which you would compare other brands on the market. However, there could be an element of faulty analogy involved here if you do not do your research. A television with a plasma screen involves a new technology. Has ACR Company kept up with the development of this new technology? Before a purchase is made involving thousands of dollars, you really need to know what companies are now the leaders in the development of this new product.

2. There certainly is a grave danger involved in this decision. Mrs. Riley may not know all the facts about Mrs. Habley's illness when comparing her own symptoms to what she observes in Mrs. Habley. This is an example of reasoning by a faulty analogy. Mrs. Riley should see a doctor first.

3. While Sarah and her mother are similar in many respects, there are certainly conditions in Sarah's life, other than her relation to her mother, that will help to determine her success in studying Geometry. Quality of instruction might be better for Sarah. Interest in the subject might be more acute with Sarah, if her life experiences, both in and out of school, have fostered an interest in ideas associated with Geometry. Assuming that she will do poorly in the subject simply because her mother did, is an example of reasoning by faulty analogy.

4. Britt is not a criminal simply because his father has been in trouble. This is reasoning by faulty analogy. Britt might have to work extra hard at overcoming his father's reputation, but he should be judged by the values he has displayed in his own life.

5. What economic forces are present in states where a sales tax has been an adequate source of funding for the state's obligations, without placing an unfair burden on the residents of the state? An example would be a state with a large amount of tourist trade, where the visitors pay a large portion of the taxes collected in the state. And, is the politician willing to set aside other forms of tax assessment once his new sales tax is instituted? Unless the politician can show that his state has economic similarities to states where a sales tax is adequate, he is using a faulty analogy to promote his idea.

6. This is a faulty analogy since, while some properties exist between a rectangle and a square that are similar, other properties exist which are different. We have to take into account that, in a rectangle, all four sides are not necessarily equal, and therefore the diagonals of a rectangle would not necessarily have the same relationship to the angles of the rectangle as the diagonals of a square have to the angles of the square. All facts about both geometric figures must be considered.

7. Consider this situation written in the form of a conditional statement. If you are a junior at Lincoln High School, then you take the Lincoln High School driver's education class. Consider the converse of this statement. If you take the Lincoln High School driver's education class, then you are a junior at Lincoln High School. This is not necessarily true. Ross Baker could be taking the driver's education class and not even be a student at Lincoln High School. Perhaps Ross is out of school but has a poor driving record and has been assigned by a judge to take a driver's education class to improve his driving knowledge and skills. Perhaps Ross attends another neighboring school that does not offer a driver's education class. Other scenarios may exist where Ross would take the class but not be a member of the Lincoln High School junior class.

8. Monica needs to be more aware of the “science” involved here. Consider this situation written in the form of a conditional statement. If the temperature of water is 212 degrees, then the water is boiling. (This is true at sea level.) Consider the converse of this statement. If the water is boiling, then the temperature of the water is 212 degrees. The atmospheric pressure on Pikes Peak is less than at sea level so water will boil at a lower temperature.

9. Consider the situation written in the form of a conditional statement. If a car runs out of gas, then the car will not start. Consider the converse of this statement. If a car will not start, then it is out of gas. The first statement is true. The converse is not necessarily true. There are a number of reasons why a car will not start. Having no gasoline is only one of them.

10. Consider the situation written in the form of a conditional statement. If a student attends school regularly, then the student gets good grades. We would generally consider that statement to be true. A student who attends school regularly will probably still have to give some effort in the classes being taken in order to get good grades, but we would expect this type of student to put forth that effort. Now consider the converse of this statement. If a student gets good grades, then the student attends school regularly. In this case, we would have to admit that the converse is certainly not always true. Perhaps Julie gets good grades because she is a very capable student and knows how to study independently. Other scenarios do exist where students get good grades and do not attend school.

11. Consider the inverse of the given conditional statement. If a student does not plan to be an engineer, then the student should elect not to study mathematics in high school. In stating the inverse, we are negating the hypothesis (denying the premise), and implying that we can now, logically, negate the conclusion. Engineering is the not the only occupation where a knowledge of mathematics is required. There are many areas of study where mathematics is not only necessary, but required. The nursing, teaching, or architectural professions are only a few. Once again, we see that, just because a given conditional is true, the inverse of that conditional is not necessarily true. We cannot successfully argue by denying the premise.

12. Consider the inverse of the given conditional statement. If a quadrilateral is not a trapezoid, then the quadrilateral does not have two parallel sides. In stating the inverse, we are negating the hypothesis (denying the premise), and implying that we can now, logically, negate the conclusion. Trapezoids are not the only quadrilaterals with two parallel sides. Parallelograms, squares, rectangles, and rhombi all have two parallel sides. Again, we see that, just because a given conditional is true, the inverse of that conditional is not necessarily true. We cannot successfully argue by denying the premise.

13. Consider the inverse of the given conditional statement. If all the angles of a quadrilateral are not equal, then the quadrilateral is not a parallelogram. In stating the inverse, we are negating the hypotheses (denying the premise), and implying that we can now, logically, negate the conclusion. A rhombus is a quadrilateral which does not have all angles the same measure, but yet is part of the family of parallelograms. One instance where this inverse is true is a kite, a quadrilateral which does not have all angles the same measure, and, in fact, is not a parallelogram. (Note: In this question, we might recognize somewhat of a problem in that we have to recognize, and accept the fact, that a rectangle is a member of the family of parallelograms) So, once again, we see that we cannot successfully argue a point by denying the premise.