

Geometry: A Complete Course (with Trigonometry)

Module D – Instructor's Guide with Detailed Solutions for Progress Tests

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VideoText Interactive

Geometry: A Complete Course (with Trigonometry)
Module D - Instructor's Guide with Detailed Solutions for Progress Tests

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Letter from the author . . .

Wow! Module C was really something wasn't it?

I hope you're not too "worn out" from all of the demonstrations. You probably have never given so much meticulous attention to detail in preparing an argument. In fact, you may even have become a little discouraged when you didn't quite get all of the statements and all of the reasons down in the same order, or in the same words I used. That really is okay. Not only are there other ways to prove some of those fundamental theorems, but there really isn't an absolutely correct way to word the reasons. In any case, the point is that you were getting better and better at seeing **the need for detailed, logical arguments**, when trying to "prove" something. Of course, I hope you weren't hesitant to call the help-line if you were having trouble.

Anyway, you can now understand why I told you that **Module C is the "pivotal unit" in the course**. And, as you now move into Module D, which investigates the **application of those fundamental theorems**, specifically to triangles, you will find the demonstrations to be generally more concise, and easier to construct. In other words, you will see, much more readily, exactly what elements are needed in your proofs.

So, before you begin this module, let me give you a brief overview of the focus of our study. As I said before, **we will be concerned primarily with triangles**, the simplest of the Simple Closed Plane Curves made up of straight line segments. Believe me, you will see the importance of this focus, in future modules, when we study other polygons. For now, we will be exploring the various **relationships between the different parts of triangles**, and then expanding that investigation to include two special relationships between triangles themselves. Specifically, we will be concerned with **Similarity** (the relationship between two triangles which are the same shape, but not necessarily the same size), and **Congruence** (the relationship between two triangles which are the same shape, and the same size).

And once again, in the course of our investigation, we will have to "make up" a few more rules. That's right. **We will have to accept several more postulates**, in order to prove some of the "what-ifs" we will encounter. Of course, that means you will have updated listings of definitions, postulates, and theorems, at the end of the student worktext.

So, do you think you are ready to begin? Of course you are. And I have no doubt that you will do well, since you have now had so much more experience with "logical thinking". And, as always, I encourage you to call us on the help-line, if you have any difficulty at all.

Thomas E. Clark, Author

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Program Overview

The VideoTextInteractive Geometry program addresses two of the most important aspects of mathematics instruction. First, the inquiry-based video format contributes to the engaging of students more personally in the concept development process. Through the frequent use of the pause button, you, as the instructor, can virtually require interaction and dialogue on the part of your student. As well, students who work on their own, can “simulate” having an instructor present by pausing the lesson every time a question is asked, and trying to answer it correctly before continuing. Of course, the student may answer incorrectly, but the narrator will be sure to give the right answer when the play button is pressed to resume the lesson. Right or wrong, however, the student is regularly engaging in analytical and critical thinking, and that is a healthy exercise, in and of itself. **Second, each incremental concept is explored in detail, using no shortcuts, tricks, rules, or formulas, and no step in the process is ignored.** As such, the logic and the continuity of the development assure students that they understand completely. Subsequently, learning is more efficient, and all of the required concepts (topics) of the subject can be covered with mastery. Of course, the benefits of these efforts can be seen even more clearly in a description of a typical session, as follows:

After a brief 2 or 3 sentence introduction of the concept to be considered, usually by examining the description, and the objective given at the beginning of the video lesson, you and your student can begin. **You should pause the lesson frequently**, usually every 15-20 seconds (or more often if appropriate), to engage your student in discussion. This means that, for a 5-10 minute VideoText lesson, it may take 10-15 minutes to finish developing the concept. Dialogue is a cornerstone. In addition, during this time, **your student should probably not be allowed to take notes.** Students should not have their attention divided, or they risk missing important links. Neither should you be dividing your attention, by looking at notes, or writing on a pad, or an overhead projector. **Everyone should be concentrating on concept development and understanding.** Please understand that a student who is accustomed to working alone, or can be motivated to study independently, has, with the VideoText, a powerful resource to explore and master mathematical concepts by simulating the dialogue normally encountered with a “live” instructor. And, because of the extensive detail of the explanations, along with the computer generated graphics, and animation, students are never shortchanged when it comes to the insight necessary to fully comprehend.

Once the concept is developed, and the VideoText lesson is completed, you can then **employ the Course Notes to review, reinforce, or to check on your student's comprehension.** These Course Notes are replications of the essential content that was viewed in the VideoText lesson, illustrating the same terms, diagrams, problems, numbers, and logical sequences. In fact, at this time, if your student needs a little more help, he or she can use the Course Notes while viewing the lesson again, using them as a guide, to re-examine the concept. **The key here is that students concentrate on understanding first, and take care of documentation later.**

Please understand that it is not the intent of the program to let the VideoText lesson completely take the place of personal instruction or interaction. Actually, **the video should never tell your students anything that hasn't been considered or discussed (while the lesson is paused), and it should never answer questions that have not already been considered and resolved.** As such, it becomes a “new breed” of chalkboard or overhead projector, whereby you, as the teacher, or your student working alone, can “write”, simply by pressing the “play” button. This is a critical point to be understood, and should

serve to help you examine all of the materials and strategies from the proper perspective.

Next, your student can begin to do some work independently, either by your personal introduction of additional examples from the WorkText, or by the student immediately going to the WorkText on his or her own. **The primary feature of the WorkText**, beside providing problem banks with which students can work on mastery, is that **objectives are restated, important terms are reviewed, and additional examples are considered**, in noticeable detail, **taking students, once again, through the logic of the concept development process**. The premise here is simple. When students work with an instructor, whether doing exercises on their own, or working through them with other students, they are usually concentrating more on “how to do” the problems. Then, when they leave the instructor, they simply don't take the discussion of the concept with them. The goal of this program is to provide a resource which will help students “re-live” the concept development on their own, whether for review, or for additional help. That is the focus of the Student WorkText.

Having completed the exercises for the lesson being considered, your **student is now ready to use the detail in the Solutions Manual to check work and engage in error analysis**. Again, it is essential to a student's understanding that he or she find mistakes, correct them, and be required to give some explanation, either verbal or in writing, to you as the instructor. In fact, at this stage, you might even **consider grading your student only on the completion of the work**, not on its accuracy. Remember, this is the first time the student has tried to demonstrate understanding of a concept, and he or she may still need some fine-tuning. So, because this is part of the initial learning process, **the focus should be on a careful analysis of the logic behind the work, not just the answers**. Finally, **it is time to assess your student's mastery** of the concept behind the work. **Just be sure you are not testing on the same day the exercises were completed**. Short-term memory can trick you into thinking that you “have it”, when, in fact, you are just remembering what you did moments before. A more accurate evaluation can be made on the next day, before moving on to the next lesson. Further, the quizzes and tests in the program often utilize **open-response questions which will require your student to state, in writing, his or her understanding of the concept**. This often reveals much more about a student's understanding than just checking to see if an answer on a test is correct. Remember too, that there are **two versions of every quiz and test**, allowing you to retest, if necessary, in order to make sure that your student has mastered the concept.

Of course, just as with the WorkText, there are detailed **solutions for all of the quiz and test problems, in the Instructor's Guide**. Again, your student should be required to analyze problems that were missed, and explain why the problem should have been done differently. It is simply a fact that one of the most powerful and effective teaching tools you can employ, is to **ask your students to “articulate” to you what their thinking was**, as they worked toward a given answer.

As you can see, the highly interactive quality of this program, affords students a much greater opportunity than usual to grow mathematically, at a personal level, and develop confidence in their ability. That can have a tremendous impact on a student's future pursuits, especially in an age where applications of mathematics are so important.

Scope and Sequence Rationale

There are two basic premises which drive concept development in Geometry, and these two essentials shape the logical scope and sequence of geometric content.

First, it is generally understood that **Geometry is the study of spatial relations**. In the same way that Algebra is the study of numerical relations (equations and inequalities), and Calculus is concerned primarily with rates of change, Geometry is a comprehensive exploration of “shapes” (as sets of points), the measurements associated with those shapes, and the relationships that can be established between those shapes. As such, no treatment of Geometry should ever investigate those relationships only individually, or in isolation. This is especially noticeable with traditional textbooks, which generally use a format which addresses them in different “chapters”. In the VideoText Interactive Geometry course, **concepts are discussed from a “Unit” perspective, pursuing and connecting, in an exhaustive way, all of the outcomes associated with various possibilities for a specific relationship**. Of course, as much as is possible, students need to “see” those relationships, and experience the “motion”, or “transformation”, necessary to clearly illustrate the concept. It really is impossible to put a value on the benefits of visualization, in life in general, and in Geometry in particular. So, in the VideoText Interactive Geometry program, **computer-generated graphics are used extensively, along with animation and color-sequencing**, in order that students can actually see the relationships develop.

The second premise is that geometric concepts should be studied **utilizing all of the power and conviction that both inductive and deductive reasoning can bring to the table**. In other words, it is always desirable, and helpful, for students to “experiment”, inductively, with a geometric relationship, in an effort to come to some general conclusion. Once that general conclusion has been arrived at, however, it is even more convincing if the student is able to “prove”, deductively, that the conclusion absolutely must follow, logically, from the given information. No, formal proof is not often asked for in everyday life. On the other hand, the exercise of developing that kind of thinking is invaluable, not only in some specific job-related activities, but, more generally, in the daily problem-solving situations that confront us. The VideoText Interactive Geometry program is formatted in such a way that formal proof is a cornerstone.

Unit I, then, focuses on a complete preparation for students to begin a formal study of Geometry by “re-teaching” of all of the basic geometric concepts for which students have simply memorized the appropriate term, definition, or formula. That means we must re-establish that **Mathematics in general, and Geometry in particular, is a language**, with parts of speech and sentence structure. We must develop, in detail, the concepts associated with **building geometric shapes**. We must investigate, again in detail, the concepts dealing with the **measurement of those shapes**. Finally, we must thoroughly develop the principles of inductive and deductive reasoning, giving significant attention to the dynamics of mathematical deductive logic, which are the building blocks that students will use to **construct formal proofs**.

In Unit II, we begin the actual study of “Plane Geometry” by developing all of the necessary terms, definitions, and assumptions we will be using as a basis for studying geometric relationships. In other words, we draw on the analogy that studying any area of Mathematics is like “playing a game”. We must first determine **which basic elements will be “undefined”** in our Geometry, or accepted without definition. We must then determine which basic elements can be formally defined, using those unde-

Quiz Form A

Name _____

Class _____ Date _____ Score _____

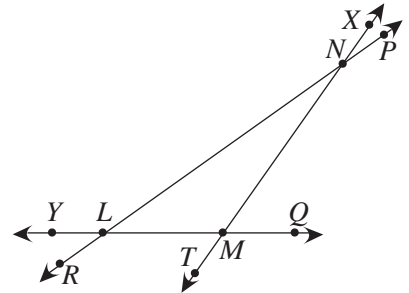
Unit IV - Triangles

Part A - Basic Definitions

Lesson 1 - Triangle Parts

Lesson 2 - Triangle Types

Use the diagram to the right to answer each of problems 1-6.



1. Names the vertices of the triangle.

Point L, Point M, Point N

2. Name the sides of the triangle.

\overline{LM} , \overline{MN} , \overline{LN}

3. Name the angles of the triangle.

$\angle NLM$, $\angle NML$, $\angle MNL$

$\angle XMQ$ ($\angle NMQ$), $\angle PNT$ ($\angle PNM$), $\angle RLQ$ ($\angle RLM$)

4. Name the exterior angles of the triangle.

$\angle TML$ ($\angle TMY$), $\angle XNR$ ($\angle XNL$), $\angle PLY$ ($\angle NLY$)

5. For $\angle NLM$:

a) Name the two sides of the angle

\overrightarrow{LN} , \overrightarrow{LM}

b) Name the two side of the triangle that include the angle.

\overline{LN} , \overline{LM}

c) Name the side of the triangle opposite the angle.

\overline{NM}

6. For \overline{MN} :

a) Name the two angles of the triangle that include this side. $\angle MNL$, $\angle NML$

b) Name the angle of the triangle which is opposite this side. $\angle NLM$

Quiz Form A

Name _____

Class _____ Date _____ Score _____

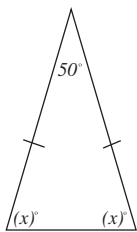
Unit IV - Triangles Part B - Basic Theorems

Lesson 1 - Theorem 25: "If you have any given triangle, then the sum of the measures of its angles is 180"

Lesson 2 - Theorem 26: "If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measure of the two remote interior angles."

1. The vertex angle of an isosceles triangle measures 50° . The base angles are congruent. Sketch a figure illustrating this triangle. Then find the measures of the base angles.

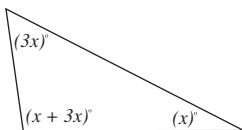
Base angle: 65°



$$\begin{aligned}x + x + 50 &= 180 \\2x &= 130 \\x &= 65\end{aligned}$$

2. The measure of one angle of a triangle is three times the measure of another and the third angle's measure is equal to their sum. Sketch a figure illustrating this triangle. Then find the measures of all three angles.

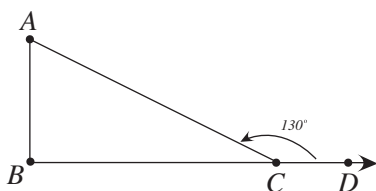
First angle: 67.5°
Second angle: 22.5°
Third angle: 90°



$$\begin{aligned}x + 3x + (x + 3x) &= 180 & 3x &= 67.5 \\8x &= 180 & x + 3x &= 90 \\x &= 22.5\end{aligned}$$

3. In $\triangle ABC$, the exterior angle at C measures 130° , and $m\angle A = 50^\circ$. Sketch a figure illustrating this triangle. Find $m\angle B$.

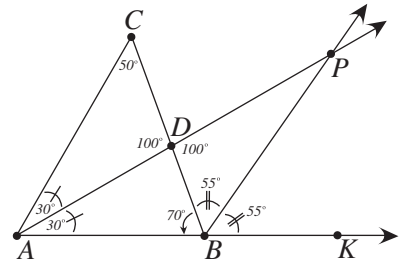
Measure $\angle B$: 80°



$$\begin{aligned}m\angle ACD &= m\angle A + m\angle B \\130 &= 50 + m\angle B \\80 &= m\angle B\end{aligned}$$

—Continued—

4. The side \overline{AB} of $\triangle ABC$ is extended to K. The bisector of $\angle A$ intersects \overline{BC} at D, and meets the bisector of $\angle CBK$ outside $\triangle ABC$ at P. Also, $m\angle A = 60^\circ$ and $m\angle ABC = 70^\circ$. Sketch a figure illustrating this situation. Find the measure of $\angle P$: 25°.
 How do the measures of $\angle C$ and $\angle P$ compare? $m\angle P$ is $\frac{1}{2} m\angle C$



First, find $m\angle C$:

$$m\angle A + m\angle ABC + m\angle C = 180$$

$$60 + 70 + m\angle C = 180$$

$$m\angle C = 50$$

Second, find $m\angle BDP$

$$m\angle CAD + m\angle ACD + m\angle CDA = 180$$

$$30 + 50 + m\angle CDA = 180$$

$$m\angle CDA = 100 \quad m\angle BDP = 100$$

Third, find $m\angle CBP$

$$m\angle ABC + m\angle CBK = 180$$

$$70 + m\angle CBK = 180$$

$$m\angle CBK = 110$$

$$\frac{1}{2} \cdot m\angle CBK = m\angle CBP = 55$$

Finally, find $m\angle P$

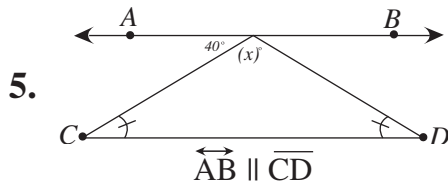
$$m\angle BDP + m\angle CBP + m\angle P = 180$$

$$100 + 55 + m\angle P = 180$$

$$m\angle P = 25$$

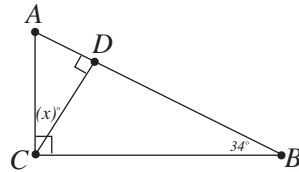
—Continued—

In problems 5-10, find the value of x in each of the accompanying figures, using the given information.



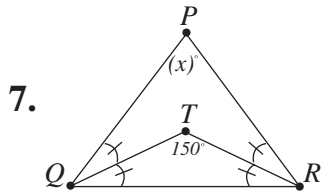
$x = \underline{100}$

$m\angle C = 40^\circ$
 (Lines \parallel , Alternate Interior \angle 's \cong)
 $m\angle D = 40^\circ$
 $m\angle C + m\angle D + x = 180$
 $40 + 40 + x = 180$
 $x = 100$



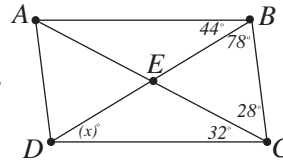
$x = \underline{34}$

$m\angle A + m\angle B + m\angle ACB = 180$
 $m\angle A + 34 + 90 = 180$
 $m\angle A = 56$
 $m\angle A + x + m\angle ADC = 180$
 $56 + x + 90 = 180$
 $x = 34$



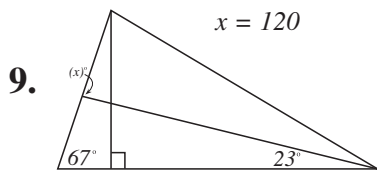
$x = \underline{120}$

$m\angle TQR + m\angle TRQ + m\angle T = 180$
 $m\angle TQR + m\angle TRQ + 150 = 180$
 $m\angle TQR + m\angle TRQ = 30$
 $m\angle TQR = m\angle TRQ$ (so each angle is 15)
 $m\angle TQR = m\angle TRQ = m\angle PQT = m\angle PRT$
 $m\angle TQR + m\angle TQP = m\angle PQR$
 $15 + 15 = 30$
 $m\angle TRQ + m\angle TRP = m\angle PRQ$
 $15 + 15 = 30$
 $m\angle PQR + m\angle PRQ + x = 180$
 $30 + 30 + x = 180$
 $x = 120$



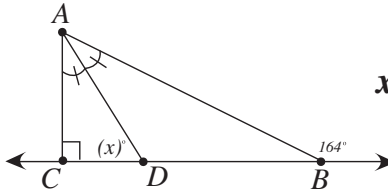
$x = \underline{42}$

$m\angle BCA + m\angle ACD = m\angle BCD$
 $28 + 32 = m\angle BCD$
 $60 = m\angle BCD$
 $m\angle DBC + m\angle BCD + x = 180$
 $78 + 60 + x = 180$
 $138 + x = 180$
 $x = 42$



$x = \underline{90}$

$x = 67 + 23$
 $x = 90$



$x = \underline{53}$

$164 = m\angle BAC + m\angle ACB$ $x + m\angle CAD + m\angle ACD = 180$
 $164 = m\angle BAC + 90$ $x + 37 + 90 = 180$
 $74 = m\angle BAC$ $x = 53$
 $m\angle BAC = m\angle CAD + m\angle DAB$
 $74 = m\angle CAD + m\angle DAB$
 $m\angle CAD = m\angle DAB$
 $m\angle CAD = 37$

—Continued—

In problems 6-9, determine whether each pair of ratios can be used to form a proportion.

6. $\frac{5}{8}$ and $\frac{15}{24}$? yes

$$\frac{15}{24} = \frac{\cancel{3} \cdot 5}{\cancel{3} \cdot 2 \cdot 2 \cdot 2} = \frac{5}{8}$$

$\frac{5}{8}$ and $\frac{15}{24}$ form the proportion $\frac{5}{8} = \frac{15}{24}$
since both equal $\frac{5}{8}$

7. $\frac{4}{9}$ and $\frac{12}{27}$? yes

$$\frac{12}{27} = \frac{\cancel{3} \cdot 2 \cdot 2}{\cancel{3} \cdot 3 \cdot 3} = \frac{4}{9}$$

$\frac{4}{9}$ and $\frac{12}{27}$ form the proportion $\frac{4}{9} = \frac{12}{27}$
since both equal $\frac{4}{9}$

8. $\frac{3x}{5}$ and $\frac{6x}{10}$? yes

$$\frac{6x}{10} = \frac{\cancel{2} \cdot 3 \cdot x}{\cancel{2} \cdot 5} = \frac{3x}{5}$$

$\frac{3x}{5}$ and $\frac{6x}{10}$ form the proportion $\frac{3x}{5} = \frac{6x}{10}$
since both equal $\frac{3x}{5}$

9. $\frac{xy}{3}$ and $\frac{14}{6xy}$? no

$$\frac{14}{6xy} = \frac{\cancel{2} \cdot 7}{\cancel{2} \cdot 3xy} = \frac{7}{3xy}$$

$\frac{xy}{3}$ and $\frac{7}{3xy}$ are not equivalent, so a proportion
cannot be formed between $\frac{xy}{3}$ and $\frac{14}{6xy}$

10. Name the means, extremes, and fourth proportional in $6 : x = 10 : y$.

Means: x and 10

Extremes: 6 and y

Fourth Proportional: y

11. Two complementary angles have a ratio of 3 to 2. What is the measure of each angle? 36°, 54°

$$3x + 2x = 90$$

$$5x = 90$$

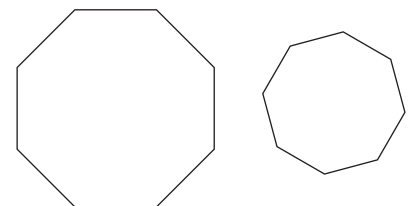
$$x = 18$$

$$2 \cdot x = 36$$

$$3 \cdot x = 54$$

12. Do the two given figures appear to be similar? yes

Explain why. They seem to be the same shape, even though not the same size.



—Continued—

4. Two corresponding sides of two similar pentagons measure 4 and 6. If the perimeter of the larger pentagon is 15, find the perimeter of the smaller pentagon. 10

$$\begin{aligned}\frac{4}{6} &= \frac{x}{15} \\ 6 \cdot x &= 4 \cdot 15 \\ x &= \frac{4 \cdot 15}{6} = \frac{\cancel{2} \cdot 2 \cdot \cancel{2} \cdot 5}{\cancel{2} \cdot \cancel{3}} \\ x &= 10\end{aligned}$$

5. The sides of a pentagon measure 7, 8, 10, 11, and 12 inches respectively. Find the perimeter of a similar pentagon if its longest side measures 14 inches. 56 inches

$$\text{Perimeter: } 7 + 8 + 10 + 11 + 12$$

Compare longest side to longest side.

$$\begin{aligned}\frac{12}{14} &= \frac{48}{x} \\ 14 \cdot 48 &= 12 \cdot x \\ 672 &= 12x \\ \frac{672}{12} &= \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 2 \cdot \cancel{2} \cdot 7}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2}} = \frac{8 \cdot 7}{1} = 56\end{aligned}$$

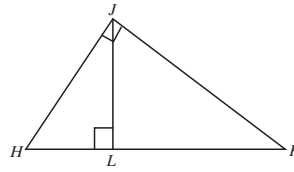
6. Two similar hexagons have perimeters of 80 and 120 meters respectively. If one side of the smaller hexagon is 15, find the measure of the corresponding side of the larger hexagon. $22\frac{1}{2}$ meters

$$\begin{aligned}\frac{80}{120} &= \frac{15}{x} \\ 120 \cdot 15 &= 80 \cdot x \\ \frac{120 \cdot 15}{80} &= x \\ \frac{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{2} \cdot 3 \cdot 5}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{2}} &= x \\ \frac{45}{2} \text{ or } 22\frac{1}{2} &= x\end{aligned}$$

17. Given: Right $\triangle HJK$ with right angle HJK .

$$\overline{JL} \perp \overline{HK}; \angle K = \angle HJL$$

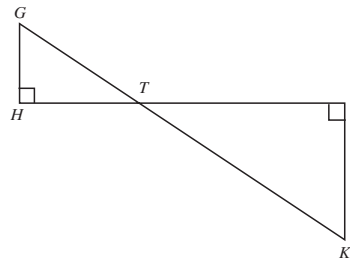
Prove: $\triangle JKL \sim \triangle HJL$



STATEMENT	REASON
1. Right $\triangle HJK$ with Right Angle HJK .	1. Given
2. $\overline{JL} \perp \overline{HK}$	2. Given
3. $\angle HJL$ is a right angle	3. Definition of Perpendicular Lines.
4. $\angle JKL$ is a right angle	4. Definition of Perpendicular Lines.
5. $\angle HJL \cong \angle JKL$	5. Theorem 11 – If you have right angles, then those right angles are congruent.
6. $\angle K \cong \angle HJL$	6. Given
8. $\triangle JKL \sim \triangle HJL$	8. Postulate Corollary 12a – If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar. (AA Postulate Corollary)

18. Given: $\overline{GH} \perp \overline{HJ}; \overline{JK} \perp \overline{HJ}$

Prove: $\frac{HT}{JT} = \frac{GT}{KT}$



STATEMENT	REASON
1. $\overline{GH} \perp \overline{HJ}; \overline{JK} \perp \overline{HJ}$	1. Given
2. $\overline{GH} \parallel \overline{JK}$	2. Theorem 22 – If two lines are perpendicular to a third line, then the two lines are parallel.
3. $\angle GHT \cong \angle KJT$	3. Theorem 16 – If two parallel lines are cut by a transversal, the alternate interior angles are congruent.
4. $\angle HTG \cong \angle JTK$	4. Theorem 15 – If two lines intersect, then the vertical angles formed are congruent.
5. $\triangle GTH \sim \triangle KJT$	5. Postulate Corollary 12a – If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the triangles are similar. (AA Postulate Corollary)
6. $\frac{HT}{JT} = \frac{GT}{KT}$	6. If two polygons are similar, then the measures of corresponding sides are proportional.

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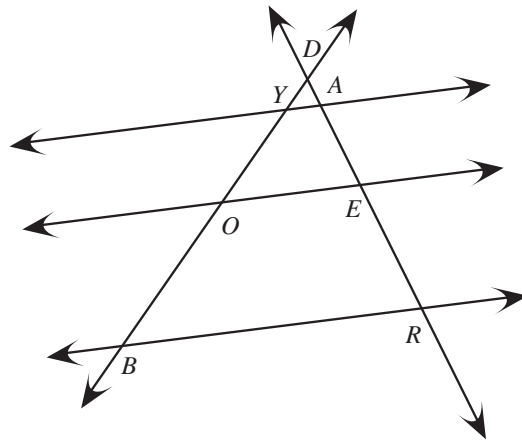
In the figure to the right, $\overline{AY} \parallel \overline{EO} \parallel \overline{RB}$. Use this figure to complete each of the proportions in problems 11 - 14.

11. $\frac{YO}{OB} = \frac{AE}{?}$? = $\frac{ER}{?}$

12. $\frac{YB}{OB} = \frac{?}{ER}$? = $\frac{AR}{?}$

13. $\frac{?}{AE} = \frac{YB}{YO}$? = $\frac{AR}{?}$

14. $\frac{DY}{YO} = \frac{DA}{?}$? = $\frac{AE}{?}$



Using the figure to the right, and the given information in problems 15 and 16 to determine if $\overline{QT} \parallel \overline{PS}$. Answer yes or no.

15. $PR = 30, PQ = 9, RT = 12, RS = 18$ Answer: no

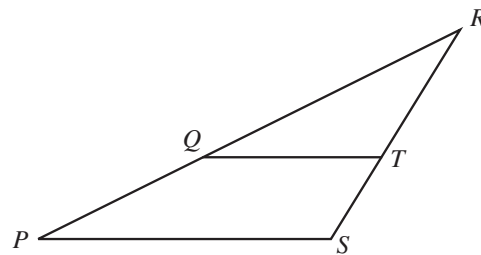
$$\frac{PR}{PQ} = \frac{SR}{ST} \quad \frac{3 \cdot 10}{3 \cdot 3} = \frac{3 \cdot 6}{1 \cdot 6}$$

$$\frac{30}{9} = \frac{18}{ST} \quad \frac{10}{3} \neq \frac{3}{1}$$

$RS - RT = ST$ NO!

$$18 - 12 = 6$$

$$\frac{30}{9} = \frac{18}{6}$$



16. $RP = 13.5, RQ = 6.3, TR = 4.2, SR = 9.0$ Answer: yes

$$\frac{RP}{RQ} = \frac{RS}{RT} \quad \frac{\cancel{3} \cdot 4.5}{\cancel{3} \cdot 2.1} = \frac{\cancel{3} \cdot 4.5}{\cancel{3} \cdot 2.1}$$

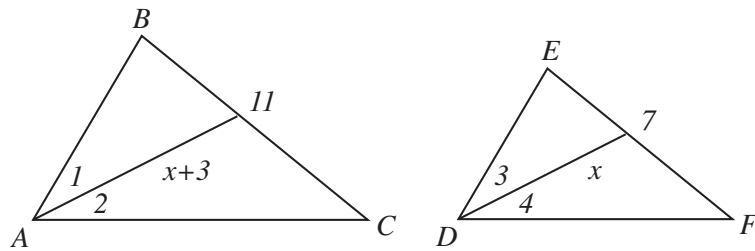
$$\frac{13.5}{6.3} = \frac{9.0}{4.2} \quad \frac{45}{21} = \frac{45}{21}$$

YES!

$$\frac{135}{63} = \frac{90}{42}$$

—Continued—

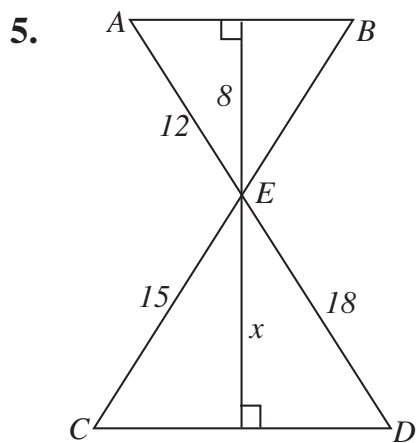
4. In the diagram below, $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. Find x .



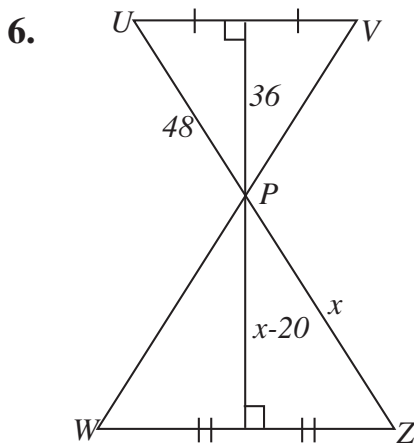
$$\begin{aligned} \frac{x+3}{x} &= \frac{11}{7} \\ x \cdot 11 &= (x+3) \cdot 7 \\ 11x &= 7x+21 \\ 4x &= 21 \\ x &= \frac{21}{4} \end{aligned}$$

$x = \underline{\underline{\frac{21}{4}}}$

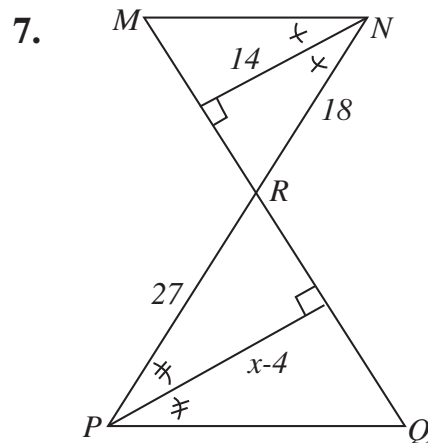
In problems 5, 6, and 7, study each diagram, apply the appropriate theorem or corollary, and find x .



$x = \underline{\underline{12}}$



$80x = \underline{\hspace{2cm}}$



$25x = \underline{\hspace{2cm}}$

$$\begin{aligned} \frac{8}{x} &= \frac{12}{18} \\ x \cdot 12 &= 8 \cdot 18 \\ x &= \frac{8 \cdot 18}{12} \\ x &= \frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2 \cdot 3 \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{3}} \\ x &= 12 \end{aligned}$$

(Theorem 29)

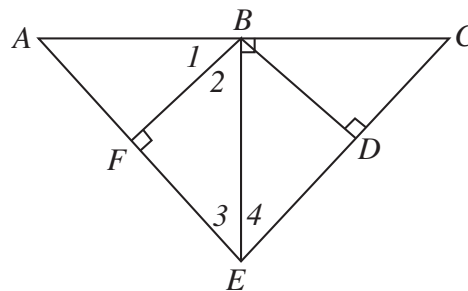
$$\begin{aligned} \frac{36}{x-20} &= \frac{48}{x} \\ (x-20) \cdot 48 &= 36x \\ 48x - 960 &= 36x \\ -960 &= -12x \\ 80 &= x \end{aligned}$$

(Corollary 29b)

$$\begin{aligned} \frac{14}{x-4} &= \frac{18}{27} \\ (x-4) \cdot 18 &= 14 \cdot 27 \\ 18x - 72 &= 378 \\ 18x &= 450 \\ x &= \frac{450}{18} \\ x &= \frac{\cancel{2} \cdot \cancel{3} \cdot \cancel{3} \cdot 5 \cdot 5}{\cancel{2} \cdot \cancel{3} \cdot \cancel{3}} \\ x &= 25 \end{aligned}$$

(Corollary 29a)

Use the diagram to the right, and the given information in problems 7 and 8, to find x .



7. $\triangle ABE \sim \triangle AEC$

$x = \frac{16}{3}$

$m\angle 3 = m\angle 4$

$m\angle 1 = m\angle 2$

$AB = 4, FB = 3, EB = 4, AE = x$

$$\begin{aligned} \frac{BF}{EB} &= \frac{AB}{AE} \\ \frac{3}{4} &= \frac{4}{AE} \\ (4)(4) &= 3 \cdot AE \\ 16 &= 3 \cdot AE \\ \frac{16}{3} &= AE \end{aligned}$$

8. $\triangle CEA \sim \triangle CBE$

$x = \frac{9}{5}$

B is the midpoint of AC

D is the midpoint of CE

$BD = 4, CE = 6, CB = 5, BE = x + 3$

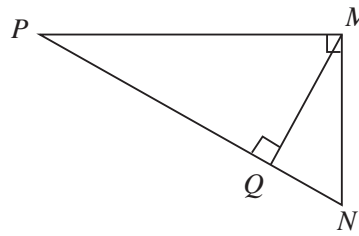
(Note: Use only these segments to solve)

$$\begin{aligned} \frac{DB}{BE} &= \frac{CB}{CE} \\ \frac{4}{x+3} &= \frac{5}{6} \\ (x+3) \cdot 5 &= 4 \cdot 6 \\ 5x+15 &= 24 \\ 5x &= 9 \\ x &= \frac{9}{5} \end{aligned}$$

Unit IV, Part D, Lesson 4, Quiz Form A
—Continued—

Name _____

For problems 3 – 11, use the right triangle shown to find the missing value.



3. $PQ = 9, QN = 4, MQ = \underline{\quad 6 \quad}$

$$\frac{9}{MQ} = \frac{MQ}{4}$$

$$(MQ)(MQ) = 9 \cdot 4$$

$$(MQ)^2 = 36$$

$$MQ = 6$$

4. $QN = 3, MQ = 9, PQ = \underline{\quad 27 \quad}$

$$\frac{x}{9} = \frac{9}{3}$$

$$(9)(9) = x \cdot 3$$

$$81 = x \cdot 3$$

$$27 = x$$

5. $PM = 12, PQ = 9, PN = \underline{\quad 16 \quad}$

$$\frac{9}{12} = \frac{12}{PN}$$

$$(12)(12) = 9 \cdot PN$$

$$144 = 9 \cdot PN$$

$$16 = PN$$

6. $MN = 8, QN = 6, PN = \underline{\quad \frac{32}{3} \quad}$

$$\frac{6}{8} = \frac{8}{PN}$$

$$8 \cdot 8 = 6 \cdot PN$$

$$64 = 6 \cdot PN$$

$$\frac{64}{6} = PN$$

$$\frac{32}{3} = PN$$

7. $PN = 75, PQ = 72, MN = \underline{\quad 15 \quad}$

$$QN = PN - PQ$$

$$QN = 3$$

$$\frac{3}{MN} = \frac{MN}{75}$$

$$(MN)(MN) = 3 \cdot 75$$

$$(MN)^2 = 225$$

$$MN = \sqrt{225}$$

$$MN = 15$$

8. $MQ = 4, PQ = 10, QN = \underline{\quad \frac{8}{5} \quad}$

$$\frac{10}{4} = \frac{4}{QN}$$

$$4 \cdot 4 = 10 \cdot QN$$

$$16 = 10 \cdot QN$$

$$\frac{16}{10} = QN$$

$$\frac{2 \cdot 2 \cdot 2 \cdot 2}{2 \cdot 5} = QN$$

$$\frac{8}{5} = QN$$

9. $PN = 13, PM = 12, PQ = \underline{\quad \frac{144}{13} \quad}$

$$\frac{13}{12} = \frac{12}{PQ}$$

$$12 \cdot 12 = 13 \cdot PQ$$

$$144 = 13 \cdot PQ$$

$$\frac{144}{13} = PQ$$

10. $PN = 16, QN = 4, QM = \underline{\quad 4\sqrt{3} \quad}$

$$\frac{PQ}{QM} = \frac{QM}{QN}$$

$$PN - QN = PQ$$

$$16 - 4 = 12$$

$$\frac{12}{QM} = \frac{QM}{4}$$

$$(QM)(QM) = 12 \cdot 4$$

$$QM^2 = 48$$

$$QM = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$$

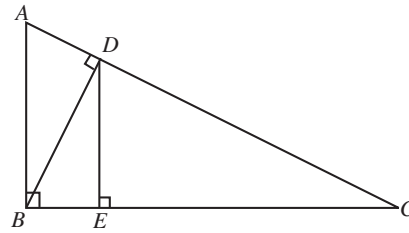
$$QM = \sqrt{2^2 \cdot 2^2 \cdot 3}$$

$$QM = \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{3}$$

$$QM = 2 \cdot 2 \cdot \sqrt{3}$$

$$QM = 4\sqrt{3}$$

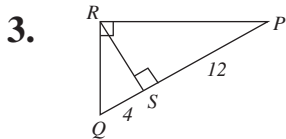
13. Given: $\triangle ABC$ is a right triangle
 $\overline{BD} \perp \overline{AC}$
 $\overline{DE} \perp \overline{BC}$
 Prove: $\triangle ADB \sim \triangle DEC$



STATEMENT	REASON
1. $\triangle ABC$ is a right triangle 2. $\overline{BD} \perp \overline{AC}$ 3. BD is an altitude 4. $\triangle ADB \sim \triangle BDC$	1. Given 2. Given 3. Definition of Altitude 4. Theorem 30 - If you have the altitude to the hypotenuse of a right triangle, then it forms two triangles that are similar to each other, and to the original triangle.
5. $\angle BDC$ is a right angle 6. $\triangle BDC$ is a right triangle 7. $\overline{DE} \perp \overline{BC}$ 8. $\triangle DEC \sim \triangle BDC$ 9. $\triangle ADB \sim \triangle DEC$	5. Definition of Perpendicular Lines 6. Definition of Right Triangles 7. Given 8. Theorem 30 9. Postulate Corollary 12c - If two triangles are similar to a third triangle, then the two triangles are similar to each other.

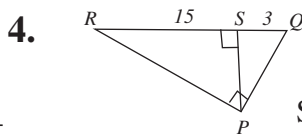
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In problems 3 - 5, use the diagram shown, and find the length of the altitude to the hypotenuse.



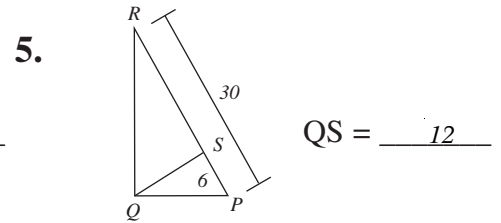
RS = $4\sqrt{3}$

From $\frac{QS}{RS} = \frac{RS}{SP} \quad \frac{4}{RS} = \frac{RS}{12}$
 $(RS)(RS) = 4 \cdot 12$
 $(RS)^2 = 48$
 $RS = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$
 $RS = \sqrt{2^2 \cdot 2^2 \cdot 3}$
 $RS = \sqrt{2^2} \cdot \sqrt{2^2} \cdot \sqrt{3}$
 $RS = 2 \cdot 2 \cdot \sqrt{3}$
 $RS = 4\sqrt{3}$



SP = $3\sqrt{5}$

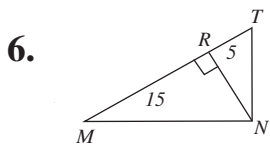
From $\frac{RS}{SP} = \frac{SP}{SQ} \quad \frac{15}{SP} = \frac{SP}{3}$
 $(SP)(SP) = 15 \cdot 3$
 $(SP)^2 = 45$
 $SP = \sqrt{45}$
 $SP = \sqrt{9 \cdot 5}$
 $SP = \sqrt{9} \cdot \sqrt{5}$
 $SP = 3\sqrt{5}$



QS = 12

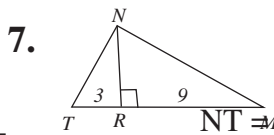
From $\frac{SP}{QS} = \frac{QS}{RS} \quad \frac{6}{QS} = \frac{QS}{24}$
 $(QS)(QS) = 6 \cdot 24$
 $(QS)^2 = 144$
 $QS = \sqrt{144}$
 $QS = 12$

In problems 6 - 8, use the diagram shown, and find the length of each leg of the right triangle.



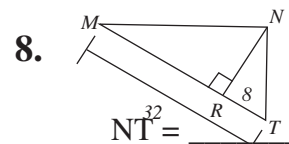
NT = 10
 NM = $10\sqrt{3}$

From $\frac{RT}{NT} = \frac{NT}{TM} \quad \frac{5}{NT} = \frac{NT}{20}$
 $(NT)(NT) = 5 \cdot 20$
 $(NT)^2 = 100$
 $NT = \sqrt{100}$
 $NT = 10$
 $\frac{15}{NM} = \frac{NM}{20}$
 $(NM)(NM) = 15 \cdot 20$
 $(NM)^2 = 300$
 $NM = \sqrt{300}$
 $NM = \sqrt{3} \cdot \sqrt{100}$
 $NM = 10\sqrt{3}$



NT = 6
 NM = $6\sqrt{3}$

From $\frac{RT}{NT} = \frac{NT}{MT} \quad \frac{3}{NT} = \frac{NT}{12}$
 $(NT)(NT) = 3 \cdot 12$
 $(NT)^2 = 36$
 $NT = \sqrt{36}$
 $NT = 6$
 $\frac{9}{NM} = \frac{NM}{12}$
 $(NM)(NM) = 12 \cdot 9$
 $(NM)^2 = 108$
 $NM = \sqrt{108}$
 $NM = \sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$
 $NM = \sqrt{2^2} \cdot \sqrt{3^2} \cdot \sqrt{3}$
 $NM = 2 \cdot 3 \cdot \sqrt{3}$
 $NM = 6\sqrt{3}$



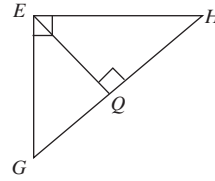
NT = 16
 NM = $16\sqrt{3}$

From $\frac{RT}{NT} = \frac{NT}{MT} \quad \frac{8}{NT} = \frac{NT}{32}$
 $(NT)(NT) = 8 \cdot 32$
 $(NT)^2 = 8 \cdot 32$
 $NT = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2}$
 $NT = \sqrt{2^8}$
 $NT = 2^4$
 $NT = 16$
 $\frac{24}{NM} = \frac{NM}{32}$
 $(NM)(NM) = 24 \cdot 32$
 $(NM)^2 = 24 \cdot 32$
 $NM = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$
 $NM = \sqrt{2^8 \cdot 3}$
 $NM = 2^4 \cdot \sqrt{3}$
 $NM = 16\sqrt{3}$

Unit IV, Part D, Lesson 4, Quiz Form B
—Continued—

Name _____

Use the figure at the right for problems 9 – 12.



9. $EG = 3, EH = 4, HG = 5. EQ = \underline{\frac{12}{5}}$

10. $EQ = 4, GH = 10. GQ = \underline{2 \text{ or } 8}$

$$\triangle EQG \sim \triangle HEG$$

$$\frac{EQ}{EH} = \frac{EG}{HG}$$

$$\frac{EQ}{4} = \frac{3}{5}$$

$$3 \cdot 4 = EQ \cdot 5$$

$$12 = 5 \cdot EQ$$

$$\frac{12}{5} = EQ$$

$$\frac{GQ}{EQ} = \frac{EQ}{HQ}$$

$$\frac{GQ}{4} = \frac{4}{10 - GQ}$$

$$\frac{x}{4} = \frac{4}{10 - x}$$

$$4 \cdot 4 = x(10 - x)$$

$$16 = 10x - x^2$$

$$x^2 - 10x + 16 = 0$$

$$(x - 2)(x - 8) = 0$$

$$x - 2 = 0 \quad \text{or} \quad x - 8 = 0$$

$$x = 2 \quad \text{or} \quad x = 8$$

11. $HG = 15, EG = 9, EQ = \frac{36}{5}. EH = \underline{12}$

12. $EQ = 8, \frac{HQ}{QG} = \frac{2}{1}. HQ = \underline{8\sqrt{2}}$

$$\triangle GQE \sim \triangle GEH$$

$$\frac{EQ}{EH} = \frac{EG}{GH}$$

$$\frac{36}{5} = \frac{9}{15}$$

$$\frac{36}{5} \cdot \frac{15}{1} = 9 \cdot EH$$

$$\frac{36 \cdot \cancel{5} \cdot 3}{\cancel{5}} = 9 \cdot EH$$

$$108 = 9 \cdot EH$$

$$\frac{108}{9} = EH$$

$$\frac{\cancel{9} \cdot 12}{\cancel{9}} = EH$$

$$12 = EH$$

$$\frac{HQ}{EQ} = \frac{EQ}{QG}$$

$$\frac{2x}{8} = \frac{8}{x}$$

$$8 \cdot 8 = 2x \cdot x$$

$$64 = 2x^2$$

$$32 = x^2$$

$$\sqrt{32} = x$$

$$\sqrt{16} \cdot 2 = x$$

$$\sqrt{16} \cdot \sqrt{2} = x$$

$$8\sqrt{2} = x$$

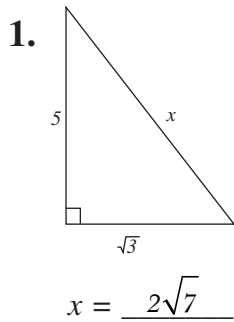
Unit IV - Triangles

Part D - Similarity – Part 2 (Triangles and Their Parts)

Lesson 5 - Theorem 31: "If you have a given right triangle, then the square of the measure of the hypotenuse, is equal to the sum of the squares of the measures of the two legs." (The Pythagorean Theorem)

Lesson 6 - Applying Pythagoras to 3-Dimensional Figures

For problems 1 – 5, find x in the given figures. Express radicals in simplest form.



$$a^2 + b^2 = c^2$$

$$(\sqrt{3})^2 + 5^2 = x^2$$

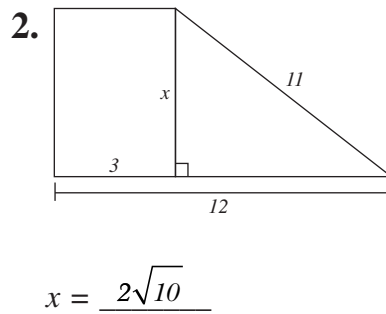
$$3 + 25 = x^2$$

$$28 = x^2$$

$$\sqrt{28} = x$$

$$\sqrt{4 \cdot 7} = x$$

$$2\sqrt{7} = x$$



$$a^2 + b^2 = c^2$$

$$9^2 + x^2 = 11^2$$

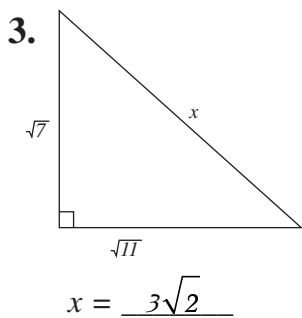
$$81 + x^2 = 121$$

$$x^2 = 40$$

$$x = \sqrt{40}$$

$$x = \sqrt{4 \cdot 10}$$

$$x = 2\sqrt{10}$$



$$a^2 + b^2 = c^2$$

$$(\sqrt{7})^2 + (\sqrt{11})^2 = x^2$$

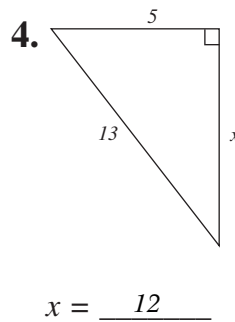
$$7 + 11 = x^2$$

$$18 = x^2$$

$$\sqrt{18} = x$$

$$\sqrt{9 \cdot 2} = x$$

$$3\sqrt{2} = x$$



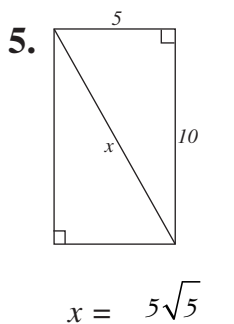
$$a^2 + b^2 = c^2$$

$$5^2 + x^2 = 13^2$$

$$25 + x^2 = 169$$

$$x^2 = 144$$

$$x = 12$$



$$a^2 + b^2 = c^2$$

$$5^2 + 10^2 = x^2$$

$$25 + 100 = x^2$$

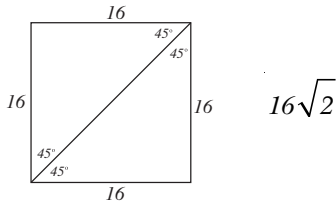
$$125 = x^2$$

$$\sqrt{25 \cdot 5} = x$$

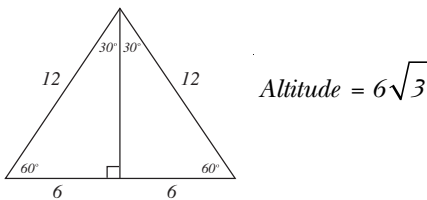
$$5\sqrt{5} = x$$

—Continued—

17. If the perimeter of square is 64 inches, how long is its diagonal? $16\sqrt{2}$

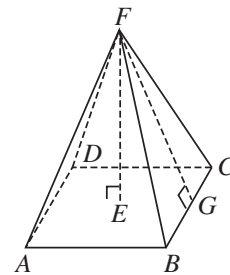


18. Find the length of the altitude of an equilateral triangle if a side of the triangle is 12 cm? $Altitude = 6\sqrt{3}$



19. The base of this right pyramid is a square, 6 inches on a side. Point G is the mid-point of \overline{BC} . The altitude of the pyramid, FE, is 8 inches. Find FC and the slant height FG.

FG = $\sqrt{73}$ FC = $\sqrt{82}$



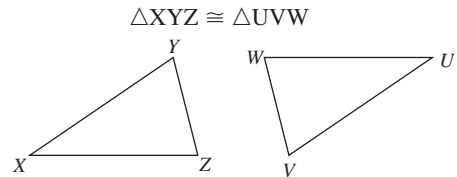
$$\begin{aligned} (FE)^2 + (EG)^2 &= (FG)^2 & (FG)^2 + (GC)^2 &= (FC)^2 \\ EG &= \frac{1}{2} DC & (\sqrt{73})^2 + 3^2 &= (FC)^2 \\ EG &= \frac{1}{2} \cdot 6 = 3 & 73 + 9 &= (FC)^2 \\ 8^2 + 3^2 &= (FG)^2 & 82 &= (FC)^2 \\ 73 &= FG^2 & \sqrt{82} &= FC \\ \sqrt{73} &= FG \end{aligned}$$

Unit IV - Triangles

Part E - Congruence - Part 1 (General Geometric Relationship)

Lesson 1 - Definition

Use the pair of triangles to the right, and the given information, for problems 1 – 8.



1. Name the three pairs of corresponding angles. $\angle X$ corresponds to $\angle U$; $\angle Y$ corresponds to $\angle V$; $\angle Z$ corresponds to $\angle W$. Note: $\angle YXZ$ corresponds to $\angle VUW$ is also a correct response. However “ $\angle YXZ$ corresponds to $\angle WUV$ ” is an incorrect response since corresponding vertices are not in the correct order.

2. Name the three pairs of corresponding sides. \overline{XY} corresponds to \overline{UV} ; \overline{YZ} corresponds to \overline{VW} ; \overline{ZX} corresponds to \overline{WU} . Note: \overline{XY} corresponds to \overline{VU} is an incorrect response, technically speaking.

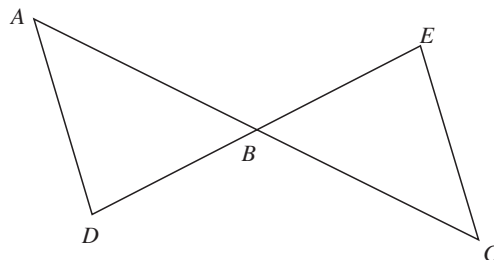
Determine whether each statement is correct or incorrect.

3. $\triangle YZX \cong \triangle VWU$ correct (corresponding parts are congruent)
4. $\triangle XZY \cong \triangle UVW$ incorrect (this could be correct, but we don't know the specific measures.)
5. $\overline{XY} \cong \overline{VU}$ incorrect (because the names do not reflect the corresponding parts, this is technically incorrect. However, the two segments are congruent, so the answer could also be “correct”)
6. $\angle Y \cong \angle V$ correct (these are corresponding angles)
7. Which statement is correct? b
- a) $\triangle YZX \cong \triangle UVW$
 b) $\triangle YZX \cong \triangle VWU$
 c) $\triangle ZXY \cong \triangle WVU$
8. Which statement is not correct? c
- a) $\overline{XZ} \cong \overline{UW}$
 b) $\angle Y \cong \angle V$
 c) $\overline{XZ} \cong \overline{UV}$

Unit IV, Part E, Lesson 1, Quiz Form A
—Continued—

Name _____

Use the pair of congruent triangles to the right, for problems 9 – 14.



9. $\overline{AD} \cong$ _____ \overline{CE}

10. $\overline{BE} \cong$ _____ \overline{BD}

11. $\angle ABD \cong$ _____ $\angle CBE$

12. $\overline{BC} \cong$ _____ \overline{BA}

13. $\triangle BDA \cong$ _____ $\triangle BEC$

14. $\triangle CEB \cong$ _____ $\triangle ADB$

Given that $\triangle MOP \cong \triangle HAT$, complete each statement in problems 15 – 20.

15. $\angle M \cong$ _____ $\angle H$

16. $\overline{OP} \cong$ _____ \overline{AT}

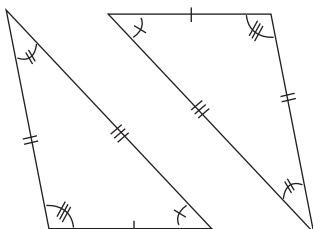
17. $m\angle P =$ _____ $m\angle T$

18. $\overline{MP} \cong$ _____ \overline{HT}

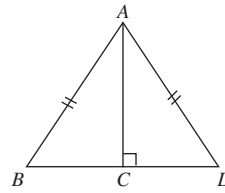
19. $\angle A \cong$ _____ $\angle O$

20. $\overline{HA} \cong$ _____ \overline{MO}

21. Sketch and label a pair of obtuse triangles to illustrate the definition of congruent triangles. Mark all six relationships on the triangles. (answers will vary)



22. Given: $\overline{AB} \cong \overline{AD}$
 $\overline{AC} \perp \overline{BD}$
 \overline{AC} bisects \overline{BD}
 \overline{AC} bisects $\angle A$



Prove: $\triangle ABC \cong \triangle ADC$ using the definition of congruent triangles.

STATEMENT	REASON
1. $\overline{AB} \cong \overline{AD}$	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflexive Property for Congruence
3. \overline{AC} bisects \overline{BD}	3. Given
4. $\overline{BC} \cong \overline{DC}$	4. Definition of Segment Bisector
5. $\overline{AC} \perp \overline{BD}$	5. Given
6. $\angle ACB$ is a right angle	6. Definition of Perpendicular
7. $\angle ACD$ is a right angle	7. Definition of Perpendicular
8. $\angle ACB \cong \angle ACD$	8. Theorem 11 - If you have right angles, then those right angles are congruent.
9. \overline{AC} bisects $\angle A$	9. Given
10. $\angle BAC \cong \angle DAC$	10. Definition of Angle Bisector
11. $\angle ABC \cong \angle ADC$	11. Corollary 25a - If two angles of one triangle are congruent to two angles of another triangle, then the third pair of triangles are congruent.
12. $\triangle ABC \cong \triangle ADC$	12. Two triangles are congruent if and only if there is a correspondence between the vertices such that each pair of corresponding sides and each pair of corresponding angles are congruent.

Unit IV - Triangles

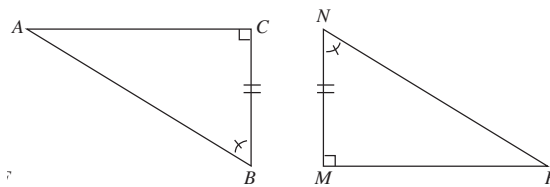
Part E - Congruence - Part 1 (General Geometric Relationship)

Lesson 2 - Postulate 13: Triangle Congruence

Lesson 3 - Congruence Postulate Corollaries

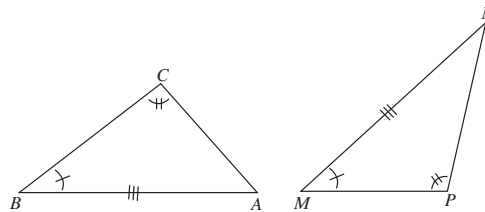
For problems 1 – 4, state the given congruence relationship in your own words and label the diagram appropriately.

1. Postulate Corollary 13c – LA Postulate Corollary: If one leg and acute angle of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the triangles are congruent.



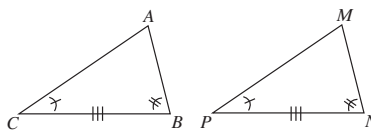
Note: Labeling may vary, but points must correspond

2. Postulate Corollary 13a – AAS Postulate Corollary: If two angles and a non-included side of one triangle are congruent to the corresponding angles and non-included side of another triangle, then the triangles are congruent.



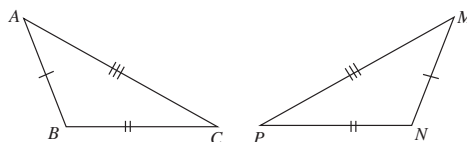
Note: Labeling may vary, but points must correspond

3. Postulate 13 – ASA Congruence Assumption: If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the two triangles are congruent.



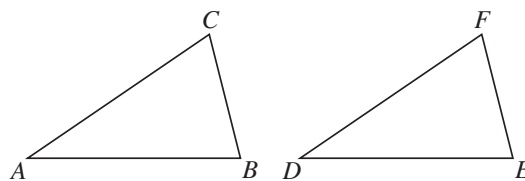
Note: Labeling may vary, but points must correspond

4. Postulate 13 – SSS Congruence Assumption: If the three sides of a triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent.



Note: Labeling may vary, but points must correspond

Referring to the triangles to the right, for problems 5 – 10, name the postulate or postulate corollary which would show $\triangle ABC \cong \triangle DEF$.



5. $\angle C \cong \angle F, \overline{AC} \cong \overline{DF}, \overline{BC} \cong \overline{EF}$

Postulate 13 – SAS Congruence Assumption

6. $\angle C \cong \angle F, \angle D \cong \angle A, \overline{AB} \cong \overline{DE}$

Postulate Corollary 13a – AAS Postulate Corollary

7. $\angle C$ and $\angle F$ are right angles, $\overline{AB} \cong \overline{DE}, \overline{BC} \cong \overline{EF}$

Postulate Corollary 13e – HL Postulate Corollary

8. $\angle C$ and $\angle F$ are right angles, $\angle E \cong \angle B, \overline{DF} \cong \overline{AC}$

Postulate Corollary 13c – LA Postulate Corollary

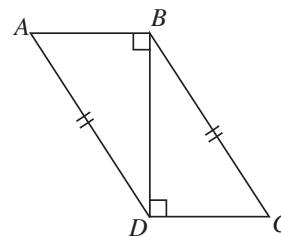
9. $\angle A \cong \angle D, \overline{AB} \cong \overline{DE}, \overline{AC} \cong \overline{DF}$

Postulate 13 – SAS Congruence Assumption

10. $\angle B$ and $\angle E$ are right angles, $\overline{CB} \cong \overline{FE}, \overline{AB} \cong \overline{DE}$

Postulate Corollary 13d – LL Postulate Corollary

11. Given: $\overline{BD} \perp \overline{AB}; \overline{BD} \perp \overline{DC}$
 $\overline{AD} \cong \overline{CB}$



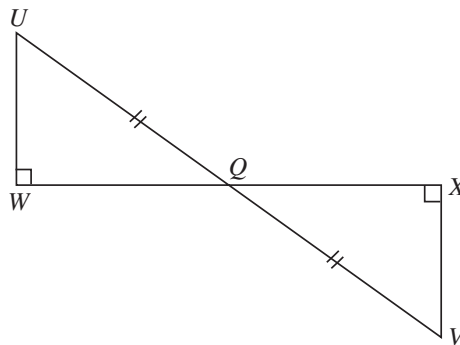
Prove: $\triangle ABD \cong \triangle CDB$

STATEMENT	REASON
1. $\overline{BD} \perp \overline{AB}$	1. Given
2. $\overline{BD} \perp \overline{DC}$	2. Given
3. $\angle ABD$ is a right angle	3. Definition of Perpendicular Line Segments
4. $\angle CDB$ is a right angle	4. Definition of Perpendicular Line Segments
5. $\triangle ABD$ is a right triangle	5. Definition of Right Triangle
6. $\triangle CDB$ is a right triangle	6. Definition of Right Triangle
7. $\overline{BD} \cong \overline{DB}$	7. Reflexive Property for Congruent Segments
8. $\overline{AD} \cong \overline{CB}$	8. Given
9. $\triangle ABD \cong \triangle CDB$	9. Postulate Corollary 13e – If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

Repeat problem 13 using a different postulate assumption or postulate corollary.

14. Given: $\overline{XW} \perp \overline{UW}$; $\overline{XW} \perp \overline{XV}$
 $\overline{UQ} \cong \overline{VQ}$

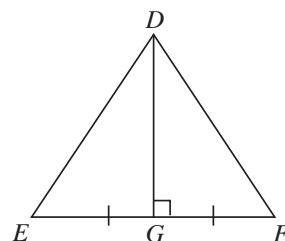
Prove: $\triangle UWQ \cong \triangle VXQ$



STATEMENT	REASON
1. $\overline{XW} \perp \overline{UW}$	1. Given
2. $\overline{XW} \perp \overline{XV}$	2. Given
3. $\overline{UW} \parallel \overline{XV}$	3. Theorem 22 - If two lines are perpendicular to a third line, then the two lines are parallel
4. $\angle WUQ \cong \angle XVQ$	4. Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
5. $\overline{UQ} \cong \overline{VQ}$	5. Given
6. $\angle WQU \cong \angle XQV$	8. Theorem 15 - If two lines intersect, then the vertical angles formed are congruent.
7. $\triangle UWQ \cong \triangle VXQ$	9. Postulate 13 - If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the two triangles are congruent. (ASA Congruence Assumption)

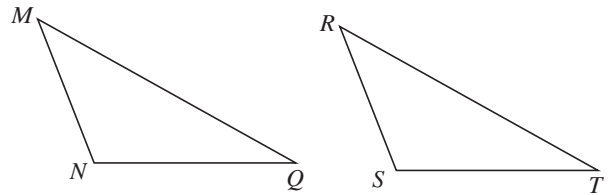
15. Given: \overline{DG} is the perpendicular bisector of \overline{EF}

Prove: $\triangle DEG \cong \triangle DFG$



STATEMENT	REASON
1. $\overline{DG} \perp \overline{EF}$	1. Given
2. $\angle DGE$ is a right angle	2. Definition of Perpendicular
3. $\angle DGF$ is a right angle	3. Definition of Perpendicular
4. $\triangle DFE$ is a right triangle	4. Definition of Right Triangle
5. $\triangle DFE$ is a right triangle	5. Definition of Right Triangle
6. \overline{DG} bisects \overline{EF}	6. Given
7. $\overline{EG} \cong \overline{FG}$	7. Definition of Bisector of a Line Segment.
8. $\overline{DG} \cong \overline{DG}$	8. Reflexive Property of Congruence
9. $\triangle DEG \cong \triangle DFG$	9. Postulate Corollary 13d - If two legs of one right triangle are congruent to the corresponding legs of another right triangle, then the two right triangles are congruent.

Referring to the two triangles to the right, for problems 5 – 7, name the postulate or postulate corollary which would show $\triangle MNQ \cong \triangle RST$.



5. $\angle M \cong \angle R$, $\overline{NQ} \cong \overline{ST}$, $\angle Q \cong \angle T$

Postulate Corollary 13a – AAS Postulate Corollary

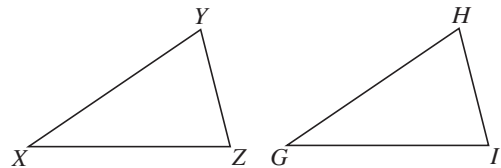
6. $\overline{MN} \cong \overline{RS}$, $\overline{MQ} \cong \overline{RT}$, $\overline{NQ} \cong \overline{ST}$

Postulate 13 – SSS Congruence Assumption

7. $\angle N \cong \angle S$, $\angle Q \cong \angle T$, $\overline{NQ} \cong \overline{ST}$

Postulate 13 – ASA Congruence Assumption

Referring to the triangles to the right for, problems 8 – 13, fill in the blanks with the required angle or side so that the indicated congruence assumption or postulate corollary justifies $\triangle XYZ \cong \triangle GHI$.



8. SAS Congruence Assumption; $\angle Y \cong \angle H$; $\overline{XY} \cong \overline{GH}$; $\overline{YZ} \cong \overline{HI}$.

9. LA Postulate Corollary; $\angle Y \cong \angle H$; $m\angle Z = m\angle I = 90$; $\overline{YZ} \cong \overline{HI}$ or $\overline{ZX} \cong \overline{IG}$.

10. ASA Congruence Assumption; $\angle X \cong \angle G$; $\overline{XY} \cong \overline{GH}$; $\angle Y \cong \angle H$.

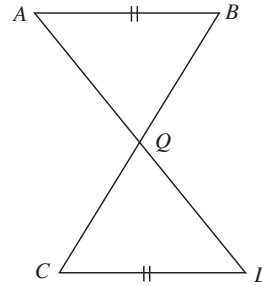
11. HL Postulate Corollary; $m\angle Y = m\angle H = 90$; $\overline{YZ} \cong \overline{HI}$; $\overline{XZ} \cong \overline{GI}$.

12. AAS Postulate Corollary; $\angle X \cong \angle G$; $\angle Z \cong \angle I$; $\overline{YZ} \cong \overline{HI}$ or $\overline{XY} \cong \overline{GH}$.

13. HA Postulate Corollary; $m\angle Y = m\angle H = 90$; $\angle X \cong \angle G$; $\overline{XZ} \cong \overline{GI}$.

14. Given: $\overline{AB} \cong \overline{DC}$; $\overline{AB} \parallel \overline{DC}$

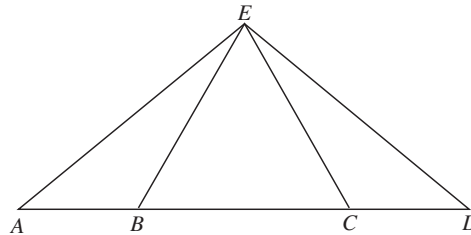
Prove: $\triangle ABQ \cong \triangle DCQ$



STATEMENT	REASON
1. $\overline{AB} \cong \overline{DC}$	1. Given
2. $\overline{AB} \parallel \overline{DC}$	2. Given
3. $\angle B \cong \angle C$	3. Theorem 16 – If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
4. $\angle A \cong \angle D$	4. Theorem 16 – If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
5. $\triangle ABQ \cong \triangle DCQ$	5. Postulate 13 – If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the two triangles are congruent. (ASA Congruence Assumption)

15. Given: $\triangle ADE$ is an Isosceles Triangle
 $\overline{EC} \perp \overline{AE}$; $\overline{EB} \perp \overline{DE}$; $\overline{AB} \cong \overline{DC}$

Prove: $\triangle AEC \cong \triangle DEB$



STATEMENT	REASON
1. $\triangle ADE$ is an isosceles triangle	1. Given
2. $\overline{AE} \cong \overline{DE}$	2. Definition of Isosceles Triangle
3. $\overline{EC} \perp \overline{AE}$	3. Given
4. $\overline{EB} \perp \overline{DE}$	4. Given
5. $\angle AEC$ is a right angle	5. Definition of Perpendicular Line Segments
6. $\angle DEB$ is a right angle	6. Definition of Perpendicular Line Segments
7. $\triangle AEC$ is a right triangle	7. Definition of Right Triangle
8. $\triangle DEB$ is a right triangle	8. Definition of Right Triangle
9. $\overline{AB} \cong \overline{DC}$	9. Given
10. $\overline{BC} \cong \overline{CB}$	10. Reflexive Property of Segment Congruence
11. $\overline{AC} \cong \overline{DB}$	11. Postulate 6 – Ruler – Fourth Assumption – Segment Addition
12. $\triangle AEC \cong \triangle DEB$	12. Postulate Corollary 13d – If the hypotenuse and one leg of a right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the two right triangles are congruent. (HL Postulate Corollary)

Unit IV - Triangles

Part F - Congruence – Part 2 (Applications)

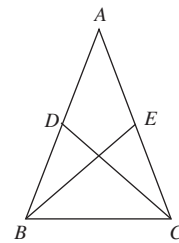
Lesson 1 - Overlapping Triangles

Lesson 2 - Using the Definition of Congruence

Lesson 3 - Theorem 32 - "If two given triangles are both congruent to a third triangle, then the two given triangles are congruent to each other."

1. Name two pairs of overlapping triangles that appear to be congruent in the figure at the right.

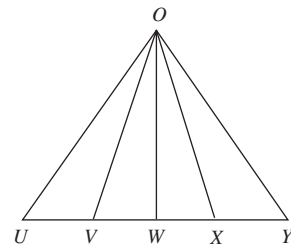
_____ $\triangle DBC$ and $\triangle ECB$; $\triangle ABE$ and $\triangle ACD$



Use the figure to the right for problems 2 and 3.

2. Name two different triangles that overlap $\triangle VOY$ and contain \overline{OU} . _____ $\triangle UOW$, $\triangle UOX$

3. Name two different triangles that overlap $\triangle UOX$ and contain \overline{OY} . _____ $\triangle VOY$, $\triangle WOY$

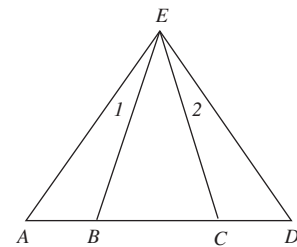


Use the figure to the right for problem 4.

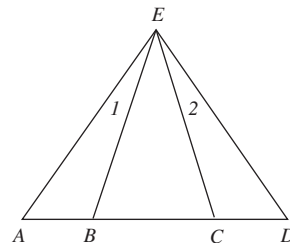
4. $AB = CD$, $AC = 3x+5$, $BD = 4x - 3$, $BC = x + 2$. Find AB and CD . _____ $AB = 19$, $CD = 19$

$AC = DB$	AC	BC	BD
$3x + 5 = 4x - 3$	$3x + 5$	$x + 2$	$4x - 3$
$8 = x$	$3 \cdot 8 + 5$	$8 + 2$	$4 \cdot 8 - 3$
	$24 + 5$	10	$32 - 3$
$AC - BC = AB$	29		29
$29 - 10 = AB$			
$19 = AB$			

$DB - BC = CD$
$29 - 10 = CD$
$19 = CD$



Use the figure to the right for problem 5.

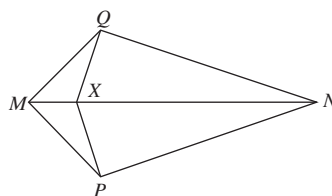


5. $\angle 1 \cong \angle 2$, $m\angle AEC = 14x - 3$, $m\angle DEB = 17x - 15$, $AE = 4x + 7$,
 $DE = 3x + 11$, $BE = 3x + 7$, $CE = 5x - 1$. Why is $\triangle AEC \cong \triangle DEB$?

$\triangle AEC \cong \triangle DEB$ by S.A.S. Assumption. Since $AE = DE$, $CE = BE$, and $\angle AEC \cong \angle DEB$, using the properties of angle addition.

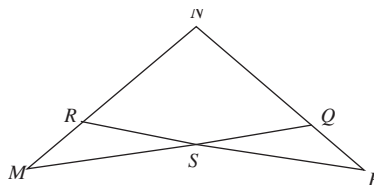
$m\angle AEC = m\angle DEB$	AE	DE	BE	CE
$14x - 3 = 17x - 15$	$4x + 7$	$3x + 11$	$3x + 7$	$5x - 1$
$12 = 3x$	$4 \cdot 4 + 7$	$3 \cdot 4 + 11$	$3 \cdot 4 + 7$	$5 \cdot 4 - 1$
$4 = x$	$16 + 7$	$12 + 11$	$12 + 7$	$20 - 1$
	23	23	19	19

6. Given: $\overline{MQ} \cong \overline{MP}$; $\angle QMX \cong \angle PMX$
Prove: $\angle NQX \cong \angle NPX$

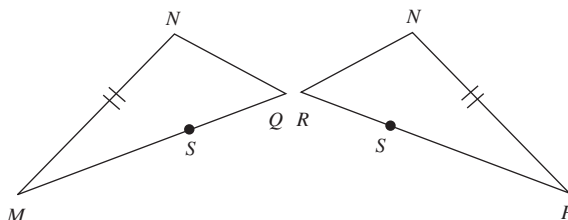


STATEMENT	REASON
1. $\overline{MQ} \cong \overline{MP}$	1. Given
2. $\angle QMX \cong \angle PMX$	2. Given
3. $\overline{MN} \cong \overline{MN}$	3. Reflexive Property for Congruence
4. $\triangle QMN \cong \triangle PMN$	4. Postulate 13 - SAS Congruence Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the two triangles are congruent.
5. $\overline{QN} \cong \overline{PN}$	5. C.P.C.T.C.
6. $\angle QNM \cong \angle PNM$	6. C.P.C.T.C.
7. $\overline{NX} \cong \overline{NX}$	7. Reflexive Property for Congruence
8. $\triangle QNX \cong \triangle PNX$	8. Postulate 13 - SAS Congruence Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the two triangles are congruent.
9. $\angle NQX \cong \angle NPX$	9. C.P.C.T.C.

8. Given: $\angle MRS \cong \angle PQS$; $\overline{NM} \cong \overline{NP}$
 Prove: $\angle NRP \cong \angle NQM$



STATEMENT	REASON
1. $\angle MRS$ and $\angle NRS$ are supplementary	1. Theorem 10 - If the exterior sides of two adjacent angles are opposite rays, then the two angles are supplementary.
2. $\angle PQS$ and $\angle NQS$ are supplementary	2. Theorem 10 - If the exterior sides of two adjacent angles are opposite rays, then the two angles are supplementary.
3. $\angle MRS \cong \angle PQS$	3. Given
4. $\angle NRS \cong \angle NQS$	4. Theorem 14 - If two angles are supplementary to the same angle or congruent angles, then they are congruent to each other.
5. $\angle N \cong \angle N$	5. Reflexive Property for Congruence
6. $\overline{NM} \cong \overline{NP}$	6. Given
7. $\triangle NQM \cong \triangle NRP$	7. Postulate Corollary 13a - AAS - If two angles and a non-included side of one triangle are congruent to the corresponding two angles and non-included side of another triangle, then the two triangles are congruent.
8. $\angle NRP \cong \angle NQM$	8. C.P.C.T.C



Unit IV - Triangles

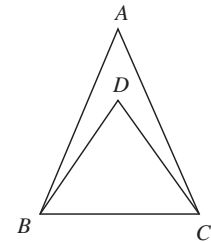
Part F - Congruence – Part 2 (Applications)

Lesson 4 - Theorem 33 - "If two sides of a triangle are congruent, then the angles opposite them are congruent."

Lesson 5 - Theorem 34 - "If two angles of a triangle are congruent, then the sides opposite them are congruent."

Complete each of the statements in problems 1-3, referring to the figure to the right. Then state, in your own words, the theorem you are using to justify your answer.

1. If $\angle CBD \cong \angle BCD$, then $\overline{BD} \cong \overline{CD}$.
 Theorem 34 - "If two angles of a triangle are congruent, then the sides opposite them are congruent."

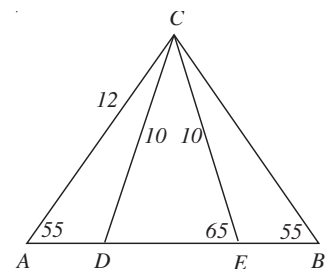


2. If $m\angle CBD + m\angle DBA = m\angle BCD + m\angle DCA$,
 then $\overline{AB} \cong \overline{AC}$.
 Theorem 34 - "If two angles of a triangle are congruent, then the sides opposite them are congruent."

3. If $\overline{AB} \cong \overline{AC}$, then $\angle ABC \cong \angle ACB$.
 Theorem 33 - "If two sides of a triangle are congruent, then the angles opposite them are congruent."

Use the diagram to the right, and Theorem 33 or Theorem 34, to complete each statement in problems 4 and 5.

4. $m\angle CDB = 65^\circ$.
 5. $CB = 12$.



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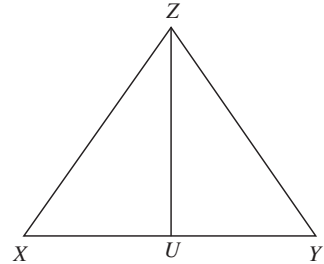
In the figure at the right, $\triangle XYZ$ is isosceles, with $\overline{XZ} \cong \overline{YZ}$. Also, $ZY = 8$, $m\angle Y = 40^\circ$, and \overline{ZU} bisects $\angle XZY$. Use this information for problems 6-9.

6. $ZX = \underline{\quad 8 \quad}$.

7. $m\angle X = \underline{\quad 40^\circ \quad}$.

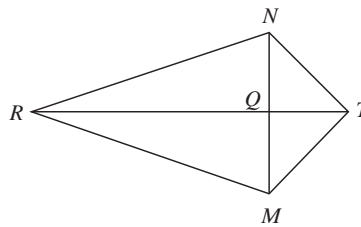
8. $\triangle XZU \cong \triangle YZU$. Why? (Answers will vary) Postulate Corollary 13 - ASA Assumption (or SAS, AAS)

9. $\overline{XU} \cong \overline{YU}$. Why? C.P.C.T.C.



10. Given: $\overline{RM} \cong \overline{RN}$
 $\overline{NQ} \cong \overline{MQ}$

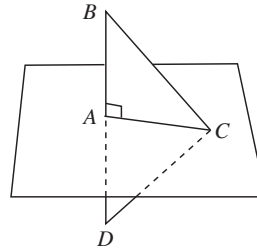
Prove: $\angle TNQ \cong \angle TMQ$



STATEMENT	REASON
1. $\overline{RM} \cong \overline{RN}$	1. Given
2. $\overline{NQ} \cong \overline{MQ}$	2. Given
3. $\overline{RQ} \cong \overline{RQ}$	3. Reflexive Property for Congruence
4. $\triangle RNQ \cong \triangle RMQ$	4. Postulate 13 - SSS Congruence Assumption - If three sides of a triangle are congruent to the corresponding sides of another triangle, then the two triangles are congruent.
5. $\angle NRT \cong \angle MRT$	5. C.P.C.T.C.
6. $\overline{RT} \cong \overline{RT}$	6. Reflexive Property for Congruence
7. $\triangle NRT \cong \triangle MRT$	7. Postulate 13 - SAS Congruence Assumption - If two sides and the included angle of one triangle are congruent to the corresponding sides and included angle of another triangle, then the two triangles are congruent.
8. $\overline{NT} \cong \overline{MT}$	8. C.P.C.T.C.
9. $\angle TNQ \cong \angle TMQ$	9. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.

—Continued—

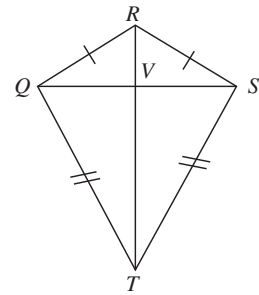
In the figure at the right, \overline{AC} is contained in plane M , and \overline{BD} intersects plane M at point A . Also, \overline{AC} is the perpendicular bisector of \overline{BD} .



- 12. Prove: $\triangle BAC \cong \triangle DAC$
- 13. Prove: $\angle B \cong \angle D$
- 14. Prove: $\triangle BDC$ is isosceles

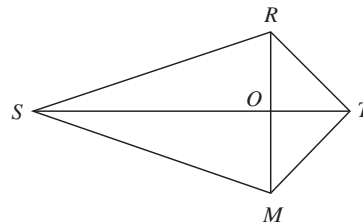
STATEMENT	REASON
1. \overline{AC} is contained in plane M	1. Given
2. \overline{BD} intersects plane M at point A	2. Given
3. \overline{AC} is the perpendicular bisector of \overline{BD}	3. Given
4. $\angle BAC$ is a right angle	4. Definition of Perpendicular
5. $\angle DAC$ is a right angle	5. Definition of Perpendicular
6. $\triangle BAC$ is a right triangle	6. Definition of a Right Triangle
7. $\triangle DAC$ is a right triangle	7. Definition of a Right Triangle
8. $\overline{BA} \cong \overline{DA}$	8. Definition of Segment Bisector
9. $\overline{AC} \cong \overline{AC}$	9. Reflexive Property of Congruence
10. $\triangle BAC \cong \triangle DAC$	10. Postulate Corollary 13d - If the two legs of a right triangle are congruent to the two legs of another right triangle, then the two right triangles are congruent.
11. $\angle B \cong \angle D$	11. C.P.C.T.C
12. $\overline{BC} \cong \overline{DC}$	12. Theorem 34 - If two angles of a triangle are congruent, then the sides opposite them are congruent. (also C.P.C.T.C.)
13. $\triangle BDC$ is isosceles	13. A triangle is an isosceles triangle, if and only if, it has at least two congruent sides.

—Continued—



In the diagram at the right, $\overline{RQ} \cong \overline{RS}$ and $\overline{TQ} \cong \overline{TS}$. Use this information for problems 8 – 11.

8. If $m\angle RQS = 6x+10$, and $m\angle RSQ = 9x - 17$, then $m\angle QRS = \underline{\hspace{2cm} 52^\circ \hspace{2cm}}$.
9. If $\overline{QS} \cong \overline{RQ}$, then $m\angle RSQ = \underline{\hspace{2cm} m\angle SRQ \hspace{2cm}}$. (Note: You might also conclude $m\angle RSQ = 60^\circ$ since all three sides of $\triangle RQS$ are congruent.)
10. If $\angle QVT \cong \angle SVT$, then $\triangle QVT \cong \triangle SVT$. Why? Postulate Corollary 13c.
11. $\angle TQS \cong \angle TSQ$. Why? Theorem 33.



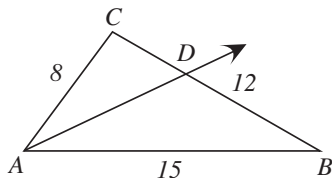
12. Given: \overline{ST} bisects $\angle RTM$
 \overline{ST} bisects $\angle RSM$
 Prove: $\angle TRM \cong \angle TMR$

STATEMENT	REASON
1. \overline{ST} bisects $\angle RTM$	1. Given
2. \overline{ST} bisects $\angle RSM$	2. Given
3. $\angle RST \cong \angle MST$	3. Definition of Angle Bisector
4. $\overline{TS} \cong \overline{TS}$	4. Reflexive Property for Congruence
5. $\triangle RTS \cong \triangle MTS$	7. Postulate 13 - ASA Congruence Assumption - If two angles and the included side of one triangle are congruent to the corresponding angles and included side of another triangle, then the two triangles are congruent.
6. $\overline{RT} \cong \overline{MT}$	8. C.P.C.T.C.
7. $\angle TRM \cong \angle TMR$	9. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.

—Continued—

5. The lengths of the sides of a triangle are 8, 12, and 15 inches respectively, as shown in the figures below. Find the lengths of the segments into which each side is divided by the bisector of the opposite angle.

Case 1 $\frac{96}{23}$ and $\frac{180}{23}$



$$\frac{AC}{AB} = \frac{CD}{BD}$$

$$\frac{8}{15} = \frac{x}{12-x}$$

$$15 \cdot x = 8(12-x)$$

$$15x = 96 - 8x$$

$$23x = 96$$

$$x = \frac{96}{23} = CD$$

$$12 - x = BD$$

$$12 - \frac{96}{23} = BD$$

$$\frac{276 - 96}{23} = BD$$

$$\frac{180}{23} = BD$$

check:

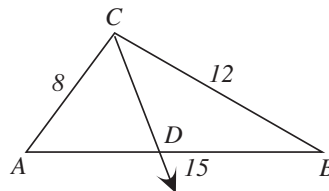
$$\frac{CD}{BD} = \frac{\frac{96}{23}}{\frac{180}{23}}$$

$$\frac{CD}{BD} = \frac{96}{180}$$

$$\frac{CD}{BD} = \frac{8 \cdot 12}{15 \cdot 12}$$

$$\frac{CD}{BD} = \frac{8}{15} = \frac{AC}{AB}$$

Case 2 6 and 9



$$\frac{CA}{CB} = \frac{AD}{BD}$$

$$\frac{8}{12} = \frac{x}{15-x}$$

$$12 \cdot x = 8(15-x)$$

$$12x = 120 - 8x$$

$$20x = 120$$

$$x = 6 = AD$$

$$15 - x = BD$$

$$15 - 6 = BD$$

$$9 = BD$$

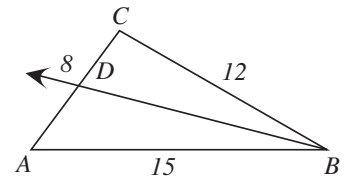
check:

$$\frac{AD}{BD} = \frac{6}{9}$$

$$\frac{6}{9} = \frac{2 \cdot 3}{3 \cdot 3} = \frac{2}{3} = \frac{AD}{BD}$$

$$\frac{8}{12} = \frac{2 \cdot 4}{3 \cdot 4} = \frac{2}{3} = \frac{CA}{CB}$$

Case 3 $\frac{32}{9}$ and $\frac{40}{9}$



$$\frac{BC}{BA} = \frac{CD}{AD}$$

$$\frac{12}{15} = \frac{x}{8-x}$$

$$15 \cdot x = 12(8-x)$$

$$15x = 96 - 12x$$

$$27x = 96$$

$$x = \frac{96}{27} = \frac{32 \cdot 3}{9 \cdot 3} = \frac{32}{9} = CD$$

$$8 - x = AD$$

$$8 - \frac{32}{9} = AD$$

$$\frac{72 - 32}{9} = AD$$

$$\frac{40}{9} = AD$$

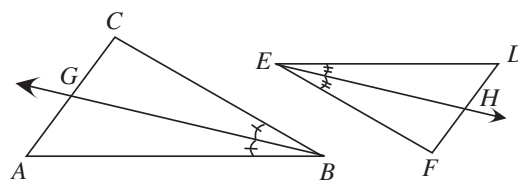
check:

$$\frac{CD}{AD} = \frac{\frac{32}{9}}{\frac{40}{9}}$$

$$\frac{CD}{AD} = \frac{32}{40}$$

$$\frac{CD}{AD} = \frac{4 \cdot 8}{5 \cdot 8} = \frac{4}{5}$$

$$\frac{BC}{BA} = \frac{12}{15} = \frac{4 \cdot 3}{5 \cdot 3} = \frac{4}{5}$$



5. Given: $\triangle ABC \sim \triangle DEF$
 \overrightarrow{BG} bisects $\angle ABC$; \overrightarrow{EH} bisects $\angle DEF$
 Prove: $\frac{CG}{GA} = \frac{FH}{HD}$

STATEMENT	REASON
1. $\triangle ABC \sim \triangle DEF$	1. Given
2. $\frac{AB}{DE} = \frac{BC}{EF}$	2. If two triangles are similar, then corresponding sides are proportional
3. \overrightarrow{BG} bisects $\angle ABC$; \overrightarrow{EH} bisects $\angle DEF$;	3. Given
4. $\frac{AB}{BC} = \frac{AG}{GC}$; $\frac{DE}{EF} = \frac{DH}{HF}$	4. Theorem 35 - If a ray bisects an angle of a triangle, then it divides the opposite side into two segments that have the same ratio as the two other sides.
5. $\frac{AB}{BC} = \frac{DE}{EF}$	5. Switch the means in proportion for statement 2. (Multiply by $\frac{DE}{EC}$ on both sides of step 2.)
6. $\frac{AG}{GC} = \frac{DH}{HF}$	6. Substitution
7. $\frac{HF}{DH} = \frac{GC}{AG}$	7. Multiplication Property for Equality
8. $\frac{GC}{AG} = \frac{HF}{DH}$	8. Symmetry Property of Equality

Unit IV - Triangles

Part G - Congruence – Part 3 (Triangle Inequalities)

Lesson 1 - Theorem 37 - "If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle." (Exterior Angle Inequality Theorem)

Lesson 2 - Theorem 38 - "In a given triangle, if two sides are not congruent, then the angles opposite those sides are not congruent."

Lesson 3 - Theorem 39 - "In a given triangle, if two angles are not congruent, then the sides opposite those angles are not congruent."

Lesson 4 - Theorem 40 - "In a given triangle, the sum of the lengths of any two sides, is greater than the length of the third side."

For each of problems 1 and 2, write an inequality that relates only x and y .

1. $x = 47 + y$ $x > y$

2. $y - x = 25$ $x < y$

For each of problems 3 and 4, write an inequality that relates only $m\angle 1$ and $m\angle 2$

3. $m\angle 1 - 160 = m\angle 2$ $m\angle 1 > m\angle 2$

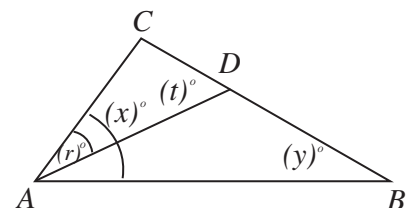
4. $m\angle 2 = m\angle 1 + 65$ $m\angle 1 < m\angle 2$

5. In the figure to the right, $AB > BC > AC$ and $r = t$.
Fill in each of the following blanks with $>$, $<$, or $=$.

y _____ $<$ _____ x

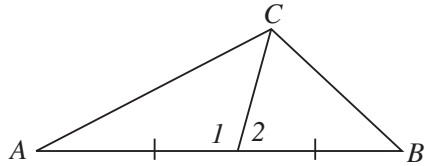
t _____ $>$ _____ y

r _____ $>$ _____ y

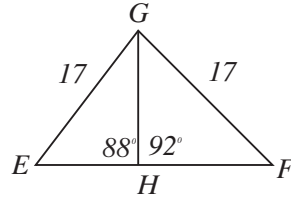


What conclusion can you draw from the given information and the given diagram in problems 9 - 12, by applying the Hinge Theorems.

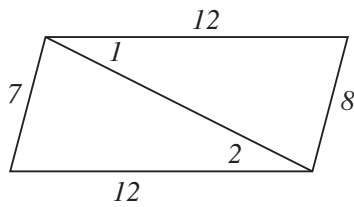
9. $m\angle 1 > m\angle 2$ $AC > BC$



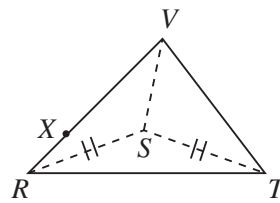
10. $EH > FH$



11. $m\angle 1 > m\angle 2$

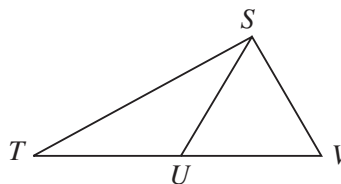


12. $SR = ST; VX = VT$ $\angle VSR > \angle VST$



13. Given: $\overline{TU} \cong \overline{US} \cong \overline{VS}$

Prove: $ST > SV$



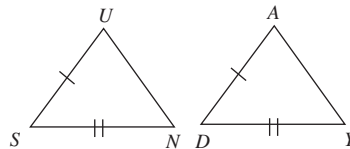
STATEMENT	REASON
1. $\overline{TU} \cong \overline{US}$	1. Given
2. $\angle STU \cong \angle TSU$	2. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent
3. $m\angle STU = m\angle TSU$	3. Definition of Congruent Angles
4. $\overline{US} \cong \overline{VS}$	4. Given
5. $\angle SUV \cong \angle SVU$	5. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent
6. $m\angle SUV = m\angle SVU$	6. Definition of Congruent Angles
7. $m\angle SUV > m\angle STU$	7. Theorem 36 - If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle.
8. $m\angle SVU > m\angle STU$	8. Substitution
9. $ST > SV$	9. Theorem 38 - If the measure of one angle of a triangle is greater than the measure of a second angle of the triangle, then the measure of the side opposite the larger angle is greater than the measure of the side opposite the smaller angle.

In problem 6, state one or more conclusions that can be drawn from the given information. Give a reason for each conclusion.

6. Given: $\triangle UNS$ and $\triangle AYD$; $\overline{SU} \cong \overline{DA}$; $\overline{SN} \cong \overline{DY}$; $UN < AY$

Conclusion: $\angle S \neq \angle D$

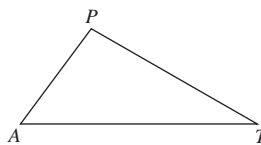
Conclusion: $m\angle S < m\angle D$



Reason: Hinge Theorem (Side-to-Angle Version) - If two sides of one triangle are congruent to two sides of a second triangle, and the length of the third side of the first triangle is greater than the length of the third side of the second triangle, then the measure of the angle opposite the third side of the first triangle, is greater than the measure of the angle opposite the third side of the second triangle.

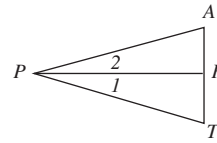
7. Given: $\triangle ATP$

Prove: $AT > TP - PA$



STATEMENT	REASON
1. $\triangle ATP$	1. Given
2. $AT + PA > TP$	2. Theorem 40 - If you have the sum of the measures of two sides of a triangle, then that sum is greater than the measure of the third side of the triangle.
3. $AT > TP - PA$	3. Subtraction Property for Inequalities (i.e., Addition Property where the Additive Inverse is added to both sides)

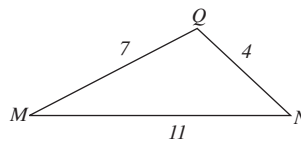
8. Given: $\overline{PA} \cong \overline{PT}$; $m\angle 1 > m\angle 2$



Prove: $RT > AR$

STATEMENT	REASON
1. $\overline{PA} \cong \overline{PT}$	1. Given
2. $m\angle 1 > m\angle 2$	2. Given
3. $\overline{PR} \cong \overline{PR}$	3. Reflexive Property for Congruence
4. $RT > AR$	4. Hinge Theorem (Angle-to-Angle Version) - If two sides of one triangle are congruent to two sides of a second triangle and the measure of the included angle of the first triangle is greater than the measure of the included angle of the second triangle, then the length of the side opposite the included angle of the first triangle is greater than the length of the side opposite the included angle of the second triangle.

9. What is wrong with this picture?

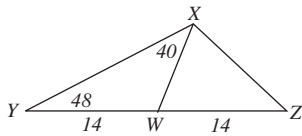


Explain.

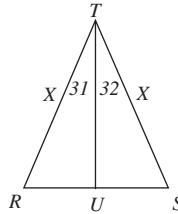
Theorem 40 - If you have the sum of the measures of two sides of a triangle, then that sum is greater than the measure of the third side of the triangle.

For problems 10 – 12, use each accompanying figure, and fill in the blanks with $<$, $=$, or $>$.

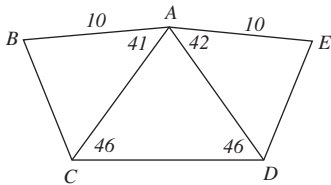
10. XY $>$ XZ ; XW $>$ 14



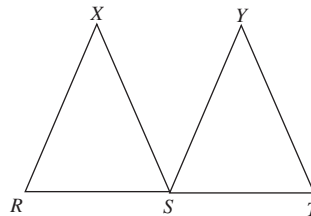
11. RU $<$ US



12. BC $<$ DE



13. Given: $XS > YS$; $\overline{RX} \cong \overline{TY}$
 S is midpoint of RT



Prove: $m\angle R > m\angle T$

STATEMENT	REASON
1. S is midpoint of RT	1. Given
2. $\overline{RS} \cong \overline{TS}$	2. Definition of Midpoint
3. $RS = TS$	3. Definition of Congruence
4. $\overline{RX} \cong \overline{TY}$	4. Given
5. $RX = TY$	5. Definition of Congruence
6. $XS > YS$	6. Given
7. $m\angle R > m\angle T$	7. Hinge Theorem (Side-to-Angle Version) - If two sides of one triangle are congruent to two sides of a second triangle, and the length of the third side of the first triangle is greater than the length of the third side of the second triangle, then the measure of the angle opposite the third side of the first triangle, is greater than the measure of the angle opposite the third side of the second triangle.

Unit IV - Triangles

1. Write true or false for the following statements

(A-2) True

a. An equilateral triangle is also an isosceles triangle.

(A-2) False

b. A right triangle may also be an acute triangle.

(A-2) False

c. All isosceles triangles are equiangular triangles.

(A-1) False

d. A right triangle has only one altitude.

(B-1) False

e. If $\triangle DEF$ is a right triangle and the measure of one acute angles is 65, then the measure of the other acute angle is 35.

(A-2, B-1)

2. $\triangle ABC$ is isosceles with $AB = AC$. The base angles are $\angle B$ and $\angle C$.

(A-2, B-1, F-4)

3. $\triangle DEF$ is isosceles with base angles measuring 25 degrees. The measure of the vertex angle is 130.

(F-4)

4. $\triangle MNQ$ is equilateral. $m\angle Q =$ 60.

(A-1)

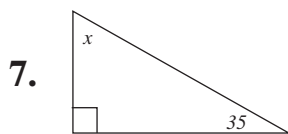
5. An altitude of a triangle is a segment drawn from a vertex and perpendicular to the opposite side of the triangle.

(E-3)

6. The HL Congruence Postulate states that two right triangles are congruent if the hypotenuse and leg of one are congruent to the corresponding parts of the other.

(B-1)

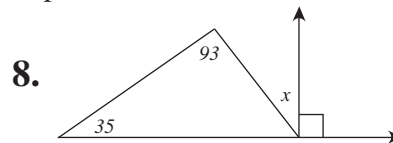
Find the measure of the indicated angles in problems 7 - 9.



$$x = \underline{\quad 55 \quad}$$

$$90 - 35 = x$$

$$55 = x$$

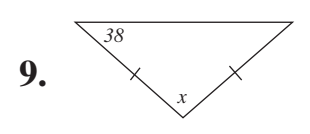


$$x = \underline{\quad 38 \quad}$$

$$180 - 35 - 93 = 52$$

$$90 - 52 = x$$

$$38 = x$$



$$x = \underline{\quad 104 \quad}$$

$$180 - 38 - 38 = x$$

$$104 = x$$

—Continued—

(B-1, F-4)

10. The measures of the base angles of an isosceles triangle are $3x + 18$ and $5x - 2$. The vertex angle measures $9x - 6$. Write two different open sentences that could be used to find the measure of each angle. Find the measure of each angle.

1) $3x + 18 = 5x - 2$ $20 = 2x$ $10 = x$	$3x + 18$ $3(10) + 18$ $30 + 18$ $48 = \text{angle 1}$	$\angle 1 = \underline{\hspace{2cm} 48 \hspace{2cm}}$ $\angle 2 = \underline{\hspace{2cm} 48 \hspace{2cm}}$ $\angle 3 = \underline{\hspace{2cm} 84 \hspace{2cm}}$
--	---	---

2) $3x + 18 + 5x - 2 + 9x - 6 = 180$ $17x + 10 = 180$ $17x = 170$ $x = 10$	$5x - 2$ $5(10) - 2$ $50 - 2$ $48 = \text{angle 2}$	$9x - 6$ $9(10) - 6$ $90 - 6$ $84 = \text{angle 3}$
---	--	--

(C-2)

11. If $\frac{a}{b} = \frac{4}{5}$, write three other proportions using a, b, 4 and 5.

$\frac{b}{a} = \frac{5}{4}$ or $\frac{a}{4} = \frac{b}{5}$ or $\frac{4}{a} = \frac{5}{b}$ or $\frac{b+a}{a} = \frac{5+4}{4}$ or $\frac{b-a}{a} = \frac{5-4}{4}$ or $\frac{b}{a} = \frac{b+5}{a+4}$

(C-2)

12. Solve for x: $\frac{5}{7} = \frac{2}{x}$ $x = \underline{\hspace{2cm} \frac{14}{5} \hspace{2cm}}$

(C-2)

13. Solve for x: $\frac{x}{x+9} = \frac{5}{8}$ $x = \underline{\hspace{2cm} 15 \hspace{2cm}}$

$(7)(2) = (5)(x)$
 $14 = 5x$
 $\frac{14}{5} = x$

$(x+9)(5) = (x)(8)$
 $5x + 45 = 8x$
 $45 = 3x$
 $15 = x$

(C-2)

14. Find the geometric mean (call it x) between 6m and 24m. $x = \underline{\hspace{2cm} 12m \hspace{2cm}}$

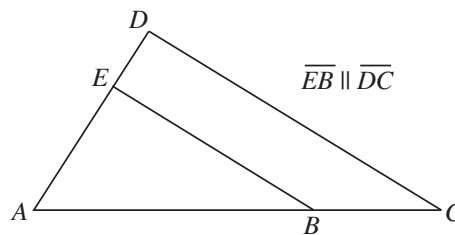
$\frac{6m}{x} = \frac{x}{24m}$
 $x \cdot x = (6m)(24m)$
 $x^2 = 144m^2$
 $x = 12m$

(C-2)

15. Find the fourth proportional to 5, 35, and 9. The fourth proportional is 63

$\frac{5}{35} = \frac{9}{x}$
 $(35)(9) = (5)(x)$
 $315 = 5x$
 $63 = x$

Use the figure to the right for problems 16 - 20.



(D-2)

16. $AB = 6, BC = 4. AE = 3, ED = \underline{\hspace{2cm}2\hspace{2cm}}$.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

$$\frac{6}{4} = \frac{3}{x}$$

$$(4)(3) = (6)(x)$$

$$12 = 6x$$

$$2 = x$$

(D-2)

17. $AB = 9, BC = 6, ED = 4, AE = \underline{\hspace{2cm}6\hspace{2cm}}$.

$$\frac{AB}{BC} = \frac{AE}{ED}$$

$$\frac{9}{6} = \frac{x}{4}$$

$$(6)(x) = (9)(4)$$

$$6x = 36$$

$$x = 6$$

(D-2)

18. $BC = 3, AC = 10, AE = 2\frac{1}{3}, ED = \underline{\hspace{2cm}1\hspace{2cm}}$.

$$\frac{BC}{AB} = \frac{ED}{AE}$$

$$\frac{3}{7} = \frac{ED}{2\frac{1}{3}}$$

$$(7)(ED) = \left(\frac{3}{1}\right)\left(\frac{7}{3}\right)$$

$$7 \cdot ED = \frac{21}{3}$$

$$ED = 1$$

(D-2)

19. $AE = 4, ED = 2, BE = 7, DC = \underline{\hspace{2cm}10\frac{1}{2}\hspace{2cm}}$.

$$\frac{AD}{AE} = \frac{DC}{EB}$$

$$\frac{AE+ED}{AE} = \frac{DC}{EB}$$

$$\frac{4+2}{4} = \frac{DC}{7}$$

$$\frac{6}{4} = \frac{DC}{7}$$

$$(4)(DC) = (6)(7)$$

$$(4)(DC) = 42$$

$$DC = \frac{42}{4} = \frac{2 \cdot 21}{2 \cdot 2} = 10\frac{1}{2}$$

(D-2)

20. $AC = 12, AB = 8, AE = \underline{\hspace{2cm}6\frac{2}{3}\hspace{2cm}}, AD = 10$

$$\frac{AB}{AC} = \frac{AE}{AD}$$

$$\frac{8}{12} = \frac{AE}{10}$$

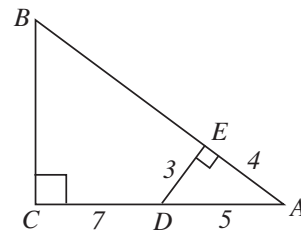
$$(12)(AE) = (8)(10)$$

$$AE = \frac{2 \cdot 2 \cdot 2 \cdot 10}{2 \cdot 2 \cdot 3} = \frac{20}{3}$$

$$AE = 6\frac{2}{3}$$

—Continued—

For problems 21-23, triangle ABC is a right triangle, as shown to the right, with $\angle C$ as the right angle. $\overline{DE} \perp \overline{AB}$, $AE = 4$, $ED = 3$, $AD = 5$, and $CD = 7$.



(D-1)

21. State why $\triangle ADE \sim \triangle ABC$. Postulate Corollary 12b - If an acute angle of one right triangle is congruent to an acute angle of another right triangle, then the right triangles are similar.

(D-3)

22. Fill in each box to complete the following extended proportion:

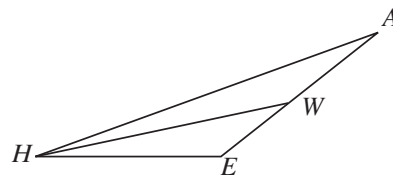
$$\frac{AD}{AB} = \frac{AE}{\boxed{AC}} = \frac{ED}{\boxed{CB}}$$

(D-3)

23. Find AC: 12
 Find BC: 9
 Find AB: 15

$$\begin{aligned} AC &= 12 \text{ (AD+DC or 5+7)} \\ \frac{AE}{AC} &= \frac{ED}{BC} & \frac{AE}{AC} &= \frac{AD}{AB} \\ \frac{4}{12} &= \frac{3}{BC} & \frac{4}{12} &= \frac{5}{AB} \\ (12)(3) &= (4)(BC) & (12)(5) &= (4)(AB) \\ 36 &= (4)(BC) & 60 &= (4)(AB) \\ \frac{4 \cdot 4}{4} &= BC & \frac{4 \cdot 15}{4} &= AB \\ 9 &= BC & 15 &= AB \end{aligned}$$

For problems 24 and 25, \overline{HW} bisects $\angle H$, as shown in the figure to the right.



(F-6)

24. If $HE = 10$, $EW = 4$, and $HA = 14$, Find WA . 5.6

$$\begin{aligned} \frac{HE}{HA} &= \frac{EW}{WA} & 56 &= (10)WA \\ \frac{10}{14} &= \frac{4}{WA} & 5.6 &= WA \\ (14)(4) &= (10)(WA) \end{aligned}$$

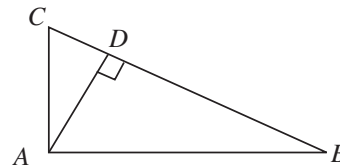
(F-6)

25. If $HE = 8$, $HA = 12$, and $EA = 10$, Find EW and WA . 4; 6

$$\begin{aligned} \frac{HE}{HA} &= \frac{EW}{WA} & EW &= 4 \\ \frac{8}{12} &= \frac{x}{10-x} & WA &= 10-x \\ (12)(x) &= (8)(10-x) & WA &= 10-4 \\ 12x &= 80-8x & WA &= 6 \\ 20x &= 80 \\ x &= 4 \end{aligned}$$

—Continued—

For problems 26–29, $\triangle ABC$ is a right triangle with altitude \overline{AD} and hypotenuse \overline{BC} , as shown in the figure to the right.



(D-4)

26. $CD = 3$, $AD = 9$. Find BD . 27

$$\begin{aligned} \frac{CD}{AD} &= \frac{AD}{BD} \\ \frac{3}{9} &= \frac{9}{BD} \\ (9)(9) &= (3)(BD) \\ 81 &= (3)(BD) \\ \frac{3 \cdot 27}{3} &= BD \\ 27 &= BD \end{aligned}$$

(D-4)

27. $CD = 1$, $CB = 12$. Find AC . $2\sqrt{3}$

$$\begin{aligned} \frac{CD}{AC} &= \frac{AC}{CB} \\ \frac{1}{AC} &= \frac{AC}{12} \\ (AC)(AC) &= (1)(12) \\ (AC)^2 &= 12 \\ AC &= \sqrt{12} = \sqrt{4 \cdot 3} \\ AC &= 2\sqrt{3} \end{aligned}$$

(D-4)

28. $AB = 8$, $BD = 6$. Find BC . $\frac{32}{3}$

$$\begin{aligned} \frac{BC}{AB} &= \frac{AB}{BD} \\ \frac{BC}{8} &= \frac{8}{6} \\ (8)(8) &= (BC)(6) \\ 64 &= (BC)(6) \\ \frac{2 \cdot 32}{2 \cdot 3} &= BC \\ \frac{32}{3} &= BC \end{aligned}$$

(D-4)

29. $CD = 4$, $DB = 16$, Find AC $4\sqrt{5}$. Find AB $8\sqrt{5}$. Find AD 8.

$\begin{aligned} \frac{CD}{AC} &= \frac{AC}{CB} \\ \frac{CD}{AC} &= \frac{AC}{CD+DB} \\ \frac{4}{AC} &= \frac{AC}{4+16} \\ (AC)(AC) &= (4)(4+16) \\ (AC)^2 &= 80 \\ AC &= \sqrt{80} = \sqrt{16 \cdot 5} \\ AC &= 4\sqrt{5} \end{aligned}$	$\begin{aligned} \frac{DB}{AB} &= \frac{AB}{BC} \\ \frac{DB}{AB} &= \frac{AB}{BD+DC} \\ \frac{16}{AB} &= \frac{AB}{16+4} \\ (AB)(AB) &= (16)(16+4) \\ (AB)^2 &= (16)(20) = 320 \\ AB &= \sqrt{320} = \sqrt{64 \cdot 5} \\ AB &= 8\sqrt{5} \end{aligned}$	$\begin{aligned} \frac{CD}{AD} &= \frac{AD}{DB} \\ \frac{4}{AD} &= \frac{AD}{16} \\ (AD)(AD) &= (4)(16) \\ (AD)^2 &= 64 \\ AD &= \sqrt{64} \\ AD &= 8 \end{aligned}$
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—Continued—

(D-5)

30. If the measures of the three sides of a triangle are 10, 12, and 15, is the triangle a right triangle? No
 If not, explain. The given triangle is not a right triangle. $a^2 + b^2 > c^2$. So, the triangle is an acute triangle.

$$a^2 + b^2 = c^2$$

$$10^2 + 12^2 = 15^2$$

$$100 + 144 = 225$$

(D-5) $244 > 225$

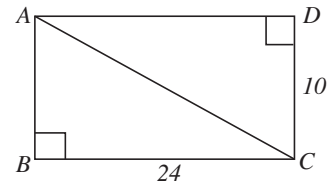
31. In rectangle ABCD, as shown to the right, find AC. 26

$$(DC)^2 + (AD)^2 = (AC)^2 \quad \sqrt{676} = AC$$

$$(10)^2 + (24)^2 = (AC)^2 \quad \sqrt{4 \cdot 169} = AC$$

$$100 + 576 = (AC)^2 \quad 2 \cdot 13 = AC$$

$$676 = (AC)^2 \quad 26 = AC$$



(D-5)

32. The length of the hypotenuse of a 45-45-90 triangle is $\sqrt{2}$ times the length of a leg.

(D-5)

33. The length of the longer leg of a 30-60-90 triangle is $\frac{\sqrt{3}}{2}$ times the length of the hypotenuse.

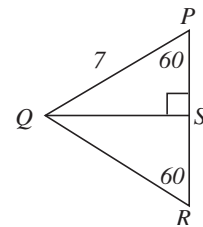
(D-5)

34. In $\triangle PRQ$ shown to the right, $\overline{QS} \perp \overline{PR}$. If $PQ = 7$, Find PS $\frac{7}{2}$.

Find SQ $\frac{7\sqrt{3}}{2}$.

$$PS = \frac{1}{2} \cdot 7 \text{ or } \frac{7}{2}$$

$$SQ = \frac{\sqrt{3}}{2} \cdot 7 \text{ or } \frac{7\sqrt{3}}{2}$$



(D-5)

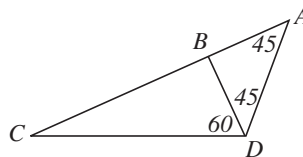
35. $AB = 1$ in the figure to the right.

Find BD 1.

Find AD $\sqrt{2}$.

Find CB $\sqrt{3}$.

Find DC 2.



start with

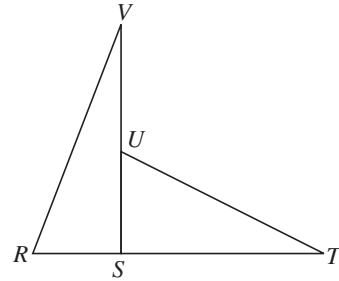
$$AB = BD = 1$$

—Continued—

(E-3)

40. Given: $\overline{VS} \perp \overline{RT}$; $\overline{VR} \cong \overline{UT}$; $\angle V \cong \angle T$

Prove: $\overline{RS} \cong \overline{US}$

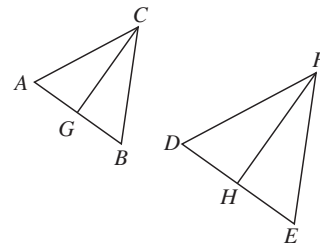


STATEMENT	REASON
1. $\overline{VS} \perp \overline{RT}$	1. Given
2. $\angle VSR$ and $\angle VST$ are right angles	2. Definition of Perpendicular
3. $\triangle VSR$ and $\triangle VST$ are right triangles	3. Definition of Right Triangles
4. $\overline{VR} \cong \overline{UT}$	4. Given
5. $\angle V \cong \angle T$	5. Given
6. $\triangle RSV \cong \triangle UST$	6. Postulate Corollary 13b - HA Congruence Postulate - If the hypotenuse and acute angle of one right triangle are congruent to the hypotenuse and corresponding acute angle of another right triangle, then the triangles are congruent
7. $\overline{RS} \cong \overline{US}$	7. C.P.C.T.C.

(D-1)

41. Given: $\angle A \cong \angle D$; $\angle B \cong \angle E$;
G is the midpoint of \overline{AB} ; H is the midpoint of \overline{DE}

Prove: $\frac{CG}{FH} = \frac{AC}{DF}$



STATEMENT	REASON
1. $\angle A \cong \angle D$	1. Given
2. $\angle B \cong \angle E$	2. Given
3. $\triangle ABC \sim \triangle DEF$	3. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar.
4. G is the midpoint of \overline{AB}	4. Given
5. H is the midpoint of \overline{DE}	5. Given
6. \overline{CG} is a median	6. Definition of a Median
7. \overline{FH} is a median	7. Definition of a Median
8. $\frac{CG}{FH} = \frac{AC}{DF}$	8. Corollary 29b - If two triangles are similar, then the measures of corresponding medians are in the same ratio as the measures of corresponding sides.

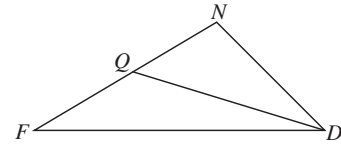
Unit IV, Unit Test Form A
—Continued—

Name _____

(D-1)

42. Given: $\triangle FDN$ in which $\angle F \cong \angle NDQ$

Prove: $\triangle NQD \sim \triangle NDF$

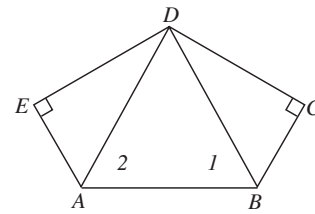


STATEMENT	REASON
<ol style="list-style-type: none"> 1. $\triangle FDN$ in which $\angle F \cong \angle NDQ$ 2. $\angle N \cong \angle N$ 3. $\triangle NQD \sim \triangle NDF$ 	<ol style="list-style-type: none"> 1. Given 2. Reflexive Property for Congruence 3. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar

(E-3)

43. Given: $\angle 1 \cong \angle 2$; $\overline{AE} \cong \overline{BC}$;
 $\overline{DE} \perp \overline{AE}$; $\overline{DC} \perp \overline{BC}$

Prove: $\overline{ED} \cong \overline{CD}$

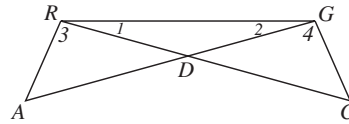


STATEMENT	REASON
<ol style="list-style-type: none"> 1. $\angle 1 \cong \angle 2$ 2. $AD \cong BD$ 3. $\overline{AE} \cong \overline{BC}$ 4. $\overline{DE} \perp \overline{AE}$ 5. $\overline{DC} \perp \overline{BC}$ 6. $\angle E$ is a right angle 7. $\angle C$ is a right angle 8. $\triangle AED$ is a right triangle 9. $\triangle BCD$ is a right triangle 10. $\triangle AED \cong \triangle BCD$ 11. $\overline{ED} \cong \overline{CD}$ 	<ol style="list-style-type: none"> 1. Given 2. Theorem 34 - If two angles of a triangle are congruent, then the sides opposite them are congruent 3. Given 4. Given 5. Given 6. Definition of Perpendicular 7. Definition of Perpendicular 8. Definition of Right Triangles 9. Definition of Right Triangles 10. Postulate Corollary 13e - HI Congruence Postulate - If a hypotenuse and one leg of a right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent. 11. C.P.C.T.C.

(F-4)

44. Given: $\angle ARG \cong \angle CGR$; $\overline{DG} \cong \overline{DR}$

Prove: $m\angle 3 = m\angle 4$

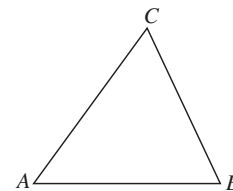


STATEMENT	REASON
1. $\angle ARG \cong \angle CGR$	1. Given
2. $m\angle ARG \cong m\angle CGR$	2. Definition of Congruent Angles
3. $m\angle 1 + m\angle 3 = m\angle ARG$	3. Postulate 7 - Protractor - Fourth Assumption - Angle Addition
4. $m\angle 2 + m\angle 4 = m\angle CGR$	4. Postulate 7 - Protractor - Fourth Assumption - Angle Addition
5. $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$	5. Substitution
6. $\overline{DG} \cong \overline{DR}$	6. Given
7. $\angle 1 \cong \angle 2$	7. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
8. $m\angle 1 = m\angle 2$	8. Definition of Congruence
9. $m\angle 3 = m\angle 4$	9. Subtraction Property for Equality

(G-3)

45. Given: $m\angle B > m\angle A$

Prove: $AC > BC$



STATEMENT	REASON
1. $m\angle B > m\angle A$	1. Given
2. $AC > BC$	2. Corollary 39a - If the measure of one angle of a triangle is greater than the measure of a second angle of the triangle, then the measure of the third side opposite the larger angle is greater than the measure of the side opposite the smaller angle.

Unit IV - Triangles

In all of problems 1–50, choose the letter for the correct response.

(A-2)

1. A triangle with no congruent sides is _____ *c* _____.
- a. Equiangular b. Obtuse
c. Scalene d. Isosceles

(A-2)

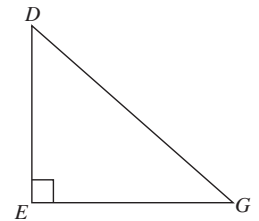
2. An acute triangle is one with _____ *b* _____ acute angle(s).
- a. two b. three
c. at least one d. one

(A-2)

3. An equiangular triangle is also _____ *c* _____.
- a. right b. obtuse
c. equilateral d. scalene

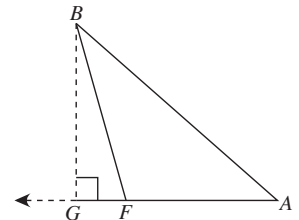
(A-2)

4. $\triangle DEG$, shown to the right, is a right triangle. \overline{DG} is called the _____ *c* _____.
- a. altitude b. leg
c. hypotenuse d. angle



(A-1)

5. In $\triangle ABF$, shown to the right, \overline{BG} is a(n) _____ *d* _____ of the triangle.
- a. side b. leg
c. hypotenuse d. altitude



(A-2)

6. An equilateral triangle must also be _____ *c* _____.
- a. scalene b. obtuse
c. acute d. right

(A-2)

7. In an isosceles triangle the base angles are _____ *d* _____.
- a. right b. acute
c. congruent d. both b and c

(D-2)

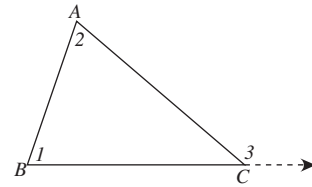
8. An auxiliary line is _____ *a* _____.
- a. a line added to a figure to help solve a problem b. an altitude
c. a hypotenuse d. a line that may be omitted from a figure

—Continued—

(A-1)

9. In $\triangle ABC$, shown to the right, $\angle 3$ is called a(n) _____ *b*.

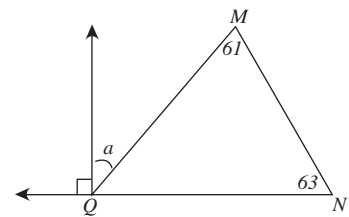
- a. remote interior angle
- b. exterior angle
- c. acute



(E-3)

10. The HA theorem states that two right triangles are congruent if _____ *c*.

- a. two angles and the side included between them are congruent
- b. each hypotenuse and a corresponding leg are congruent
- c. each hypotenuse and a corresponding angle are congruent



(B-1)

11. In $\triangle MNQ$, shown to the right, find *a*. _____ *d*

- a. 61
- b. 63
- c. 45
- d. 34

(C-2)

12. If $\frac{P}{9} = \frac{x}{y}$, then $\frac{y}{9} =$ _____ *b*.

- a. $\frac{P}{x}$
- b. $\frac{x}{P}$
- c. $\frac{y}{x}$
- d. $\frac{9}{y}$

(B-1)

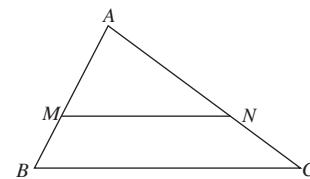
13. If the measures of the angles of a triangle are in the ratio 2:2:5, then the measure of the largest angle is _____ *c*.

- a. 80°
- b. 40°
- c. 100°
- d. 90°

(D-2)

14. If $\frac{AM}{MB} = \frac{AN}{NC}$ in the figure to the right, then _____ *b*.

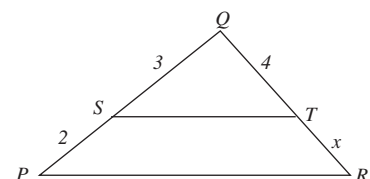
- a. $MN = \frac{1}{2} \cdot BC$
- b. $\overline{MN} \parallel \overline{BC}$
- c. $AM = \frac{1}{2} \cdot AB$
- d. $\frac{AN}{MN} = \frac{AN}{NC}$



(D-2)

15. If $\overline{ST} \parallel \overline{PR}$ in the figure to the right, then $x =$ _____ *d*.

- a. 6
- b. 2
- c. 1.5
- d. $2\frac{2}{3}$ or $\frac{8}{3}$

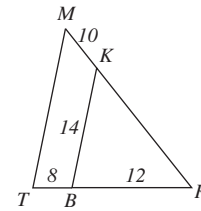


Unit IV, Unit Test Form B

Name _____

—Continued—

Use the figure to the right for problems 16 and 17.



(D-2)

16. $\overline{KB} \parallel \overline{MT}$. $KR =$ _____ *b* _____.

- a. 20
- b. 15
- c. 5
- d. 14

(D-2)

17. $\overline{KB} \parallel \overline{MT}$. $MT =$ _____ *c* _____.

- a. 28
- b. 25
- c. $\frac{70}{3}$
- d. $\frac{28}{3}$

(C-3)

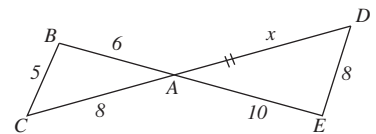
18. If $\triangle BCF \sim \triangle GHJ$, then _____ *c* _____.

- a. $\frac{CF}{BF} = \frac{GJ}{HJ}$
- b. $\frac{CF}{BF} = \frac{GH}{HJ}$
- c. $\frac{CF}{BF} = \frac{HJ}{GJ}$
- d. $\frac{CF}{BF} = \frac{JH}{JG}$

(C-3)

19. If in the figure to the right, $\triangle ABC \sim \triangle AED$, then _____ *c* _____.

- a. $\frac{6}{8} = \frac{x}{10}$
- b. $(6)(10) = 8x$
- c. $\frac{6}{10} = \frac{8}{x}$
- d. $\frac{5}{y} = \frac{8}{10}$



(C-3)

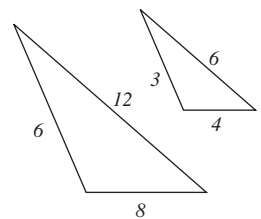
20. Which of these polygons must be similar? _____ *d* _____

- a. two isosceles triangles
- b. two rectangles
- c. two parallelograms with 65° angles
- d. two regular octagons

(D-1)

21. The triangles shown to the right are similar by the _____ *b* _____.

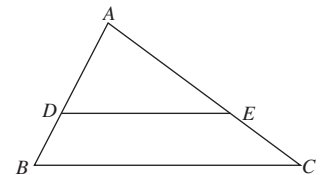
- a. Postulate Corollary 12a - AA Similarity Postulate
- b. Postulate 12 - SSS Similarity Assumption
- c. Postulate 12 - SAS Similarity Assumption



(D-1)

22. In the figure to the right, $\overline{DE} \parallel \overline{BC}$. So, $\triangle ABC \sim \triangle ADE$ because of the _____ *b* _____.

- a. Postulate 12 - SAS Similarity Assumption
- b. Postulate Corollary 12a - AA Similarity Postulate
- c. Postulate 12 - SSS Similarity Assumption
- d. Definition of Similar Triangles



(C-3)

23. $\triangle PQR \sim \triangle MNO$. $PQ = 18$, $QR = 12$, $PR = 16$, and $MN = 12$. The perimeter of $\triangle MNO$ is _____ *a* _____.

- a. $\frac{92}{3}$
- b. 36
- c. 46
- d. 40

(C-2)

24. What is the geometric mean between $\frac{3}{8}$ and 4? _____ *c* _____

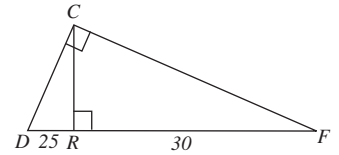
- a. $\frac{3}{2}$
- b. $\frac{3}{2}\sqrt{2}$
- c. $\frac{1}{2}\sqrt{6}$
- d. $3\sqrt{6}$
- e. $\frac{35}{16}$

—Continued—

(E-3)

25. In right triangle CFD, shown to the right, find the length of the altitude \overline{CR} . **b**

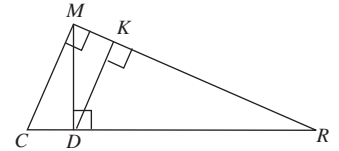
- a. $15\sqrt{5}$
- b. $5\sqrt{30}$
- c. $15\sqrt{3}$
- d. $5\sqrt{55}$



(E-3)

26. The length of \overline{DK} , in the figure to the right, is the geometric mean between the lengths of **c** .

- a. \overline{DR} and \overline{MR}
- b. \overline{MK} and \overline{MD}
- c. \overline{MK} and \overline{KR}
- d. \overline{CD} and \overline{DR}



(D-5)

27. The hypotenuse and one leg of a right triangle are 29 and 21. The other leg measures **c** .

- a. $\sqrt{1282}$
- b. 36
- c. 20
- d. $5\sqrt{2}$

(D-5)

28. The legs of a right triangle are 6 and 9. The hypotenuse measures **a** .

- a. $\sqrt{117}$
- b. $3\sqrt{6}$
- c. 54
- d. 117

(F-7)

29. A triangle whose sides are 5, 12, and 13 is a(n) **c** .

- a. obtuse triangle
- b. acute triangle
- c. right triangle
- d. given measure cannot form a triangle

(F-7)

30. A triangle whose sides are 6, 11, and 15 is a(n) **a** .

- a. obtuse triangle
- b. acute triangle
- c. right triangle
- d. given measure cannot form a triangle

(D-5)

31. The longer leg of a 30-60-90 triangle is 7. The hypotenuse measures **a** .

- a. $\frac{14\sqrt{3}}{3}$
- b. $7\sqrt{3}$
- c. 14
- d. $3\sqrt{7}$

(D-5)

32. One side of a square is x . The measure of a diagonal is **a** .

- a. $x\sqrt{2}$
- b. $x\sqrt{3}$
- c. $\frac{x\sqrt{3}}{2}$
- d. $\frac{x\sqrt{2}}{3}$

(D-5)

33. Each side of an equilateral triangle is 18. The measure of an altitude is **c** .

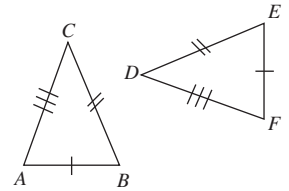
- a. 9
- b. $\frac{9\sqrt{3}}{2}$
- c. $9\sqrt{3}$
- d. $18\sqrt{3}$

—Continued—

(E-1)

34. Two statements of congruence for the two triangles shown are b and d .

- a. $\triangle ABC \cong \triangle DEF$
- b. $\triangle ABC \cong \triangle FED$
- c. $\triangle ABC \cong \triangle EDF$
- d. $\triangle BCA \cong \triangle EDF$



(E-1)

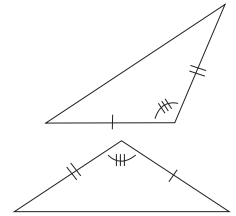
35. If $\triangle STU$ is congruent to $\triangle VWX$, then b .

- a. $\angle S \cong \angle W$
- b. $\angle T \cong \angle W$
- c. $\overline{TU} \cong \overline{VX}$
- d. $\overline{US} \cong \overline{WX}$

(E-2)

36. The two triangles shown to the right are congruent by c .

- a. ASA
- b. SSS
- c. SAS
- d. Not enough information to tell.



(E-1)

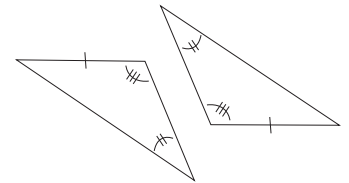
37. In $\triangle ADC$, the side included by $\angle A$ and $\angle D$ is c .

- a. \overline{DC}
- b. \overline{AC}
- c. \overline{AD}

(E-2)

38. The two triangles shown to the right are congruent by d (or a) .

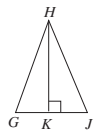
- a. ASA
- b. SSS
- c. SAS
- d. none of these



(E-2)

39. In the figure to the right, given that $\overline{GK} \cong \overline{JK}$ and $\overline{HK} \perp \overline{GJ}$, you could prove $\triangle GHK \cong \triangle JHK$ by c .

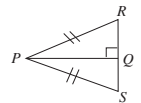
- a. ASA
- b. SSS
- c. SAS



(E-2)

40. In the figure to the right, given that $\overline{PR} \cong \overline{RS}$, and \overline{PQ} bisects \overline{RS} , you could prove $\triangle PRQ \cong \triangle PSQ$ b .

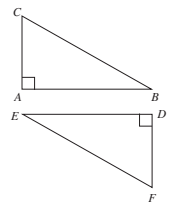
- a. ASA
- b. SSS
- c. SAS



(F-2)

41. In the figure to the right, $\triangle ABC \cong \triangle DEF$. Therefore, $\angle CBA \cong \angle FED$ because c .

- a. they are right triangles
- b. they are acute triangles
- c. CPCTC
- d. not enough information to tell



(Appendix A-1)

42. The statement “If $\triangle ABC \cong \triangle DEF$, then $\angle DEF \cong \angle ABC$ ” illustrates b .

- a. the reflexive property of congruence
- b. the symmetric property of congruence
- c. the transitive property of congruence