

# Geometry: A Complete Course (with Trigonometry)

## Module B – Instructor's Guide with Detailed Solutions for Progress Tests

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**ERRATA**  
**3/2015**



**VideoText *Interactive***

Geometry: A Complete Course (with Trigonometry)  
Module B - Instructor's Guide with Detailed Solutions for Progress Tests

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# Letter from the author . . .

Welcome to Module B of the VideoText Interactive Geometry Course. I hope you had good success in Module A, and I certainly hope you contacted us if you had questions. Please remember, we are here to help. And don't forget that a lot of your questions may be addressed in the Program Overview and the Scope and Sequence Rationale in the pages following the Table of Contents.

As was stated in numerous lessons in Module A, you have been preparing to “play the game of Plane Geometry”. To begin with, you have spent a considerable amount of time revisiting familiar areas of the subject and investigating the reasons behind the terms and rules you have learned in the past. As well, I'm sure you have encountered many new concepts, not only in the discovery of different types of Geometry, but also in our brief study of Mathematical Deductive Logic. You must remember that a general mastery of these concepts is essential, if you are to be able to understand the relationships in Plane Geometry, and be able to formally prove those relationships.

In this Module then, we will begin developing the “rules of the game”, by investigating the essential elements of Plane Geometry, based on the work done by the renowned mathematician, Euclid. The elements introduced will include Fundamental Terms, both those which will be accepted without definition (Undefined Terms) and those which will be clearly and completely explained, according to the formal rules of logic (Defined Terms). Further, we will introduce the seemingly obvious relationships between the Fundamental Terms, and which we will accept as valid, without proof (Postulates or Axioms).

Of course, this means that, when we have completed the module, we will have determined the basic rules that will guide our study. We will then be ready to prove conclusively, in our next module, the fundamental relationships (Theorems) we will use to explore the simple closed plane curves that make up Plane Geometry.

Again, as in Unit I, to make sure that every concept is mastered, and internalized, it will be essential that your student do every exercise in the Student WorkText. We want to do everything we can to cover these concepts from every perspective possible.

Thank you again for your continued interest in the VideoText Interactive mathematics programs, and good luck as you now formally begin the study of Plane Geometry. Of course, as always, call us on the help-line, should you need some assistance.

**Thomas E. Clark, Author**

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# Program Overview

The VideoTextInteractive Geometry program addresses two of the most important aspects of mathematics instruction. **First, the inquiry-based video format contributes to the engaging of students more personally in the concept development process.** Through the frequent use of the pause button, you, as the instructor, can virtually require interaction and dialogue on the part of your student. As well, students who work on their own, can “simulate” having an instructor present by pausing the lesson every time a question is asked, and trying to answer it correctly before continuing. Of course, the student may answer incorrectly, but the narrator will be sure to give the right answer when the play button is pressed to resume the lesson. Right or wrong, however, the student is regularly engaging in analytical and critical thinking, and that is a healthy exercise, in and of itself. **Second, each incremental concept is explored in detail, using no shortcuts, tricks, rules, or formulas, and no step in the process is ignored.** As such, the logic and the continuity of the development assure students that they understand completely. Subsequently, learning is more efficient, and all of the required concepts (topics) of the subject can be covered with mastery. Of course, the benefits of these efforts can be seen even more clearly in a description of a typical session, as follows:

After a brief 2 or 3 sentence introduction of the concept to be considered, usually by examining the description, and the objective given at the beginning of the video lesson, you and your student can begin. **You should pause the lesson frequently**, usually every 15-20 seconds (or more often if appropriate), to engage your student in discussion. This means that, for a 5-10 minute VideoText lesson, it may take 10-15 minutes to finish developing the concept. Dialogue is a cornerstone. In addition, during this time, **your student should probably not be allowed to take notes.** Students should not have their attention divided, or they risk missing important links. Neither should you be dividing your attention, by looking at notes, or writing on a pad, or an overhead projector. **Everyone should be concentrating on concept development and understanding.** Please understand that a student who is accustomed to working alone, or can be motivated to study independently, has, with the VideoText, a powerful resource to explore and master mathematical concepts by simulating the dialogue normally encountered with a “live” instructor. And, because of the extensive detail of the explanations, along with the computer generated graphics, and animation, students are never shortchanged when it comes to the insight necessary to fully comprehend.

Once the concept is developed, and the VideoText lesson is completed, you can then **employ the Course Notes to review, reinforce, or to check on your student's comprehension.** These Course Notes are replications of the essential content that was viewed in the VideoText lesson, illustrating the same terms, diagrams, problems, numbers, and logical sequences. In fact, at this time, if your student needs a little more help, he or she can use the Course Notes while viewing the lesson again, using them as a guide, to re-examine the concept. **The key here is that students concentrate on understanding first, and take care of documentation later.**

Please understand that it is not the intent of the program to let the VideoText lesson completely take the place of personal instruction or interaction. Actually, **the video should never tell your students anything that hasn't been considered or discussed (while the lesson is paused), and it should never answer questions that have not already been considered and resolved.** As such, it becomes a “new breed” of chalkboard or overhead projector, whereby you, as the teacher, or your student working alone, can “write”, simply by pressing the “play” button. This is a critical point to be understood, and should

serve to help you examine all of the materials and strategies from the proper perspective.

Next, your student can begin to do some work independently, either by your personal introduction of additional examples from the WorkText, or by the student immediately going to the WorkText on his or her own. **The primary feature of the WorkText**, beside providing problem banks with which students can work on mastery, is that **objectives are restated, important terms are reviewed, and additional examples are considered**, in noticeable detail, **taking students, once again, through the logic of the concept development process**. The premise here is simple. When students work with an instructor, whether doing exercises on their own, or working through them with other students, they are usually concentrating more on “how to do” the problems. Then, when they leave the instructor, they simply don't take the discussion of the concept with them. The goal of this program is to provide a resource which will help students “re-live” the concept development on their own, whether for review, or for additional help. That is the focus of the Student WorkText.

Having completed the exercises for the lesson being considered, your **student is now ready to use the detail in the Solutions Manual to check work and engage in error analysis**. Again, it is essential to a student's understanding that he or she find mistakes, correct them, and be required to give some explanation, either verbal or in writing, to you as the instructor. In fact, at this stage, you might even **consider grading your student only on the completion of the work**, not on its accuracy. Remember, this is the first time the student has tried to demonstrate understanding of a concept, and he or she may still need some fine-tuning. So, because this is part of the initial learning process, **the focus should be on a careful analysis of the logic behind the work, not just the answers**. Finally, **it is time to assess your student's mastery of the concept behind the work**. Just **be sure you are not testing on the same day the exercises were completed**. Short-term memory can trick you into thinking that you “have it”, when, in fact, you are just remembering what you did moments before. A more accurate evaluation can be made on the next day, before moving on to the next lesson. Further, the quizzes and tests in the program often utilize **open-response questions which will require your student to state, in writing, his or her understanding of the concept**. This often reveals much more about a student's understanding than just checking to see if an answer on a test is correct. Remember too, that there are **two versions of every quiz and test**, allowing you to retest, if necessary, in order to make sure that your student has mastered the concept.

Of course, just as with the WorkText, there are detailed **solutions for all of the quiz and test problems, in the Instructor's Guide**. Again, your student should be required to analyze problems that were missed, and explain why the problem should have been done differently. It is simply a fact that one of the most powerful and effective teaching tools you can employ, is to **ask your students to “articulate” to you what their thinking was**, as they worked toward a given answer.

As you can see, the highly interactive quality of this program, affords students a much greater opportunity than usual to grow mathematically, at a personal level, and develop confidence in their ability. That can have a tremendous impact on a student's future pursuits, especially in an age where applications of mathematics are so important.

# Scope and Sequence Rationale

There are two basic premises which drive concept development in Geometry, and these two essentials shape the logical scope and sequence of geometric content.

First, it is generally understood that **Geometry is the study of spatial relations**. In the same way that Algebra is the study of numerical relations (equations and inequalities), and Calculus is concerned primarily with rates of change, Geometry is a comprehensive exploration of “shapes” (as sets of points), the measurements associated with those shapes, and the relationships that can be established between those shapes. As such, no treatment of Geometry should ever investigate those relationships only individually, or in isolation. This is especially noticeable with traditional textbooks, which generally use a format which addresses them in different “chapters”. In the VideoText Interactive Geometry course, **concepts are discussed from a “Unit” perspective, pursuing and connecting, in an exhaustive way, all of the outcomes associated with various possibilities for a specific relationship**. Of course, as much as is possible, students need to “see” those relationships, and experience the “motion”, or “transformation”, necessary to clearly illustrate the concept. It really is impossible to put a value on the benefits of visualization, in life in general, and in Geometry in particular. So, in the VideoText Interactive Geometry program, **computer-generated graphics are used extensively, along with animation and color-sequencing**, in order that students can actually see the relationships develop.

The second premise is that geometric concepts should be studied **utilizing all of the power and conviction that both inductive and deductive reasoning can bring to the table**. In other words, it is always desirable, and helpful, for students to “experiment”, inductively, with a geometric relationship, in an effort to come to some general conclusion. Once that general conclusion has been arrived at, however, it is even more convincing if the student is able to “prove”, deductively, that the conclusion absolutely must follow, logically, from the given information. No, formal proof is not often asked for in everyday life. On the other hand, the exercise of developing that kind of thinking is invaluable, not only in some specific job-related activities, but, more generally, in the daily problem-solving situations that confront us. The VideoText Interactive Geometry program is formatted in such a way that formal proof is a cornerstone.

**Unit I, then, focuses on a complete preparation for students to begin a formal study of Geometry** by “re-teaching” of all of the basic geometric concepts for which students have simply memorized the appropriate term, definition, or formula. That means we must re-establish that **Mathematics in general, and Geometry in particular, is a language**, with parts of speech and sentence structure. We must develop, in detail, the concepts associated with **building geometric shapes**. We must investigate, again in detail, the concepts dealing with the **measurement of those shapes**. Finally, we must thoroughly develop the principles of inductive and deductive reasoning, giving significant attention to the dynamics of mathematical deductive logic, which are the building blocks that students will use to **construct formal proofs**.

**In Unit II, we begin the actual study of “Plane Geometry” by developing all of the necessary terms, definitions, and assumptions we will be using as a basis for studying geometric relationships**. In other words, we draw on the analogy that studying any area of Mathematics is like “playing a game”. We must first determine **which basic elements will be “undefined”** in our Geometry, or accepted



without definition. We must then determine which basic elements can be formally defined, using those undefined terms. Finally, we must **build a list of “postulates”, or conditional assumptions** which will serve as the “rules of the game”, guiding us through the investigation of relationships, in our Geometry. It is important to note, at this point, that every Plane Geometry study will, in certain ways, be unique to the philosophy of the instructor, depending on the acceptance of these fundamental terms. In other words, while the prevailing context will always be that of classical Euclidean Geometry, the lists of definitions and postulates may differ from person to person. The key, however, is that each study will rely on its own particular list of Essential Elements to prove the rest of the relationships to be investigated.

**So, in Unit III, we use the Fundamental Terms developed in Unit II, to prove Fundamental Theorems related to points, lines, rays, segments, and angles.** These theorems will be foundational to the study of Simple Closed Plane Curves, which are the primary backdrop of all studies of Plane Geometry.

At this point, since we have put in place the “rules of the game”, we can begin, and, for all practical purposes, complete, a methodic investigation of the **geometric relationships associated with Triangles (Unit IV), Other Polygons (Unit V), and Circles (Unit VI).** That then allows us to conclude our study by the investigation of several applications, internal to the study of Geometry.

**First, in Unit VII, we will engage in the classic geometric exploration of “Construction”.** This means that, with the use of only a **straight edge** (to construct lines, rays, and segments), and a **compass** (to construct circles, and arcs of circles), we will attempt to use our knowledge of geometric relationships to “build”, and “operate on”, various geometric shapes. Included will be the replication and division of line segments and angles, the building of polygons to desired specifications, and the generation of circles to desired specifications.

**Second, in Unit VIII, we will examine, in significant detail, the relationships between the various components of triangles.** This is, of course, the study of **Trigonometry**, from the Greek, meaning “tri-angle-measure”. Included are the basic relationships of **sine, cosine, and tangent**, as well as applications involving the Pythagorean Theorem, the Laws of Sines and Cosines, and several other ambiguous cases.

Please understand that the organizational argument presented here is not meant to stifle the creativity of the instructor. Neither should it prohibit the instructor from utilizing a modular approach to concept development. It does, however, serve to remedy the fragmented, isolated topic, “chapter” approach to a subject which has been traditionally presented to us in “textbooks”, without that element of developmental continuity. To that end, it speaks loudly to the curricular issues which all instructors face, and the attitudinal issues students deal with when they are presented with a new and different Mathematics course.

## Quiz Form B

Name \_\_\_\_\_

Class \_\_\_\_\_

Date \_\_\_\_\_

Score \_\_\_\_\_

### Unit II - Fundamental Terms

#### Part A - Undefined Terms

#### **Lesson 1 - In Algebra**

#### **Lesson 2 - In Geometry**

---

From the list of five mathematical parts of speech and four types of mathematical expressions, identify each of the items in problems 1 through 9 below, using the most appropriate term.

1.  $7n + 5$

Open Phrase

2. The fraction bar in  $\frac{5 + 4 + 7}{2}$

Technically it is part of the Number Symbol. But it might also be considered a Grouping Symbol or an Operation Symbol.

3.  $\sqrt{5}$

Number Symbol

4.  $\neq$

Relation Symbol

5.  $2x - 3 = 7$

Open Sentence

6. The fraction bar in  $\frac{5}{4}$

Technically it is part of the Number Symbol.  
But it might also be considered as an Operation Symbol.

7.  $2 + 9 = 5 \div 2$

Closed Sentence

8. The “ $m$ ” in  $2m + 5$

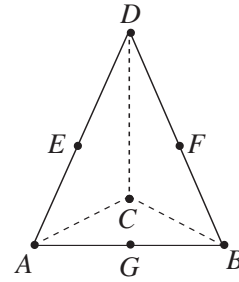
Placeholder Symbol

9.  $(8)(3) + 15$

Closed Phrase

## —Continued—

3. Refer to the figure at the right to identify each of the following sets of points as coplanar or non-coplanar.



- a) F, G, A, and C  
 b) D, B, A, and C  
 c) C, G, A, and B  
 d) A, C, E, and D

Non-Coplanar

Non-Coplanar

Coplanar

Coplanar

4. For each of the given sets of numbers, which number is between the other two?

a)  $\{ 1.8, (1.3)^2, \sqrt{3} \}$

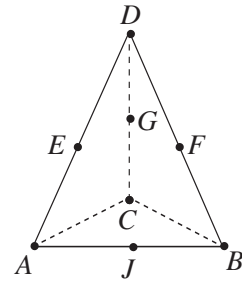
$\sqrt{3}$

b)  $\{ .713, \frac{7}{11}, \frac{3}{4} \}$

.713

—Continued—

3. Refer to the figure at the right to identify each of the following sets of points as coplanar or non-coplanar.



- a) F, G, E, and C  
 b) D, F, G, and B  
 c) A, J, E, and D  
 d) B, F, D, and E

Non-Coplanar

Coplanar

Coplanar

Coplanar

4. For each of the given sets of numbers, which number is between the other two?

a)  $\left\{ \frac{2}{3}, \frac{3}{4}, \frac{4}{5} \right\}$

$$\frac{3}{4}$$

b)  $\{ .615, .636, .513 \}$

$$.615$$

## Unit II - Fundamental Terms

### Part B - Defined Terms

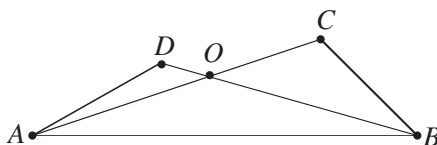
#### Lesson 3 - Definitions about Lines

#### Lesson 4 - Definitions about Rays

#### Lesson 5 - Definitions about Line Segments

1. How many endpoints does a line segment have? A ray? A line? \_\_\_\_\_ 2, 1, None

2. Name the nine line segments in the following diagram. \_\_\_\_\_  $\overline{AB}, \overline{BC}, \overline{AC}, \overline{AD}, \overline{DO}, \overline{DB}, \overline{AO}, \overline{OC}, \overline{BO}$



3. How many rays on a line may have a common end point? \_\_\_\_\_ 2

4. How many line segments may be found on a line? \_\_\_\_\_ An infinite number

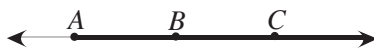
5. How many line segments on a line may have a common endpoint? \_\_\_\_\_ An infinite number

6. Two line segments lie on the same line. Can we then say that the union of the two line segments must be a line segment? (yes or no) Sketch an illustration for your answer. \_\_\_\_\_

No,  $\overline{AB} \cup \overline{CD} = \{\text{all points on } \overline{AB} \text{ or } \overline{CD} \text{ or both}\}$



7. Two rays lie on the same line. Can we then say that the union of the two rays might be a ray? (yes or no) Sketch an illustration for your answer. \_\_\_\_\_ Yes,  $\overrightarrow{AB} \cup \overrightarrow{BC} = \overrightarrow{AC}$



8. Two rays lie on the same line. Can we then say that the intersection of the two rays must be a line segment? (yes or no) Sketch an illustration for your answer. \_\_\_\_\_ No,  $\overrightarrow{AB} \cap \overrightarrow{AC} = A \text{ (a point)}$



—Continued—

9. Is it true that the union or intersection of two sets of points on a line must be a subset of the line?  
(yes or no) Explain your answer. Yes, even the empty set (if the two sets have no points in common) is a subset of every set.

10. For each of the following, if you are given the coordinates  $c$ ,  $d$ ,  $s$ , and  $t$ , for the points  $C$ ,  $D$ ,  $S$ , and  $T$  respectively, determine if  $\overline{CD} \cong \overline{ST}$ . Show your work.

	$c$	$d$	$s$	$t$		
a)	7	12	-8	-13	<u>Yes</u>	$\begin{array}{l}  12 - 7  \\  -13 - (-8)  \\ 5 \end{array}$
b)	$3\frac{1}{2}$	$-7\frac{1}{4}$	-1	$9\frac{3}{4}$	<u>Yes</u>	$\begin{array}{l} \left  -7\frac{1}{4} - 3\frac{1}{2} \right  \\ \left  9\frac{3}{4} - (-1) \right  \\ 10\frac{3}{4} \end{array}$
c)	$a - b$	$a + b$	$3b$	$5b$	<u>Yes</u>	$\begin{array}{l}  (a + b) - (a - b)  \\  5b - 3b  \\ 2b \end{array}$

11. Find the coordinate of the midpoint  $C$  of a line segment, if the coordinates of the endpoints are  $5x^2$  and  $13x^2$ .  
(Hint: Think about a number line)

$$\frac{|5x^2 - 13x^2|}{2} = \frac{8x^2}{2} = 4x^2 \quad \text{So, the coordinate of the midpoint } C \text{ is } 5x^2 + 4x^2 \text{ or } 9x^2$$

12. What is meant if we write  $AB = CD$ ? We mean that  $AB$  and  $CD$  represent the same measure (number); that is, the length of  $\overline{AB}$  is the same as the length of  $\overline{CD}$ . Here  $AB$  and  $CD$  are symbols or names for the same number.

## —Continued—

9. What is the basic difference between a ray and each of the following?

a) A line segment A ray has only one endpoint, while a line segment has two endpoints.

b) A half line A ray has one endpoint and a half-line has no endpoint (technically). It may be argued that there is no difference, since, to draw a half-line, you must start somewhere, but the strict definition of a half-line does not allow for an endpoint

c) A line A ray has one endpoint, while a line has no endpoints.

10. For each of the following, if you are given the coordinates c, d, s, and t, for the points C, D, S, and T respectively, determine if  $\overline{CD} \cong \overline{ST}$ ? Show your work.

	c	d	s	t		
a)	6	9	14	17	<u>Yes</u>	$ 9 - 6 $ $ 17 - 14 $ 3
b)	4.72	8.35	5.15	9.78	<u>No</u>	$ 8.35 - 4.72 $ 3.63 $ 9.78 - 5.15 $ 4.63
c)	$4\sqrt{7}$	$9\sqrt{7}$	0	$\sqrt{175}$	<u>Yes</u>	$ 9\sqrt{7} - 4\sqrt{7} $ $5\sqrt{7}$ $ \sqrt{175} - 0 $ $\sqrt{175}$ $\sqrt{25} \cdot \sqrt{7}$ $5\sqrt{7}$

—Continued—

11.  $AB = 7x - 2$ ,  $BC = 2x + 8$ , and B is the midpoint of  $\overline{AC}$ . Find the values of  $x$ ,  $AB$ ,  $BC$ , and  $AC$ .

$$\begin{array}{ll} 7x - 2 = 2x + 8 & \\ 7x - 2 + 2 = 2x + 8 + 2 & AB = 7(2) - 2 = 12 \\ 7x - 2x = 2x + 10 - 2x & BC = 2(2) + 8 = 12 \\ 5x = 10 & AC = 12 + 12 = 24 \\ x = 2 & \end{array}$$

12. May we write  $\overline{AB} = \overline{CD}$ ? Explain your answer. No. Even if the lengths of  $\overline{AB}$  and  $\overline{CD}$  are equal, you cannot say that the line segments, as sets of points, are equal. You would have to say that the line segments are congruent ( $\cong$ ). In fact, the only way to use the equality symbol, would be to place a lower-case  $m$  in front of the line segments, indicating that the measures of the line segments are equal ( $m\overline{AB} = m\overline{CD}$ ), or to remove the segments from over the letters, also indicating that the measures are equal ( $AB = CD$ ).
- \_\_\_\_\_
- \_\_\_\_\_



## Unit II - Fundamental Terms

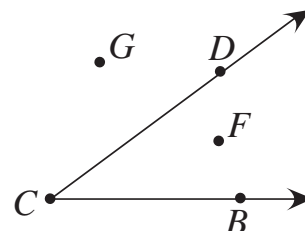
### Part B - Defined Terms

### Lesson 6 - Definitions about Angles as Sets of Points

### Lesson 7 - Definitions about Measurement of Angles

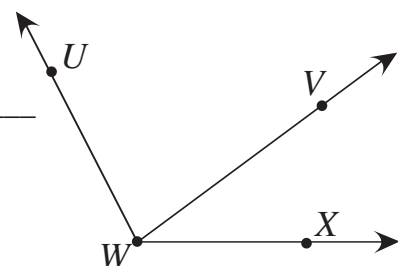
For exercises 1 through 5, refer to the angle at the right.

1. Name the angle in two ways.  $\angle BCD$ ;  $\angle DCB$
2. Name the two sides of the angle.  $\vec{CB}$ ;  $\vec{CD}$
3. Name the vertex of the angle. Point C
4. Point F is in the interior of the angle.
5. Point G is in the exterior of the angle.



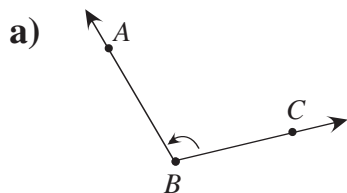
For exercises 6 through 9, refer to the figure at the right.

6. How many angles are there, in the figure? 3
7. Name the angles in the figure.  $\angle UWX$ ;  $\angle VWX$ ;  $\angle VWU$
8. Name the vertex of the angle. Point W
9. Is  $\vec{WV}$  totally in the interior of angle UWX? Explain your answer. No; the endpoint of the ray, point W, is on the angle, not in the interior of the angle.
10. In defining an angle, is it possible that the rays forming the angle might have different endpoints?  
No
11. May a page of a book serve as a model of a half plane? A perfect model of a half-plane? Explain why or why not. Yes, it may serve as a model, but not as a perfect model. A page of a book is flat, which is a required quality of a half-plane, but it has 4 edges, and a half-plane has only one edge, defined by the separation line.



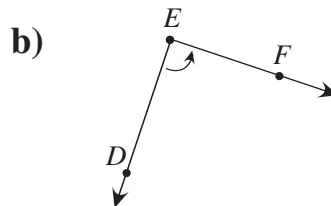
—Continued—

12. Use your protractor to measure each angle below. Write your answer, being sure to use correct notation.



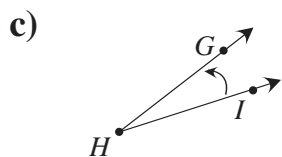
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$$m\angle ABC = 107^\circ$$



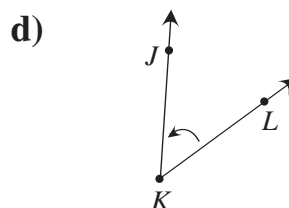
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$$m\angle DEF = 90^\circ$$



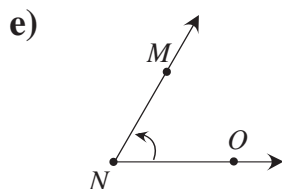
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$$m\angle GHI = 20^\circ$$



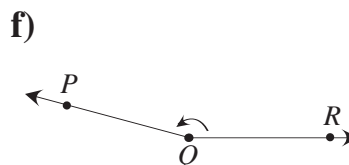
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$$m\angle JKL = 50^\circ$$



---


$$m\angle MNO = 60^\circ$$



---


$$m\angle PQR = 165^\circ$$

13. Name two examples of different size angles in your classroom. *(answers will vary)* (1) Corner of door forms a  
90° angle. (2) Pole holding American flag forms about a 60° angle with the wall.

## Unit II - Fundamental Terms

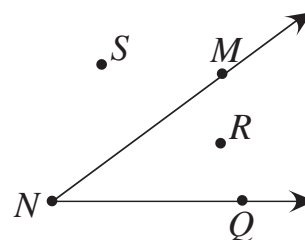
### Part B - Defined Terms

### Lesson 6 - Definitions about Angles as Sets of Points

### Lesson 7 - Definitions about Measurement of Angles

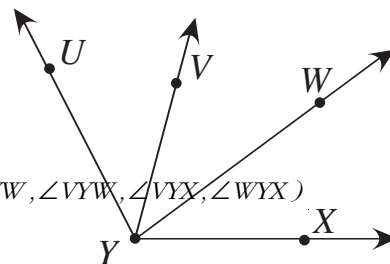
- Suppose that two half-planes in the same plane have the same edge. Describe their union and intersection. Their union is a complete plane, less the edge. Their intersection is empty since the edge is not included. The edge is not a part of either half-plane.
- May two consecutive pages of a book serve as a model of a dihedral angle? A perfect model of a dihedral angle? Explain why or why not. Yes, it may serve as a model, but not as a perfect model. While both pages are flat, which is a required quality of half-planes, they both have several edges. A half-plane has only one edge, defined by the separation line.

For exercises 3 through 6, refer to the figure at the right.



- Point R is in the interior of angle MNQ.
- Point Q is on angle MNQ.
- Point S is in the exterior of angle MNQ.
- Point N is the vertex of angle MNQ.

For exercises 7 through 11, refer to the figure at the right.



- $\overrightarrow{YV}$  forms an angle with ray  $YU$  and ray  $YW$ . (and ray  $YX$ )
- How many angles are in the figure? 6; ( $\angle UYX, \angle UYV, \angle UYW, \angle VYW, \angle VYX, \angle WYX$ )  
( $\angle UYX, \angle UYV, \angle UYW, \angle VYW, \angle VYX, \angle WYX$ )
- Name angle  $WYU$  another way.  $\angle UYW$
- Point V is in the interior of angle  $UYW$  and angle  $UYX$ .
- Point W is in the interior of angle  $UYX$  (also  $VYX$ ) but in the exterior of angle  $UYV$ .

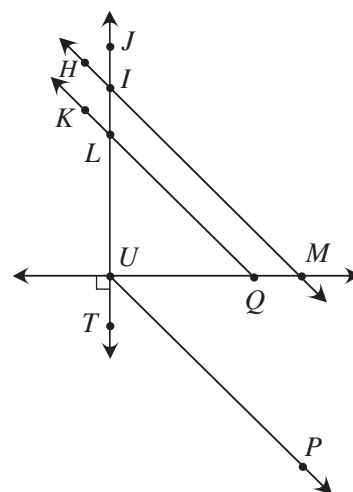
## Unit II - Fundamental Terms

### Part B - Defined Terms

### Lesson 8 - Definitions about Pairs of Angles

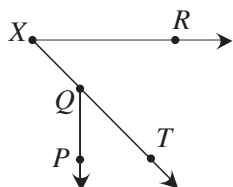
Complete each statement in exercises 1 through 6, using the figure at the right.

- $\angle HIJ$  and  $\angle TIM$  or  $\angle LIM$  are vertical angles.
- $\angle MQL$  and  $\angle UQL$  are adjacent angles.
- $\angle JIM$  and  $\angle TIM$  or  $\angle JIH$  are supplementary angles.
- $m\angle MUJ = 90^\circ$ .
- The complement of  $\angle TUP$  is  $\angle PUM$ .
- $\angle KLT$  forms a linear pair with  $\angle TLQ$  (also  $\angle KLJ$ ).



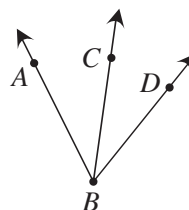
In exercises 7 and 8, explain why the angles in each pair are not adjacent.

7.  $\angle RXT$  and  $\angle TQP$



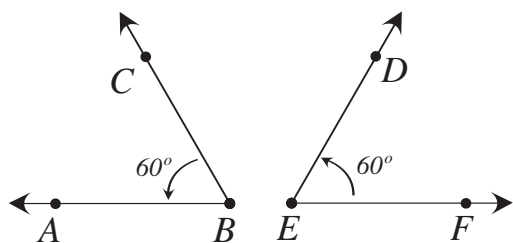
There is no common vertex, and only part of a  
common ray.

8.  $\angle ABC$  and  $\angle ABD$



The common ray,  $\vec{AB}$ , is not between  $\vec{CB}$  and  $\vec{BD}$ .

9. What is the measure of the supplement, of the complement, of a 35 degree angle? (Hint: Make a drawing)  
The complement of  $35^\circ$  is  $55^\circ$ . The supplement of 55 is  $125^\circ$ .



Use the figures above to determine whether each statement in problems 10 through 15 is true or false.

False 10.  $\angle DEF = 60^\circ$

True 11.  $m\angle ABC = 60^\circ$

False 12.  $\angle ABC = \angle DEF$

False 13.  $m\angle ABC \cong m\angle DEF$

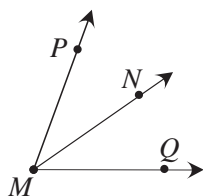
True 14.  $m\angle DEF = 60^\circ$

True 15.  $\angle ABC \cong \angle DEF$

16.  $\overrightarrow{MN}$  bisects  $\angle PMQ$

$$m\angle PMN = 3x + 8$$

$$m\angle QMN = 5x - 10$$



Find the value of  $x$ ,  
and the measure of  
each angle.

$$3x + 8 = 5x - 10$$

$$3x + 8 - 3x = 5x - 10 - 3x$$

$$0 + 8 = 2x - 10$$

$$8 + 10 = 2x - 10 + 10$$

$$18 = 2x$$

$$\frac{1}{2} \cdot 18 = \frac{1}{2} \cdot 2x$$

$$9 = x$$

$$m\angle PMN = 3x + 8$$

$$= 3(9) + 8$$

$$= 27 + 8$$

$$= 35$$

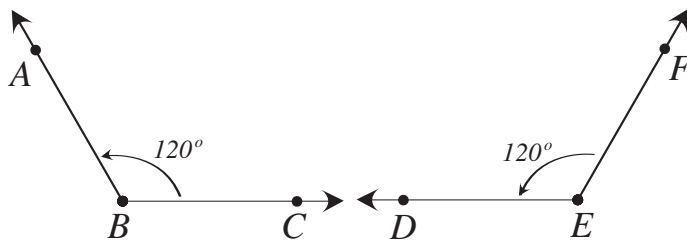
$$m\angle QMN = 5x - 10$$

$$= 5(9) - 10$$

$$= 45 - 10$$

$$= 35$$

—Continued—



The statements in problems 10 through 15 refer to the figures above. The statements are false. Rewrite each statement in the correct form, in the blank to the right.

10.  $\angle ABC = 120^\circ$   $m\angle ABC = 120^\circ$

11.  $\angle ABC = \angle DEF$   $\angle ABC \cong \angle DEF$

12.  $m\angle ABC \cong m\angle DEF$   $m\angle ABC = m\angle DEF$

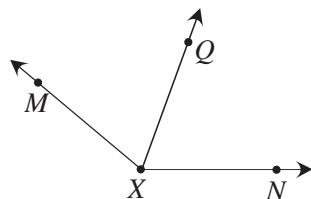
13.  $\angle DEF \cong 120^\circ$   $m\angle DEF = 120^\circ$

14.  $m\angle ABC \cong 120^\circ$   $m\angle ABC = 120^\circ$

15.  $\angle DEF = \angle ABC$   $\angle DEF \cong \angle ABC$  or  $m\angle DEF = m\angle ABC$

16.  $\overrightarrow{XQ}$  bisects  $\angle MXN$   
 $m\angle MXQ = \frac{1}{2}x + 20$   
 $m\angle NXQ = 70$

Find the value of  $x$ ,  
 and the measure of  
 $\angle MXQ$ .



$$\frac{1}{2}x + 20 = 70$$

$$\frac{1}{2}x + 20 - 20 = 70 - 20$$

$$\frac{1}{2}x + 0 = 50$$

$$2 \cdot \frac{1}{2}x = 2 \cdot 50$$

$$x = 100$$

$$\begin{aligned} m\angle MXQ &= \frac{1}{2}x + 20 \\ &= \frac{1}{2}(100) + 20 \\ &= 50 + 20 \\ &= 70 \end{aligned}$$

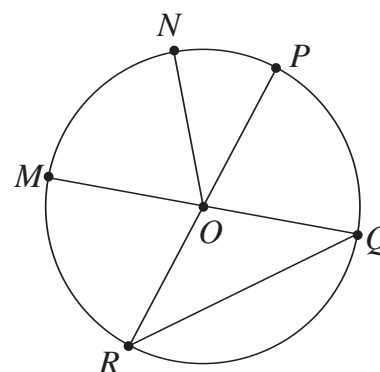
## Unit II - Fundamental Terms

### Part B - Defined Terms

### Lesson 9 - Definitions about Circles

In exercises 1 through 6, refer to the diagram at the right to determine whether the arc is a minor arc, a major arc or a semi circle of circle O.

1. RQ \_\_\_\_\_ *minor arc*
2. MRP \_\_\_\_\_ *major arc*
3. MNQ \_\_\_\_\_ *semi circle*
4. NP \_\_\_\_\_ *minor arc*
5. RN \_\_\_\_\_ *minor arc*
6. RQP \_\_\_\_\_ *semi circle*



7. In the figure for exercises 1 through 6,  $\angle NOP$  is called a \_\_\_\_\_ *central* \_\_\_\_\_ angle.
8. In the figure for exercises 1 through 6,  $\overline{OQ}$  is a \_\_\_\_\_ *radius* \_\_\_\_\_ of the circle,  $\overline{RP}$  is a \_\_\_\_\_ *diameter* \_\_\_\_\_ of the circle, point O is the \_\_\_\_\_ *center* \_\_\_\_\_ of the circle, and  $\overline{RQ}$  is called a \_\_\_\_\_ *chord* \_\_\_\_\_ of the circle.
9. Tell what fractional part of a circle is represented by the following arcs:

a) an arc of  $36^\circ$  \_\_\_\_\_  $\frac{36}{360} = \frac{36 \cdot 1}{36 \cdot 10} = \frac{1}{10}$  or *one-tenth*

b) an arc of  $160^\circ$  \_\_\_\_\_  $\frac{160}{360} = \frac{40 \cdot 4}{40 \cdot 9} = \frac{4}{9}$  or *four-ninths*

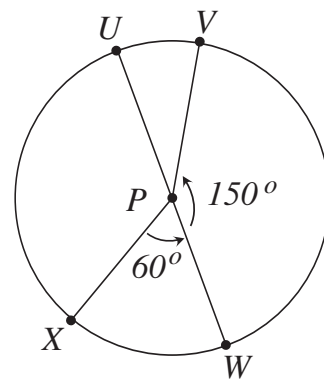
10. If an arc is  $\frac{3}{20}$  of a circle, it measures \_\_\_\_\_ *54* \_\_\_\_\_ degrees.

$$\frac{3}{20} \cdot \frac{360}{1} = \frac{3}{20} \cdot \frac{20 \cdot 18}{1} = 3 \cdot 18 = 54^\circ$$

## —Continued—

In exercise 11 through 18, refer to the figure at the right to determine the measure of each arc or angle.

- |                      |                         |
|----------------------|-------------------------|
| 11. $m\angle UPX$    | _____ $120^\circ$ _____ |
| 12. $m\widehat{XW}$  | _____ $60^\circ$ _____  |
| 13. $m\angle VPU$    | _____ $30^\circ$ _____  |
| 14. $m\widehat{VX}$  | _____ $150^\circ$ _____ |
| 15. $m\widehat{VW}$  | _____ $150^\circ$ _____ |
| 16. $m\widehat{UVW}$ | _____ $180^\circ$ _____ |
| 17. $m\widehat{UV}$  | _____ $30^\circ$ _____  |
| 18. $m\widehat{VWX}$ | _____ $210^\circ$ _____ |





## Unit II - Fundamental Terms

### Part B - Defined Terms

### Lesson 9 - Definitions about Circles

1. If two arcs of a circle are equal, are their central angles equal? Yes

2. If an arc of a circle is halved, is its central angle halved? Yes

In exercises 3 through 10, refer to the diagram at the right.

3. Name four central angles of circle P.  $\angle EPB$ ;  $\angle BPC$ ;  $\angle CPG$ ;  $\angle GPA$   
(other answers possible)

4. Name three minor arcs of circle P.  $\widehat{GC}$ ;  $\widehat{CB}$ ;  $\widehat{BE}$   
(other answers possible)

5. Name two major arcs of circle P.  $\widehat{EBA}$ ;  $\widehat{AEC}$   
(other answers possible)

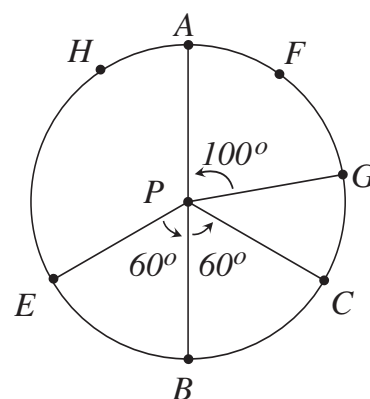
6. Name a semi circle of circle P.  $\widehat{AFB}$  (other answers possible)

7. Name a pair of arcs you think are congruent.  $\widehat{BE} \cong \widehat{BC}$

8. What is  $m\widehat{BC}$ ?  $60^\circ$

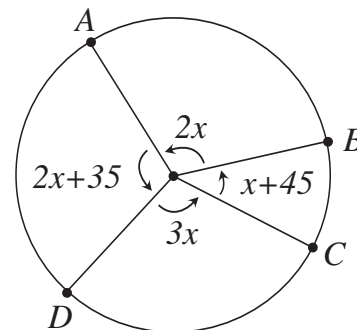
9. What is  $m\widehat{BG}$ ?  $80^\circ$

10. What is  $m\widehat{CEA}$ ?  $240^\circ$



## —Continued—

In exercises 11 through 14, refer to the diagram at the right to find the measures of the indicated arcs.



$$\begin{aligned}
 11. \quad m\widehat{AD} &= \underline{105^\circ} & m\widehat{AD} &= 2x+35 \\
 & & &= 2(35)+35 \\
 & & &= 70+35 \\
 & & &= 105^\circ
 \end{aligned}$$

$$\begin{aligned}
 12. \quad m\widehat{AB} &= \underline{70^\circ} & m\widehat{AB} &= 2x \\
 & & &= 2(35) \\
 & & &= 70^\circ
 \end{aligned}$$

$$\begin{aligned}
 2x + (2x + 35) + 3x + (x + 45) &= 360^\circ \\
 8x + 80 &= 360 \\
 8x + 80 - 80 &= 360 - 80 \\
 8x + 0 &= 280 \\
 8x &= 280 \\
 \frac{1}{8} \cdot 8x &= \frac{1}{8} \cdot 280 \\
 1 \cdot x &= 35^\circ
 \end{aligned}$$

$$\begin{aligned}
 13. \quad m\widehat{DAC} &= \underline{255^\circ} & m\widehat{DAC} &= m\widehat{DA} + m\widehat{AB} + m\widehat{BC} \\
 & & &= 2x + 35 + 2x + x + 45 \\
 & & &= 5x + 80 \\
 & & &= 5(35) + 80 \\
 & & &= 175 + 80 \\
 & & &= 255^\circ
 \end{aligned}$$

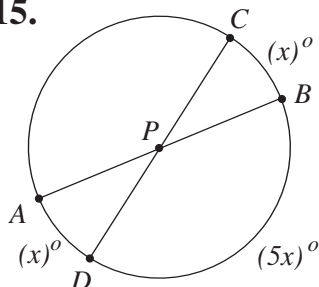
$$\begin{aligned}
 14. \quad m\widehat{CDA} &= \underline{210^\circ} & m\widehat{CDA} &= m\widehat{CD} + m\widehat{DA} \\
 & & &= 3x + 2x + 35 \\
 & & &= 3(35) + 2(35) + 35 \\
 & & &= 105 + 70 + 35 \\
 & & &= 210^\circ
 \end{aligned}$$

**Unit II, Part B, Lesson 9, Quiz Form B**  
**—Continued—**

Name \_\_\_\_\_

In exercises 15 through 18, refer to circle P to find the value of  $x$ . Then find the measure of the labeled arcs.

**15.**



$$x + 5x + x + 5x = 360$$

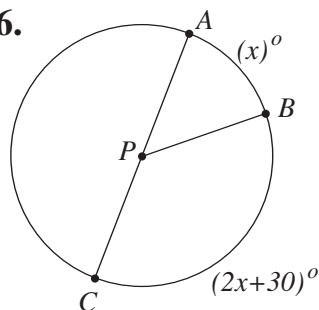
$$12x = 360$$

$$x = 30$$

$$m\widehat{AD} = m\widehat{CB} = x = 30^\circ$$

$$m\widehat{DB} = 5x = 150^\circ$$

**16.**



$$x + 2x + 30 = 180$$

$$3x + 30 = 180$$

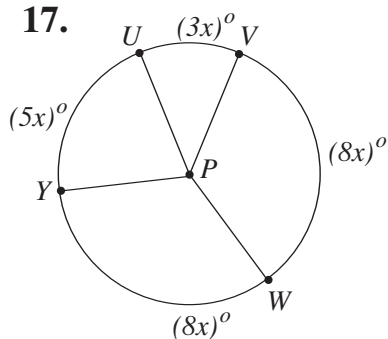
$$3x = 150$$

$$x = 50$$

$$m\widehat{AB} = x = 50^\circ$$

$$m\widehat{BC} = 2x + 30 = 2(50) + 30 = 130^\circ$$

**17.**



$$3x + 8x + 8x + 5x = 360$$

$$24x = 360$$

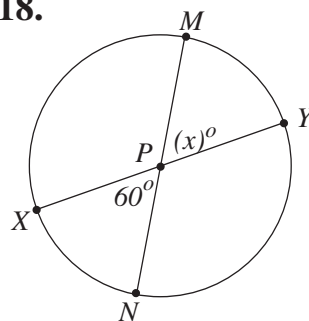
$$x = 15$$

$$m\widehat{UV} = 3x = 3(15) = 45^\circ$$

$$m\widehat{VW} = m\widehat{WY} = 8x = 8(15) = 120^\circ$$

$$m\widehat{YU} = 5x = 5(15) = 75^\circ$$

**18.**



$$x = 60$$

$$m\widehat{NX} = m\widehat{MY} = x = 60^\circ$$

$$m\widehat{NY} = m\widehat{MX} = 180 - x = 120^\circ$$

4. State the missing reason (mathematical property) for each step in the following proof.

	$\frac{3}{4} \cdot x = \frac{1}{3}$	Assumed to be true
Step 1	$\frac{4}{3} \cdot \left( \frac{3}{4} \cdot x \right) = \frac{4}{3} \cdot \frac{1}{3}$	<u>Multiplication Property of Equality</u>
Step 2	$\left( \frac{4}{3} \cdot \frac{3}{4} \right) \cdot x = \frac{4}{3} \cdot \frac{1}{3}$	<u>Associative Property of Multiplication</u>
Step 3	$1 \cdot x = \frac{4}{9}$	<u>Property of Multiplicative Inverse</u>
Step 4	$x = \frac{4}{9}$	<u>Property of One for Multiplication</u>

5. Which of the following statements are true for all real numbers a and b?  
If false, give a numerical example to show it is false.

a) If  $a < b$ , then  $a + b < 0$ .

*False*

$$\begin{array}{ll} a < b & a + b < 0 \\ 3 < 5 & 3 + 5 < 0 \\ & 8 < 0 \text{ (not true)} \end{array}$$

b) If  $a < b$ , then  $a \cdot b > 0$ .

*False*

$$\begin{array}{ll} a < b & a \cdot b > 0 \\ -3 < 4 & (-3) \cdot (4) > 0 \\ & -12 > 0 \text{ (not true)} \end{array}$$

## Unit II - Fundamental Terms

### Part C - Postulates (or Axioms)

### Lesson 1 - Need

1. Name the property of equality (reflexive, symmetric, or transitive) that allows you to conclude that each statement is true.

a)  $MN = MN$

Reflexive Property of Equality

b) If  $x = 3$ , then  $3 = x$

Symmetric Property of Equality

c) If  $m\angle R = m\angle S$  and  $m\angle S = m\angle T$ ,  
then  $m\angle R = m\angle T$

Transitive Property of Equality

2. Write a paragraph proof to show that the following is true: If  $\frac{x}{5} - 9 = 16$ , then  $x = 125$

Since  $\frac{x}{5} - 9 = 16$ , it follows that  $\frac{x}{5} = 25$  by the addition property for equality, (add 9 to both sides of equation). Multiplying both sides by 5, the multiplication property for equality leads to  $x = 125$ .

3. Use the given property in each of the following to draw a conclusion.

a) Transitive Property: If  $a = b$  and  $b = c$ , then  $a = c$

$m\angle A = m\angle C$ ,  $m\angle C = 61$

Conclusion:  $m\angle A = 61$

b) Symmetric Property: If  $m = n$ , then  $n = m$ .

$AB = CD$

Conclusion:  $CD = AB$

c) Multiplication Property: If  $a = b$  and  $c = d$ , then  $a \cdot c = b \cdot d$

$4(x + 7) = 52$ ,  $\frac{1}{4} = .25$

Conclusion:  $x + 7 = 13$

**Unit II, Part C, Lesson 1, Quiz Form B**  
**—Continued—**

Name \_\_\_\_\_

4. State the missing reason (mathematical property) for each step in the following proof.

$$-2x < 8$$

Assumed to be true

**Step 1**  $\frac{1}{-2}(-2x) > \frac{1}{-2} \cdot 8$

Negative Multiplication Property for Inequality

**Step 2**  $\left(\frac{1}{-2} \cdot -2\right) \cdot x > \frac{1}{-2} \cdot 8$

Associative Property of Multiplication

**Step 3**  $1 \cdot x > -4$

Property of Multiplicative Inverse

**Step 4**  $x > -4$

Property of One for Multiplication

5. Which of the following statements are true for all real numbers a and b?

If false, give a numerical example to show it is false.

**a)** If  $a < b$ , then  $a - b > 0$ .

*False*

$$\begin{array}{ll} a < b & a - b < 0 \\ 3 < 5 & 3 - 5 < 0 \\ & -2 < 0 \text{ (not true)} \end{array}$$

**b)** If  $a < b$ , then  $a \div b > 0$ . ( $b \neq 0$ )

*False*

$$\begin{array}{ll} a < b & a \div b > 0 \text{ (} b \neq 0 \text{)} \\ -3 < 5 & -3 \div 5 > 0 \\ & -\frac{3}{5} > 0 \text{ (not true)} \end{array}$$

## Unit II - Fundamental Terms

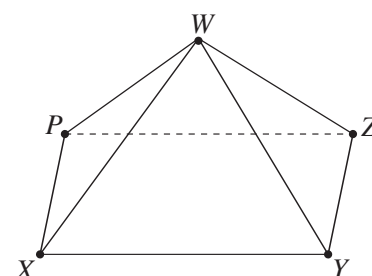
### Part C - Postulates (or Axioms)

#### Lesson 2 - Postulate 1 (Existence of Points)

#### Lesson 3 - Postulate 2 (Uniqueness of Lines, Planes and Spaces)

#### Lesson 4 - Postulate 3 (One, Two, and Three Dimensions)

In the given figure, points X, Y, P and Z lie in the same plane with point W not in plane XYPZ. Give a reason for each statement in problems 1- 5.



1. Exactly one plane contains W, X, and Y.

*If there is a plane, then there exist at least 3 different, non-collinear points on the plane.*

\_\_\_\_\_

2. Points Z and P determine a line.

*If we have 2 different points, then we have exactly one line containing those two points.*

\_\_\_\_\_

3. Any plane containing X and Y must contain  $\overleftrightarrow{XY}$ .

*If those points are in a plane, then the line containing them must be in the plane.*

\_\_\_\_\_

4. Point X is not on  $\overleftrightarrow{YZ}$ .

*For any line in a plane, there is at least one point in the plane that is not on the line.*

\_\_\_\_\_

5. Point X is not on plane WYZ.

*For every plane in space, there is at least one point in space that is not on the plane.*

\_\_\_\_\_

## Quiz Form B

Name \_\_\_\_\_

Class	Date	Score
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Date	Score
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Score \_\_\_\_\_

## Unit II - Fundamental Terms

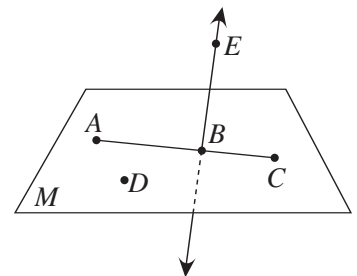
## Part C - Postulates (or Axioms)

## Lesson 2 - Postulate 1 (Existence of Points)

## Lesson 3 - Postulate 2 (Uniqueness of Lines, Planes and Spaces)

## Lesson 4 - Postulate 3 (One, Two, and Three Dimensions)

In the given figure, Plane M contains point D, and collinear points A, B, and C. Point E is not in Plane M. Give a reason for each statement in problems 1 - 5.



- 1.** There is exactly one plane containing points E, B, and A.

*There is exactly one plane through three different, non-collinear points.*

- 2.** There is exactly one line containing points D and A.

*There is exactly one line through two different points.*

- 3.** Line DC is in Plane M.

*For any two different points in a plane, the line containing them is in the plane.*

- 4.** Point D is not on  $\overleftrightarrow{AC}$ .

*For any line in a plane, there is at least one point in the plane that is not on the line.*

- 5.** Point D is not on plane ACE.

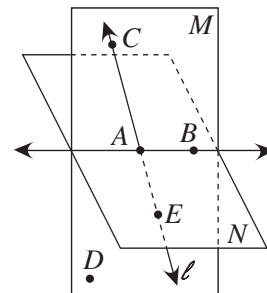
*For every plane in space, there is at least one point in space that is not on the plane.*



## —Continued—

In the given figure, plane M intersects plane N. Use this information to answer problems 6-14.

6. Name two points that determine line  $\ell$ .  $C$  and  $E$
7. Name three points that determine plane M.  $C$ ,  $E$ , and  $B$
8. Name the intersection of plane M and plane N.  $\overleftrightarrow{AB}$
9. Does  $\overleftrightarrow{AD}$  lie on plane M?  $Yes$
10. Does plane N contain any points not on  $\overleftrightarrow{AB}$ ?  $Yes$
11. Only one  $plane$  contains  $\overleftrightarrow{AB}$  and point D.
12. The plane containing points A, B, and C also contains point  $D$ .
13. Line CE and line AB intersect at point  $A$ .
14. Line CE and line AB determine plane  $M$ .



In problems 15-18, tell how many planes at most can pass through each set of points.

15. One line  $infinite$
16. Two points  $infinite$
17. Three collinear points  $infinite$
18. A line and a point not on the line  $one$

## Unit II - Fundamental Terms

### Part C - Postulates (or Axioms)

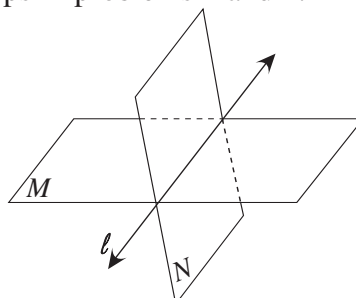
#### **Lesson 4 - Postulate 3 (One, Two, and Three Dimensions)**

#### **Lesson 5 - Postulate 4 (Separation of Lines, Planes and Spaces)**

#### **Lesson 6 - Postulate 5 (Intersection of Lines or Planes)**

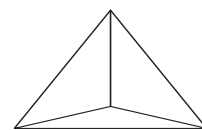
- Planes M and N are known to intersect. Describe the intersection of planes M and N? A line
- State the Postulate that supports your answer to problem 1. If two different planes intersect, then the intersection is a unique line.

- Sketch a diagram to illustrate the geometric relationships in problems 1 and 2.



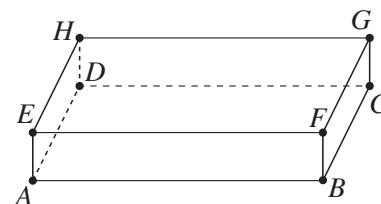
- Is it possible for three non-collinear points to all lie in each of two planes? Explain.  
No; The planes would have to be the same plane. From II-C-3, there is exactly one plane through three different, non-collinear points.
- Is it possible for two points to lie in plane X, two other points lie in a different plane Y, and for all four points to be coplanar but not collinear? Explain.  
Yes; The two points in Plane X determine a line. The two points in Plane Y determine a line. If the two lines are parallel, the 4 points will be coplanar.

- What is the greatest number of planes determined by four points, no three of which are collinear? 4 (see below)



- Two distinct lines intersect in at most 1 point.  
(1 point or 2 points)
- A plane is determined by three non-collinear points.  
(collinear or non-collinear)

To answer problems 9-12, you will have to visualize certain lines and planes not drawn specifically in the diagram.



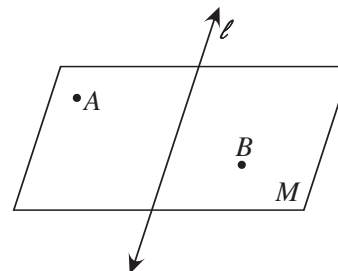
9. Name a plane that contains  $\overleftrightarrow{AC}$ . plane ABC

10. Name the intersection of plane DCFE and plane ABCD.  $\overleftrightarrow{DC}$

11. Name two lines that are not shown in the diagram and that do not intersect.  $\overleftrightarrow{AH}, \overleftrightarrow{BG}$   
Answers may vary

12. Name three planes that do not intersect  $\overleftrightarrow{EF}$ . plane DHG, plane ABC, plane ABG

In the figure at the right, line  $\ell$  separates plane M into two half-planes. Use this information to answer problem 13.



13. Point A and point B are in different half-planes. Justify this statement by using the appropriate part of the line separation postulate.

The line segment joining point A to point B will intersect separation

line in one point.

\_\_\_\_\_  
\_\_\_\_\_

## Unit II - Fundamental Terms

### Part C - Postulates (or Axioms)

#### Lesson 7 - Postulate 6 (Ruler)

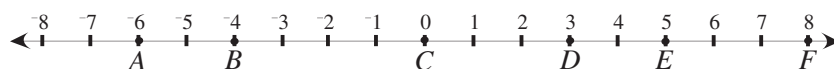
#### Lesson 8 - Postulate 7 (Protractor)

1. Complete the following relations:

a)  $|7| = \underline{7}$       c)  $|0| = \underline{0}$       e)  $|-19| - |-3| = \underline{16}$

b)  $|-7| = \underline{7}$       d)  $|6 - 9| = \underline{3}$       f)  $|9 \cdot (-6)| = \underline{54}$

2. Using the given number line, find the distance between each indicated pair of points.



a) B and A

$$\begin{aligned} & |-4 - (-6)| \\ & |2| = 2 \end{aligned}$$

b) A and C

$$\begin{aligned} & |0 - (-6)| \\ & |6| = 6 \end{aligned}$$

c) F and D

$$\begin{aligned} & |8 - 3| \\ & |5| = 5 \end{aligned}$$

d) C and F

$$\begin{aligned} & |0 - 8| \\ & |-8| = 8 \end{aligned}$$

e) C and D

$$\begin{aligned} & |0 - 3| \\ & |-3| = 3 \end{aligned}$$

f) E and A

$$\begin{aligned} & |5 - (-6)| \\ & |11| = 11 \end{aligned}$$

g) D and B

$$\begin{aligned} & |3 - (-4)| \\ & |7| = 7 \end{aligned}$$

h) A and F

$$\begin{aligned} & |-6 - 8| \\ & |-14| = 14 \end{aligned}$$

3. How many different points can be either to the right or to the left of a given point on a horizontal number line? An infinite number to the right; an infinite number to the left.

—Continued—

4. Suppose on line  $t$ , point P corresponds to the real number 2. How many different points exist on  $t$  such that the distance from P to each of these points is 12? 2 What real numbers correspond to these points? 14; -10 How many different points exist on  $t$  such that the distance from P to each of these points is a given real number  $q$ ? 2 What are the coordinates of these points?  $p + q$ ;  $p - q$

$$\begin{array}{l} 2; \quad 2 + 12 = 14; \quad 2; \quad p + q \\ 2 - 12 = -10 \quad p - q \end{array}$$

5. If  $AB = 6$  and  $AC = 38$ , is BC necessarily 32? Explain your answer. No; BC is 32 if B is between A and C.  
BC is 44 if A is between B and C.

6. In the given diagram of collinear points,  $PT = 20$ ,  $QS = 6$ , and  $PQ = QR = RS$ . Find each length asked for below.



a) QR

$$QR + RS = QS$$

$$QR + RS = QS = 6$$

$$RS + RS = 6$$

$$2 \cdot RS = 6$$

$$RS = 3$$

b) ST

$$20 - 3(3)$$

$$ST = 11$$

c) RT

$$RS + ST = RT$$

$$3 + 11 = RT$$

$$14 = RT$$

d) SP

$$SP = PQ + QR + RS$$

$$SP = 3 + 3 + 3$$

$$SP = 9$$

7. Measure the given line segment to the nearest millimeter.



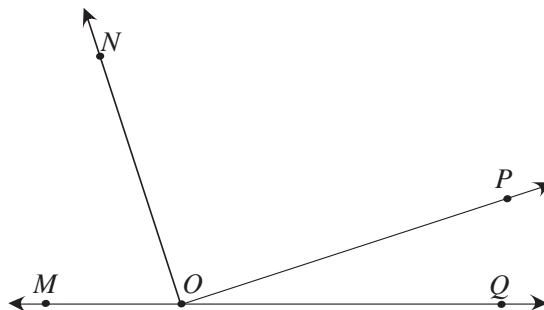
$$CD = \underline{54 \text{ mm}}$$

8. Use a protractor to determine the measure of each angle named below.

a)  $m\angle NOP = \underline{90^\circ}$

b)  $m\angle NOM = \underline{72^\circ}$

c)  $m\angle QON = \underline{108^\circ}$



—Continued—

9. If M is between L and N and  $LM = \frac{1}{2}x + 2$ ,  $MN = 3x + \frac{3}{2}$ , and  $LN = 5x + 2$ , find  $x$ ,  $LM$ ,  $MN$  and  $LN$  by using the Segment Addition Postulate.

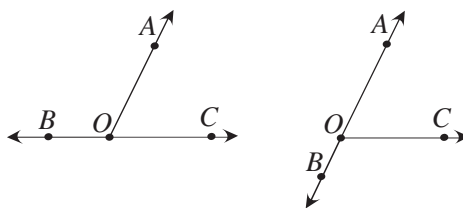
$$\begin{aligned} LM + MN &= LN \\ \left(\frac{1}{2}x + 2\right) + \left(3x + \frac{3}{2}\right) &= 5x + 2 \\ 3\frac{1}{2}x + 3\frac{1}{2} &= 5x + 2 \\ 1\frac{1}{2} &= 1\frac{1}{2}x \\ 1 &= x \end{aligned}$$

$$\begin{aligned} LM &= \frac{1}{2}(1) + 2 = 2\frac{1}{2} \\ MN &= 3(1) + \frac{3}{2} = 4\frac{1}{2} \\ LN &= 5(1) + 2 = 7 \end{aligned}$$

10. Explain why the Angle Addition Property begins with “If a ray OB lies between rays OA and OC,...”

Use a sketch of  $\angle AOB$  with ray OB not between rays OA and OC to illustrate what the relationship would be otherwise. If  $\vec{OB}$  was in the exterior of  $\angle AOC$  (in other words, not between  $\vec{OA}$  and  $\vec{OC}$ ), then

either  $\angle BOA$  or  $\angle BOC$  would be greater than  $\angle AOC$  and you couldn't add the measures properly.



## Unit II - Fundamental Terms

### Part C - Postulates (or Axioms)

#### Lesson 7 - Postulate 6 (Ruler)

#### Lesson 8 - Postulate 7 (Protractor)

1. Complete the following:

a)  $|0 - 9| = \underline{\quad 9 \quad}$       c)  $\left| \frac{3}{4} \right| = \underline{\quad \frac{3}{4} \quad}$       e)  $|-0.76| = \underline{\quad .76 \quad}$

b)  $|-5| + |3| = \underline{\quad 8 \quad}$       d)  $|(-5)(-3)| = \underline{\quad 15 \quad}$       f)  $|-13| = \underline{\quad 13 \quad}$

2. Find the distance between each pair of points having the given coordinates.

a) -3 and 10

$$\begin{aligned} |10 - (-3)| \\ |13| = 13 \end{aligned}$$

b) -2.4 and -3.1

$$\begin{aligned} |-3.1 - (-2.4)| \\ |-0.7| = 0.7 \end{aligned}$$

c)  $7\frac{1}{8}$  and  $-3\frac{1}{4}$

$$\begin{aligned} \left| -3\frac{1}{4} - 7\frac{1}{8} \right| \\ \left| \frac{-13}{4} - \frac{57}{8} \right| \\ \left| \frac{-26 + -57}{8} \right| \\ \left| \frac{-83}{8} \right| = \frac{83}{8} \text{ or } 10\frac{3}{8} \end{aligned}$$

d) 3.14 and 7.3

$$\begin{aligned} |7.3 - 3.14| \\ |4.16| = 4.16 \end{aligned}$$

e)  $\frac{5}{8}$  and  $\frac{1}{2}$

$$\begin{aligned} \left| \frac{1}{2} - \frac{5}{8} \right| \\ \left| \frac{4}{8} - \frac{5}{8} \right| \\ \left| \frac{-1}{8} \right| = \frac{1}{8} \end{aligned}$$

f)  $\sqrt{7}$  and  $\sqrt{11}$

$$\begin{aligned} |\sqrt{11} - \sqrt{7}| \\ |3.32 - 2.65| \text{ approx.} \\ |.67| = .67 \text{ approx.} \end{aligned}$$

g) 11.3 and 7.1

$$\begin{aligned} |7.1 - 11.3| \\ |-4.2| = 4.2 \end{aligned}$$

h) a and b

$$|b - a| \text{ or } |a - b|$$

3. Suppose line  $t$  contains M and Q. Does a point P always exist such that Q is between M and P? yes  
Does a point R always exist such that M is between R and Q? yes What in general does this tell us? Between any two given points there always exists another point.

## —Continued—

4. How many different points on a line can be half the distance from one given point to another given point? Explain your answer. One; The distance between any two points divided by 2 is unique.
- \_\_\_\_\_
- \_\_\_\_\_

5. If A, B, and C are collinear,  $AB = 26$ , and  $BC = 16$ , is AC necessarily 42? No Explain your answer. AC is 42 only if B is between A and C. AC is 10 if C is between A and B.
- \_\_\_\_\_
- \_\_\_\_\_

6. In the accompanying diagram of collinear points,  $PT = 26$ ,  $PR = 8$ , and  $PQ = QR = RS$ . Find each length asked for.



a) RS

$$PQ + QR = PR$$

$$PQ = QR$$

$$QR + QR = 8$$

$$2 \cdot QR = 8$$

$$QR = 4$$

$$RS = 4$$

b) PQ

$$PQ = QR = RS$$

$$PQ = 4 = 4$$

$$PQ = 4$$

c) QS

$$QR + RS = QS$$

$$4 + 4 = QS$$

$$8 = QS$$

d) QT

$$PT - PQ = QT$$

$$26 - 4 = QT$$

$$22 = QT$$

7. Measure the given line segment to the nearest millimeter.

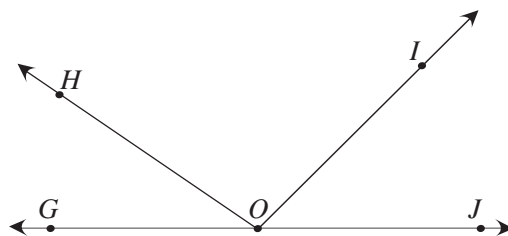
A \_\_\_\_\_ B       $AB = 82 \text{ mm}$

8. Use a protractor to determine the measure of each angle named below.

a)  $m\angle GOI = 135^\circ$

b)  $m\angle HOJ = 146^\circ$

c)  $m\angle IOH = 103^\circ$





—Continued—

9. If M is between L and N and  $LM = \frac{1}{3}x + 4$ ,  $MN = 4x + \frac{4}{3}$ , and  $LN = 4x + 6\frac{1}{3}$ , find  $x$ ,  $LM$ ,  $MN$  and  $LN$  by using the Segment Addition Postulate.

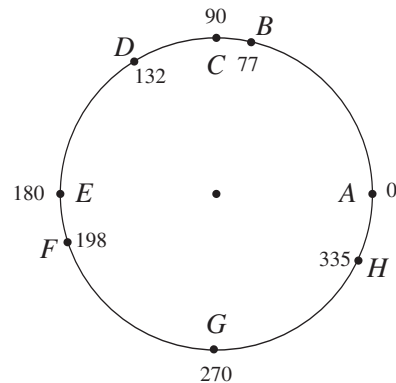
$$\begin{aligned}LM + MN &= LN \\ \left(\frac{1}{3}x + 4\right) + \left(4x + \frac{4}{3}\right) &= 4x + 6\frac{1}{3} \\ 4\frac{1}{3}x + 5\frac{1}{3} &= 4x + 6\frac{1}{3} \\ \frac{1}{3}x &= 1 \\ x &= 3\end{aligned}$$
$$\begin{aligned}LM &= \frac{1}{3}(3) + 4 = 5 \\ MN &= 4(3) + \frac{4}{3} = 13\frac{1}{3} \\ LN &= 4(3) + 6\frac{1}{3} = 18\frac{1}{3}\end{aligned}$$

10. What requirement for adding segment lengths is similar to the requirement that a ray is between two rays when adding angles? In the Segment Addition Postulate, the requirement that a point must be between two points, is similar to the requirement in the Angle Addition Postulate that a ray must be between two rays.
- \_\_\_\_\_

## —Continued—

Using the information on the circle at the right, find the distance between each pair of points in problems 3-7.

Give two distance measurements in each case.



3. B to H

4. G to C

Minor Arc	Major Arc	Minor Arc	Major Arc
$ (360-335)+77 $	$360-102$	$ 270-90 $	$360-180$
$ 25+77 $	$258^\circ$	$ 180 $	$180^\circ$
$ 102 $		$180^\circ$	
$102^\circ$			

5. D to A

6. F to D

7. C to B

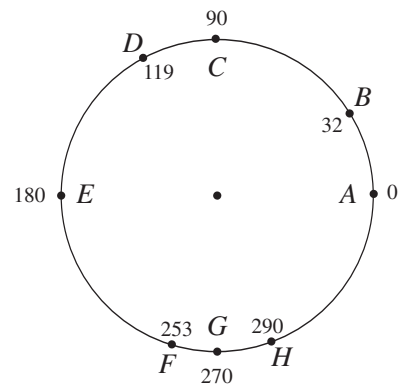
Minor Arc	Major Arc	Minor Arc	Major Arc	Minor Arc	Major Arc
$ 132-0 $	$360-132$	$ 198-132 $	$360-66$	$ 90-77 $	$360-13$
$ 132 $	$228^\circ$	$ 66 $	$294^\circ$	$ 13 $	$347^\circ$
$132^\circ$		$66^\circ$		$13^\circ$	

8. How many lines can be perpendicular to a given plane from a given line not in the given plane? infinite

9. How many lines can be perpendicular to a given line at a given point on the given line? infinite

## —Continued—

Using the given circle, find the distance between the following pairs of points. Give two distance measurements in each case.



3. D to E

*Minor Arc*

$|119 - 180|$

$|-61|$

$61^\circ$

*Major Arc*

$360 - 61$

$299^\circ$

4. C to H

*Minor Arc*

$| (360 - 290) + 90 |$

$|70 + 90|$

$|160|$

$160^\circ$

*Major Arc*

$360 - 160$

$200^\circ$

5. F to G

*Minor Arc*

$|270 - 253|$

$|17|$

$17^\circ$

*Major Arc*

$360 - 17$

$343^\circ$

6. B to E

*Minor Arc*

$|180 - 32|$

$|148|$

$148^\circ$

*Major Arc*

$360 - 148$

$212^\circ$

7. D to G

*Minor Arc*

$|270 - 119|$

$|151|$

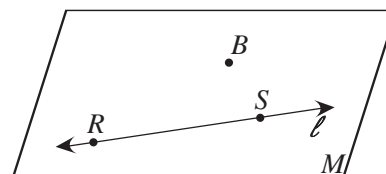
$151^\circ$

*Major Arc*

$360 - 151$

$209^\circ$

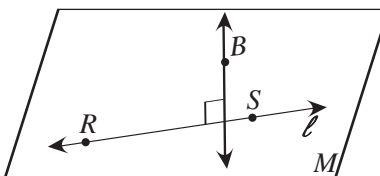
In the given figure, line  $\ell$  passes through points R and S. point B is not on line  $\ell$ . points R, S, and B are in plane M. Use this information to solve problems 8 and 9 below.



8. State the postulate that guarantees the existence of only one line through point B, which is perpendicular to line  $\ell$ .

If you have a line in a plane, and a point in the plane, not on the given line, then there is one and only one line through the point, which is perpendicular to the given line.

9. Trace the figure in Exercise 8, and complete the picture by sketching the perpendicular line through point B to line  $\ell$ .



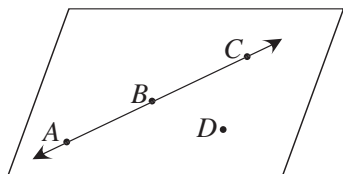
## —Continued, Page 2—

- <sup>(C-2)</sup>  
       *c* **13.** If there is space...
- <sup>(A-1)</sup>  
       *b* **14.** placeholder symbols
- <sup>(B-8)</sup>  
       *g* **15.** supplementary angles
- <sup>(C-7)</sup>  
       *x* **16.** If on a line point B lies between points A and C, then  $AB + BC = AC$ . This is called the...
- <sup>(B-9)</sup>  
       *i* **17.** arc
- <sup>(A-2)</sup>  
       *a* **18.** line
- <sup>(B-6)</sup>  
       *z* **19.** half plane
- <sup>(C-1)</sup>  
       *q* **20.** Property of Zero for Addition
- <sup>(B-8)</sup>  
       *f* **21.** angle bisector
- <sup>(B-1)</sup>  
       *o* **22.** definition
- <sup>(B-9)</sup>  
       *p* **23.** radius
- <sup>(C-8)</sup>  
       *w* **24.** The union of two distinct rays with a common endpoint is an...
- <sup>(B-7)</sup>  
       *r* **25.** acute angle
- <sup>(C-8)</sup>  
       *y* **26.** If, in a half plane, a ray OB lies between rays OA and OC, then...
- n)** The union of the separation point of a line with a half-line.
- o)** A description of a term which clearly specifies what the term means.
- p)** Distance from the center of the circle to the curve which makes the circle.
- q)** There exists a unique real number 0, such that for any real number  $a$ ,  $a + 0 = a$ .
- r)** An angle smaller than a right angle.
- s)** containing the two points is in the plane.
- t)** If  $a = b$  and  $b = c$ , then  $a = c$
- u)**  $\frac{1}{360}$  of a complete revolution.
- v)** If  $a = b$ , then  $a + c = b + c$
- w)** angle
- x)** Segment Addition Property
- y)**  $m\angle AOB + m\angle BOC = m\angle AOC$
- z)** One of the two sets of points on a plane on either side of a specific line called the separation line.

In problems 27-30, sketch and label the figures described.

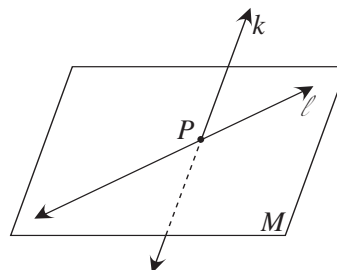
(B-2)

- 27.** Points A, B, C, and D are coplanar, but A, B, and C are the only three of those points that are collinear.



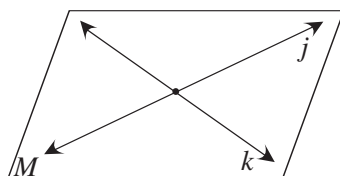
(C-6)

- 28.** Line  $\ell$ , on plane M, point P on line  $\ell$ , line k intersects plane M through point P.



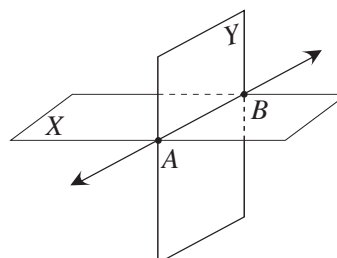
(C-4)

- 29.** Plane M contains intersecting lines j and k.

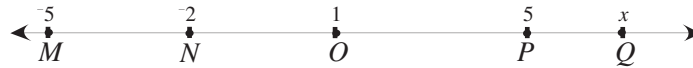


(C-6)

- 30.** Plane X and Y intersect in  $\overleftrightarrow{AB}$ .



For problems 31-34, use the figure below.



(B-4)  
**31.** Name a point in  $\overrightarrow{NP}$  that is not on  $\overline{NP}$ . point Q

(B-5)  
**32.** Complete:  $MN =$  3  $\text{ and } NO =$  3  $.$

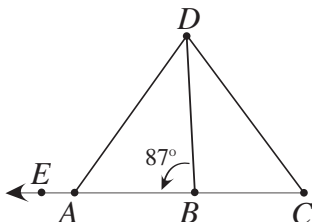
$ -2 - 5(-5) $	$ 1 - (-2) $
$ -2 + 5 $	$ 1 + 2 $
$ 3 $	$ 3 $
3	3

(B-5)  
**33.** Complete:  $\overline{MN}$  and  $\overline{NO}$  are called congruent segments.

(B-5)  
**34.** If point P is the midpoint of  $\overline{OQ}$ , find the value of  $x$ .

$ 5 - 1 $	$5 + 4 = 9$
$ 4 $	Point Q has 9 for a coordinate
4	

For problems 35-43, use the figure below.



(B-6)

35. Name three angles that have vertex D.  $\angle ADB, \angle BDC, \angle ADC$

(B-8)

36. Name two pairs of angles that form a linear pair.  $\angle EAD$  and  $\angle DAC$  or  $\angle ABD$  and  $\angle DBC$

(B-8)

37. Name a pair of adjacent angles that do not form a linear pair.  $\angle ADB$  and  $\angle CDB$

(B-7)

38.  $m\angle CBD =$   $93^\circ$ .

39. What kind of angle is  $\angle EAD$ ? *obtuse*

(C-7)

40.  $m\angle ABD + m\angle DBC =$   $180^\circ$ .

(B-5)

41. Assume  $\overrightarrow{DB}$  bisects  $\overrightarrow{AC}$ ,  $\overline{AB} = 5x - 3$  and  $BC = x + 25$ . Find the value of  $x$ . 7

$$5x - 3 = x + 25$$

$$4x = 28$$

$$x = 7$$

42.  $\angle DBC$  is an *obtuse* angle. (classify by referring to types of angles)

43.  $\angle ABD$  is an *acute* angle. (classify by referring to types of angles)

**Unit II, Fundamental Terms, Unit Test Form A**  
**—Continued, Page 6—**

Name \_\_\_\_\_

In problems 44-47, classify each statement as true or false.

(C-4)

**44.** It is possible for two intersecting lines to be non-coplanar. false

(C-4)

**45.** It is possible to locate three points in such a position that an unlimited number of planes contain all three points. true

(C-4)

**46.** Through any three points there is at least one line. false

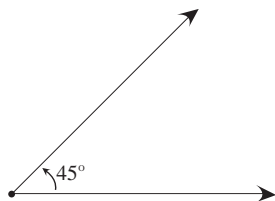
(C-4)

**47.** If points A and B lie in plane P, then so does any point of  $\overleftrightarrow{AB}$ . true

For problems 48-51, use a protractor to draw angles having the following degree measures

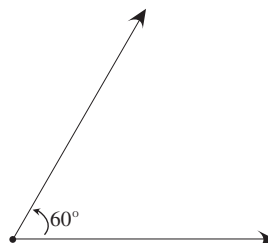
(C-8)

**48.**  $45^\circ$



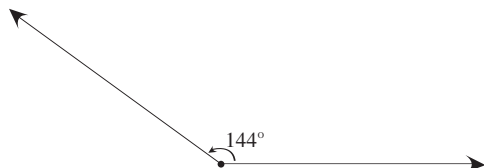
(C-8)

**49.**  $60^\circ$



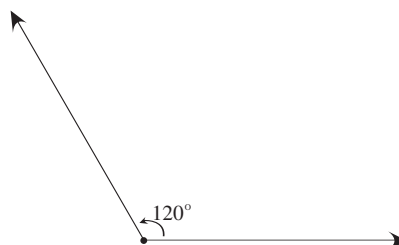
(C-8)

**50.**  $144^\circ$



(C-8)

**51.**  $120^\circ$





In the figure at the right, the following pairs of angles are complementary:  $\angle AOB$  and  $\angle BOC$ ,  $\angle COD$  and  $\angle DOE$ ,  $\angle EOF$  and  $\angle FOG$ ,  $\angle GOH$  and  $\angle HOA$ . If  $m\angle AOB = 45^\circ$ ,  $m\angle COD = 30^\circ$ ,  $\angle EOF \cong \angle FOG$ , and  $m\angle HOA = 10^\circ$ , use this information to find the measures asked for in problems 52-56.

(C-8)

52.  $m\angle BOC$  \_\_\_\_\_  $45^\circ$

(C-8)

53.  $m\angle DOE$  \_\_\_\_\_  $60^\circ$

(C-8)

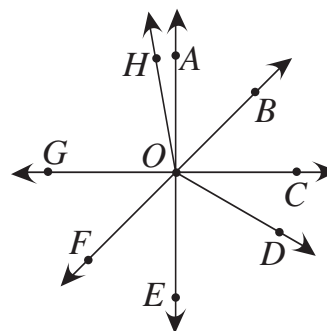
54.  $m\angle GOH$  \_\_\_\_\_  $80^\circ$

(C-8)

55.  $m\angle GOF$  \_\_\_\_\_  $45^\circ$

(C-8)

56.  $m\angle FOE$  \_\_\_\_\_  $45^\circ$



In the figure at the right,  $\overline{XY}$  and  $\overline{AB}$  are diameters. Use the information given in the figure to solve problems 57-66.

(C-9)

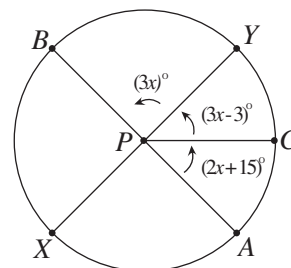
57. Find the value of  $x$ .

$$3x + 3x - 3 + 2x + 15 = 180$$

$$8x + 12 = 180$$

$$8x = 168$$

$$x = 21$$



(C-9)

58. Find  $m\angle BPY$  \_\_\_\_\_  $63^\circ$

(C-9)

60. Find  $m\angle YPC$  \_\_\_\_\_  $60^\circ$

(C-9)

62. Find  $m\widehat{BX}$  \_\_\_\_\_  $117^\circ$

(C-9)

64. Find  $m\angle XPA$  \_\_\_\_\_  $63^\circ$

(C-9)

66. Find  $m\widehat{BC}$  \_\_\_\_\_  $123^\circ$

(C-9)

59. Find  $m\widehat{YAX}$  \_\_\_\_\_  $180^\circ$

(C-9)

61. Find  $m\angle BPC$  \_\_\_\_\_  $123^\circ$

(C-9)

63. Find  $m\angle CPA$  \_\_\_\_\_  $57^\circ$

(C-9)

65. Find  $m\widehat{CA}$  \_\_\_\_\_  $57^\circ$

## —Continued, Page 2—

u <sup>(A-1)</sup> **13.** closed phrase

h <sup>(C-2)</sup> **14.** If there is a plane,...

x <sup>(B-6)</sup> **15.** edge

z <sup>(A-2)</sup> **16.** plane

g <sup>(B-9)</sup> **17.** minor arc

y <sup>(B-8)</sup> **18.** equal angles

d <sup>(B-2)</sup> **19.** betweenness (points)

k <sup>(B-7)</sup> **20.** obtuse angle

b <sup>(A-1)</sup> **21.** open sentence

o <sup>(B-5)</sup> **22.** bisector of a line segment

s <sup>(B-9)</sup> **23.** chord

n <sup>(C-1)</sup> **24.** Symmetric Property of Equality

t <sup>(C-1)</sup> **25.** Property of One  
for Multiplication

j <sup>(C-11)</sup> **26.** If you have a line in a plane  
and a point in the plane not  
on the given line, then...

**m)** Symbols that represent operations on numbers, or, more generally, symbols which indicate the actions of mathematics.

**n)** If  $a = b$ , then  $b = a$

**o)** Any point, line segment, ray, line, or plane which intersects the line segment in the midpoint of the line segment.

**p)** Exactly one space containing those four points.

**q)** Geometric tool used to measure the number of degrees in an angle.

**r)** If  $a = b$ , then  $a \cdot c = b \cdot c$

**s)** Line segment across a circle whose endpoints lie on the curve forming the circle.

**t)** There exists a unique real number 1, such that for every real number  $a$ ,  $a \cdot 1 = a$ .

**u)** Mathematical expression which contains neither a relation symbol, nor a placeholder symbol.

**v)**  $(a + b) + c = a + (b + c)$

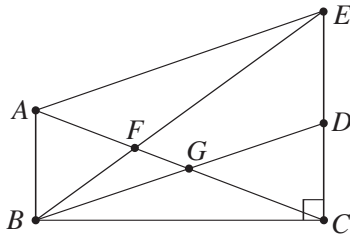
**w)** that is not on the line.

**x)** The line in a plane which separates the plane into two half planes.

**y)** Two angles with the same measure.

**z)** A geometric figure which is described as an infinite set of points, arranged as a flat surface, extending to infinity in both directions.

Answer problems 32-43, using the given figure..



(B-2)

32. Name four collinear points. A, F, G, C

(B-3)

33. Name the intersection of  $\overrightarrow{DG}$  and  $\overleftrightarrow{BC}$ . Point B

(C-1)

34. What property justifies the statement  $AF + FC = AC$ . Segment Addition Property

(B-5)

35. If  $\overline{BD}$  bisects  $\overline{AC}$ , name two congruent segments.  $\overline{AG} \cong \overline{GC}$

(B-7)

36. Name a right angle.  $\angle BCE$

(B-2)

37. Name three coplanar points. A, B, C (answers will vary)

(B-2)

38. Is it possible for an answer to exercise 37 to be incorrect? Explain. No, any combination of points in the figure will be coplanar.

(B-8)

39. Name a pair of complementary angles.  $\angle ACB$  and  $\angle ACE$

(B-8)

40. Name a pair of supplementary angles.  $\angle AGB$  and  $\angle AGD$

(B-8)

41. Name two pairs of vertical angles.  $\angle AGB$  and  $\angle DGC$ ;  $\angle AGD$  and  $\angle BGC$

(B-8)

42. If  $m\angle AFB$  is  $28^\circ$ , find  $m\angle EFC$   $28^\circ$  and  $m\angle AFE$ .  $152^\circ$

(B-8)

43. If  $m\angle ACB$  is  $12^\circ$ , find  $m\angle ACE$ .  $78^\circ$

**Unit II, Fundamental Terms, Unit Test Form B**  
**—Continued, Page 5—**

Name \_\_\_\_\_

For exercises 44-48, refer to a numberline, not pictured here, with the following information. Point A has coordinate 2 and point B has coordinate 5.



(B-5)

**44.** What is the length of  $\overline{AB}$ .  $|5 - 2|$  or  $3$

(B-5)

**45.** What is the coordinate of the midpoint of  $\overline{AB}$ ?  $\left(2 + \frac{3}{2}\right)$  or  $3\frac{1}{2}$

(B-5)

**46.** If A is the midpoint of  $\overline{PB}$ , what is the coordinate of point P?  $-1$  (3 to the left of 2)

(B-5)

**47.** What is the coordinate of a point that is on  $\overrightarrow{AB}$  and is 4 units from B.  $(5 + 4)$  or  $9$

(B-5)

**48.** What is the coordinate of a point that is 4 units from point B, but is not on  $\overrightarrow{AB}$ ?  $1$  (4 to the left of 5)

(B-2)

**49.** Is it possible for a line and a point to be non-coplanar? *no*

(C-5)

**50.** Is it possible for the intersection of two planes to consist of a segment? *no*

In the figure at the right,  $\overrightarrow{VR}$  bisects  $\angle QVS$ ,  $m\angle PVQ = 72^\circ$ ,  $m\angle TVS = 70^\circ$ , and  $\overrightarrow{VP}$  and  $\overrightarrow{VT}$  are opposite rays. Use this information to find the measures asked for in problems 51-56.

(C-8)

**51.**  $m\angle QVS$   $38^\circ$

(C-8)

**52.**  $m\angle QVR$   $19^\circ$

(C-8)

**53.**  $m\angle PVR$   $91^\circ$

(C-8)

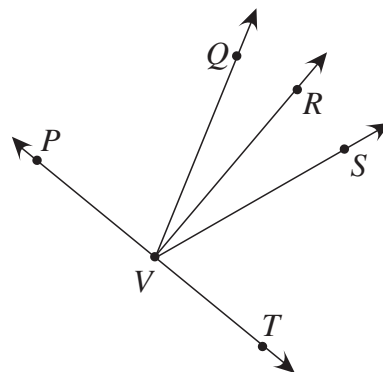
**54.**  $m\angle TVR$   $89^\circ$

(C-8)

**55.**  $m\angle PVS$   $110^\circ$

(C-8)

**56.**  $m\angle TVQ$   $108^\circ$



**Unit II, Fundamental Terms, Unit Test Form B**  
**—Continued, Page 6—**

Name \_\_\_\_\_

Complete each of the statements in problems 57-62.

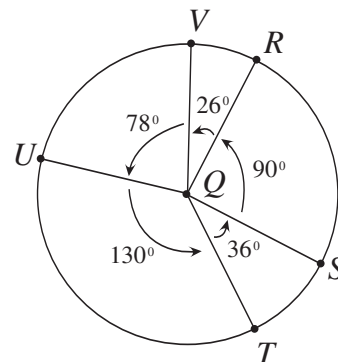
- (C-1)  
**57.**  $m\angle A = m\angle A$  is true because of the Reflexive Property of Equality.
- (B-8)  
**58.** If  $m\angle A = m\angle A$ , then is it true that  $\angle A \cong \angle A$ ? yes (definition of congruence)
- (C-1)  
**59.** If  $m\angle B = m\angle C$  and  $m\angle C = m\angle A$ , then  $m\angle B = m\angle A$ . This is an example of the Transitive Property of Equality.
- (B-8)  
**60.** With reference to exercise 59, would it be correct to say that if  $\angle B \cong \angle C$  and  $\angle C \cong \angle A$ , the  $\angle C \cong \angle A$ ? yes
- (C-1)  
**61.** If  $m\angle A = m\angle B$ , then  $m\angle B = m\angle A$  is an example of the Symmetric Property.
- (B-8)  
**62.** If  $\angle A \cong \angle B$ , then  $\angle B \cong \angle A$  must be true since if two angles have the same measure (in exercise 61) then by definition, the angles are congruent.

Tell if each of the statements in problems 63-65 is true or false..

- (C-1)  
**63.** Angle congruence is reflexive. true
- (C-1)  
**64.** Angle congruence is transitive. true
- (C-1)  
**65.** Angle congruence is not symmetric. false

In the figure at the right, the measure of each angle in circle Q is given. Use this information to find the measure of each arc in problems 66-73.

- (C-9) **66.**  $\widehat{RT}$  126°
- (C-9) **67.**  $\widehat{UR}$  104°
- (C-9) **68.**  $\widehat{VS}$  116°
- (C-9) **69.**  $\widehat{US}$  166°
- (C-9) **70.**  $\widehat{RSU}$  256°
- (C-9) **71.**  $\widehat{SUV}$  244°
- (C-9) **72.**  $\widehat{TVR}$  234°
- (C-9) **73.**  $\widehat{VTR}$  334°



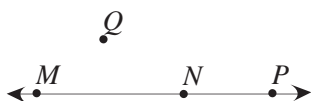
## —Continued, Page 2—

- y     <sup>(C-10)</sup> **13.** If you have a line in a plane and a point in the plane not on the given line, then...
- w     <sup>(C-1)</sup> **14.**  $a > b$  means...
- z     <sup>(B-6)</sup> **15.** half space
- c     <sup>(C-1)</sup> **16.** Commutative Property of Multiplication
- x     <sup>(C-9)</sup> **17.** If, on a circle, a point B lies between point A and C, then...
- k     <sup>(A-2)</sup> **18.** plane
- t     <sup>(C-1)</sup> **19.** Additive Inverse Property
- o     <sup>(B-8)</sup> **20.** complementary angles
- a     <sup>(B-4)</sup> **21.** separation point
- g     <sup>(B-6)</sup> **22.** angle
- p     <sup>(C-1)</sup> **23.** Transitive Property of Order
- r     <sup>(C-2)</sup> **24.** If there is a line,...
- q     <sup>(C-4)</sup> **25.** For an two different points in a plane, the line containing these points...
- j     <sup>(C-1)</sup> **26.** Reflexive Property of Equality
- n)** Represented by a straight line segment with arrowheads on each end.
- o)** Two angles whose measures total 90 degrees.
- p)** If  $a < b$  and  $b < c$ , then  $a < c$ .
- q)** is in the plane.
- r)** then there exists at least two different points on the line.
- s)** we have exactly one plane containing those three points.
- t)** For every real number  $a$ , there exists a unique real number  $(-a)$  such that  $a + (-a) = 0$ .
- u)** Mathematical expression which contains a placeholder symbol, but does not contain a relation symbol.
- v)** For three distinct coplanar rays,  $\overrightarrow{AB}$ ,  $\overrightarrow{AC}$ , and  $\overrightarrow{AD}$  with coordinates  $b$ ,  $c$ , and  $d$  respectively,  $\overrightarrow{AC}$  is between  $\overrightarrow{AB}$  and  $\overrightarrow{AD}$  if and only if  $b < c < d$  or  $d < c < b$ .
- w)**  $b < a$
- x)**  $m\widehat{AB} + m\widehat{BC} = m\widehat{AC}$
- y)** There is one and only one line through the point which is parallel to the given line.
- z)** One of the two sets of points in space on either side of a plane, called the separation plane.

**Unit II, Fundamental Terms, Unit Test Form C**  
**—Continued, Page 3—**

Name \_\_\_\_\_

For problems 27-34, use the figure shown.



(B-3)

**27.** Write three names for the line pictured.  $\overleftrightarrow{MN}$ ,  $\overleftrightarrow{NP}$ ,  $\overleftrightarrow{MP}$

(B-4)

**28.** Name the ray that is opposite  $\overrightarrow{NP}$ .  $\overrightarrow{NM}$

(B-4)

**29.** Is it correct to say that point Q lies between points M and N? Why or Why not? no; not collinear

(B-4)

**30.** Is point M on  $\overleftrightarrow{NP}$ ? no

(B-4)

**31.** Is point M on  $\overleftrightarrow{NP}$ ? yes

(B-5)

**32.** Name three line segments that are in the line pictured.  $\overline{MN}$ ,  $\overline{NP}$ ,  $\overline{MP}$

(C-7)

**33.** If  $MN = 7$ ,  $NP = 3x + 5$ , and  $MP = 18$ , what is the value of “x”? Write an equation.  $x = 2$

$$7 + (3x + 5) = 18$$

$$3x + 12 = 18$$

$$3x = 6$$

$$x = 2$$

(C-7)

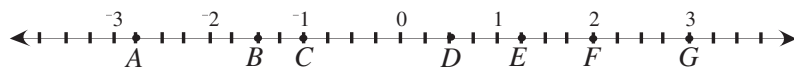
**34.** What property allows you to set up the equation you used in exercise 33? Segment Addition Property



**Unit II, Fundamental Terms, Unit Test Form C**  
**—Continued, Page 4—**

Name \_\_\_\_\_

Use the number line below for problems 35-42.



(C-7)  
**35.** Find DE

$$\left| \frac{5}{4} - \frac{2}{4} \right|$$

$$\frac{3}{4}$$

(C-7)  
**36.** Find AC

$$\left| -1 - \frac{-11}{4} \right| = \left| -1 + \frac{11}{4} \right|$$

$$\frac{7}{4}$$

(C-7)  
**37.** Find BF

$$\left| 2 - \frac{-3}{2} \right| = \left| 2 + \frac{3}{2} \right|$$

$$3\frac{1}{2}$$

(C-7)  
**38.** Find AE

$$\left| \frac{5}{4} - \frac{-11}{4} \right| = \left| \frac{5}{4} + \frac{11}{4} \right|$$

$$4$$

(C-7)  
**39.** Show that C is between B and D.

$$BC + CD = BD$$

*B, C, and D Collinear*

$$\frac{1}{2} + \frac{3}{2} = \frac{4}{2}$$

(B-5)  
**40.** Show that D is the midpoint of CF.

$$CF = 3 \quad |2 - (-1)| = |3| = 3$$

$$\text{Midpoint of } \overline{CF} \text{ is } \frac{1}{2} \left( -1 + \frac{1}{2} \cdot 3 \right) = \left( -1 + \frac{3}{2} \right) = \frac{1}{2}$$

The Coordinate of D is  $\frac{2}{4}$  or  $\frac{1}{2}$ .

\*C, D, F are Collinear.

$$\text{From C to D: } \left| \frac{1}{2} - (-1) \right| = \left| \frac{1}{2} + 1 \right| = \left| \frac{3}{2} \right| = \frac{3}{2}$$

$$\text{From D to F: } \left| 2 - \frac{1}{2} \right| = \left| \frac{3}{2} \right| = \frac{3}{2}$$

$$CD = DF$$

$$CD \cong DF$$

Therefore, D is the midpoint since points are collinear and segments are congruent.

(C-7)  
**41.** Show that F is not between C and E.

$$CF = 3 \quad |2 - (-1)| = |2 + 1| = 3$$

$$FE = \frac{3}{4} \quad \left| 2 - 1\frac{1}{4} \right| = \left| \frac{3}{4} \right| = \frac{3}{4}$$

$$CE = 2\frac{1}{4} \quad \left| \frac{5}{4} - (-1) \right| = \left| \frac{5}{4} + 1 \right| = 2\frac{1}{4}$$

Points C, E, and F are collinear.

However,  $CF + FE \neq CE$ .

$$3 + \frac{3}{4} \neq 2\frac{1}{4}$$

(B-5)  
**42.** Show that C is not the midpoint of AD.

$$AC = \frac{7}{4} \quad \left| -1 - \left( -2\frac{3}{4} \right) \right| = \left| -1 + 2\frac{3}{4} \right| = \left| \frac{-4}{4} + \frac{11}{4} \right| = \left| \frac{7}{4} \right| = \frac{7}{4}$$

$$CD = \frac{3}{2} \quad \left| \frac{2}{4} - (-1) \right| = \left| \frac{2}{4} + 1 \right| = \left| \frac{2}{4} + \frac{4}{4} \right| = \left| \frac{6}{4} \right| = \frac{6}{4} = \frac{3}{2}$$

Points A, C, and E are collinear.

but,  $AC \neq CD$ .

Use the figure at the right to find the measures asked for in problems 52-56.

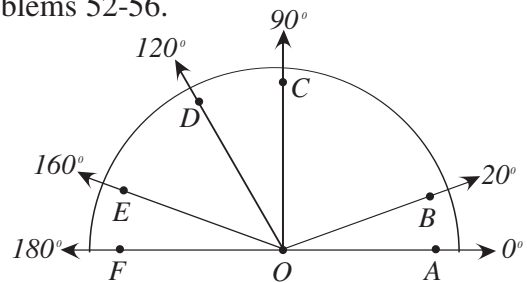
(C-8)  
**52.**  $m\angle BOC$   $|90 - 20|$  or  $70^\circ$

(C-8)  
**53.**  $m\angle EOF$   $|180 - 160|$  or  $20^\circ$

(C-8)  
**54.**  $m\angle DOE$   $|160 - 120|$  or  $40^\circ$

(C-8)  
**55.**  $m\angle COD$   $|120 - 90|$  or  $30^\circ$

(C-8)  
**56.**  $m\angle EOB$   $|160 - 20|$  or  $140^\circ$



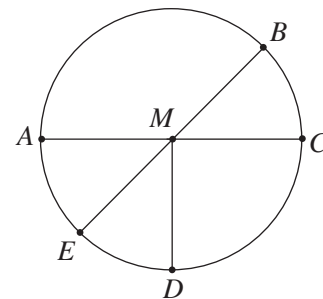
Using the diagram at the right, and your powers of observation, determine if each arc in problems 57-60 is a minor arc, major arc, or semicircle.

(C-9)  
**57.**  $\widehat{BDE}$  semicircle

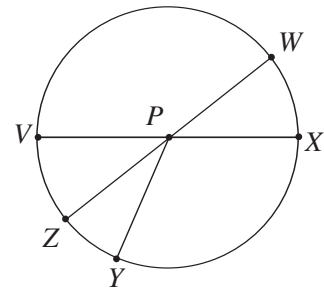
(C-9)  
**58.**  $\widehat{CAE}$  major arc

(C-9)  
**59.**  $\widehat{AE}$  minor arc

(C-9)  
**60.**  $\widehat{BCD}$  minor arc



In the diagram at the right,  $m\angle WPX = 38^\circ$ ,  $m\angle ZPY = 28^\circ$ , and  $\overline{WZ}$  and  $\overline{XV}$  are diameters. Use this information to find the measure of the arcs given in problems 61-64.



(C-9)  
**61.**  $m\widehat{YZ}$                       $28^\circ$                     

(C-9)  
**62.**  $m\widehat{VZ}$                       $38^\circ$                     

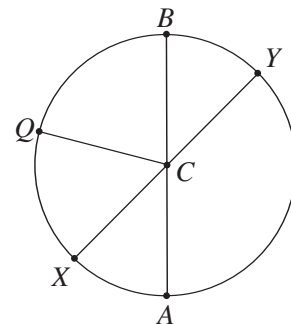
(C-9)  
**63.**  $m\widehat{WZX}$                       $322^\circ$                     

(C-9)  
**64.**  $m\widehat{WZY}$                       $208^\circ$                     

Answer problems 65-68, referring to the figure at the right, and using the following information.  
If  $m\angle BCY = 3x$ ,  $m\angle BCQ = 3x + 30$ , and  $m\angle QCX = 2x + 30$ .

(C-9)  
**65.** Find the value of  $x$ .                      $15^\circ$                     

$$\begin{aligned} 3x + (3x + 30) + 2x + 30 &= 180 \\ 8x + 60 &= 180 \\ 8x &= 120 \\ x &= 15 \end{aligned}$$



(C-9)  
**66.** Find  $m\widehat{AX}$                       $45^\circ$                     

(C-9)  
**67.** Find  $m\widehat{BQ}$                       $75^\circ$                     

(C-9)  
**68.** Find  $m\widehat{XQY}$                       $180^\circ$

**Extending Your Mind—**

Determine if the statements in problems 69-72 are true or false.

69. Two congruent circles will have congruent radii. true

70. Two circles with the same center  
will be congruent circles. false

71. If two central angles are congruent, then their  
corresponding minor arcs are congruent. false

72. If two minor arcs are congruent, then their  
corresponding central angles are congruent. true

## —Continued, Page 2—

     <sup>(B-8)</sup>  
     *b* **13.** linear pair

     <sup>(A-2)</sup>  
     *g* **14.** line

     <sup>(C-1)</sup>  
     *c* **15.** Multiplicative Inverse Property

     <sup>(C-1)</sup>  
     *x* **16.** Distributive Property of  
Multiplication Over Addition

     <sup>(B-9)</sup>  
     *m* **17.** central angle

     <sup>(C-6)</sup>  
     *j* **18.** If two different lines intersect,  
then the intersection is...

     <sup>(C-1)</sup>  
     *p* **19.** Commutative Property  
of Addition

     <sup>(C-4)</sup>  
     *w* **20.** For every plane in space, there is  
at least one point in space...

     <sup>(B-8)</sup>  
     *s* **21.** congruent angles

     <sup>(B-9)</sup>  
     *d* **22.** semi circle

     <sup>(B-6)</sup>  
     *o* **23.** dihedral angle

     <sup>(C-9)</sup>  
     *k* **24.** minor arc

     <sup>(B-9)</sup>  
     *e* **25.** center

     <sup>(B-4)</sup>  
     *v* **26.** half-line

**n)** A set of points, not all of which lie on the same line.

**o)** The union of two half planes with the same edge.

**p)**  $a + b = b + a$

**q)**  $\frac{1}{4}$  of complete rotation

**r)** Mathematical expression which contains a relation symbol,  
but does not contain a placeholder symbol.

**s)** Two angles with the same measure.

**t)** A unique line.

**u)** There exists a unique real number 0, such that for every  
real number  $a$ ,  $a \cdot 0 = 0$

**v)** One of two sets of points on either side of the separation  
point on a line.

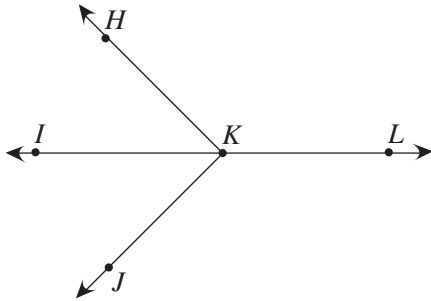
**w)** that is not on the plane.

**x)**  $a(b + c) = a \cdot b + a \cdot c$

**y)** If  $a < b$ , then  $a + c < b + c$

**z)** Symbols that group numbers together, or, more generally,  
symbols which indicate the associations of mathematics.

For problems 27-32, refer to the figure below.



(C-8)

27.  $m\angle HKI + m\angle IKJ = m\angle$  HKJ

(C-8)

28. Name the property which allows you to answer exercise 27. Angle Addition Property

(B-8)

29. If  $\angle HKI \cong \angle JKI$ , then  $\overrightarrow{KI}$  is the bisector of  $\angle$  HKJ.

(B-7)

30.  $m\angle LKI =$   $180^\circ$ , so  $\angle LKI$  is called a(n) straight angle.

(B-8)

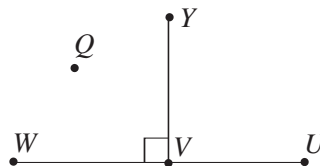
31.  $\angle IKJ$  and  $\angle JKL$  form a linear pair.

(B-8)

32. Since  $m\angle HKI + m\angle HKL =$   $180^\circ$ ,  $\angle HKI$  and  $\angle HKL$  are said to be supplementary.

(B-5)

- 33.** Indicate for each of the four statements below whether you can or cannot conclude them from the figure at the right?



- a) U, V, and W are collinear can  
 b)  $\angle UVY$  is a right angle can  
 c) V is the midpoint of  $\overline{WV}$  cannot  
 d) Q is in the interior of  $\angle WVY$  can

In problems 34-37, determine whether each of the statements given is a good definition. If not, tell why. Assume the terms in each statement are previously defined.

(B-1)

- 34.** The points lie on the same line. no; the set to which these points belong is not identified.

(B-1)

- 35.** Points are coplanar if and only if they lie in the same plane. yes

(B-1)

- 36.** Skew lines do not intersect. no; the distinguishing qualities are not identified

(B-1)

- 37.** A rhombus is a quadrilateral. no; the distinguishing qualities are not identified.

(C-2)

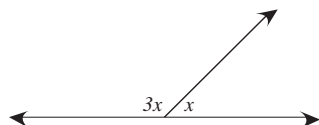
- 38.** How many planes are determined by three non-collinear points? one

(C-2)

- 39.** Suppose two different planes contain  $\overline{AB}$ . Describe the intersection of the planes. The intersection of the two planes is the line containing line segment AB.

(B-8)

- 40.** Two angles form a linear pair. The degree measure of one of the angles is three times the degree measure of the other angle. Find the degree measure of each angle.  $45^\circ$ ;  $135^\circ$



$$3x + x = 180$$

$$4x = 180$$

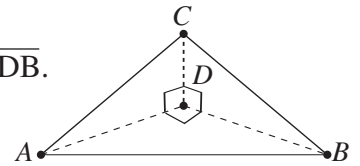
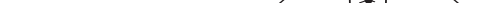
$$x = 45$$

$$3x = 135$$

In the three-dimensional figure at the right,  $\overline{AD} \perp \overline{DB}$ ,  $\overline{AD} \perp \overline{DC}$ , and  $\overline{DC} \perp \overline{DB}$ .

(B-6)

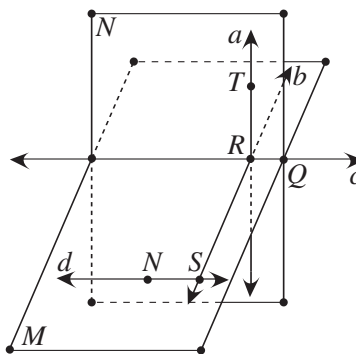
**41.** Name the dihedral angle with edge of,  $\overleftrightarrow{CD}$ .  $A - \overleftrightarrow{CD} - B$



(B-6)  
**42.** Name the edge of  $\angle B - \overleftrightarrow{AD} - C$ .  $\overleftrightarrow{AD}$

(B-6)  
**43.**  $\overleftrightarrow{DB}$  is called the edge of half-plane ADB and half-plane CDB.

For problems 44-50, use the diagram below, and find an example for each set of conditions given. Assume line  $c$  and line  $d$  are parallel.



(B-3)  
**44.** Two non-coplanar lines. line  $d$  and line  $a$  (or line  $a$  and line  $b$ )

(B-3)  
**45.** Two parallel lines. line  $d$  and  $\overleftrightarrow{RQ}$

(B-3)

**46.** Two segments that are not parallel.  $\overline{RT}$  and  $\overline{SR}$

(B-3)  
**47.** Two coplanar lines that intersect. *line d and line a (or line a and line b)*

(B-3)  
**48.** Two skew lines line  $d$  and line  $a$

(B-2)  
**49.** A line which lies in plane M                      $\overleftrightarrow{NS}$                     

(B-3)  
**50.** Line  $a$  intersects plane  $M$  in point            $R$           .



**Unit II, Fundamental Terms, Unit Test Form D**  
**—Continued, Page 6—**

Name \_\_\_\_\_

Using the diagram at the right, state whether you can reach each conclusion given in problems 51-60.

(C-8)

**51.**  $m\angle FOB = 50$  yes

(C-8)

**52.**  $m\angle AOC = 90$  yes

(C-8)

**53.**  $m\angle DOC = 180$  yes

(B-5)

**54.**  $AO = OB$  no

(B-8)

**55.**  $\angle AOC \cong \angle BOC$  yes

(B-8)

**56.**  $m\angle AOF = 130$  yes

(B-2)

**57.** Points A, O, and B are collinear. yes

(B-8)

**58.** Point C is in the interior of  $\angle AOF$  yes

(B-9)

**59.**  $\angle AOE$  and  $\angle AOD$  are adjacent angles. no

(B-4)

**60.**  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$  are opposite rays. yes

