Geometry: A Complete Course
(with Trigonometry)

Module D - Course Notes

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VideoText Interactive

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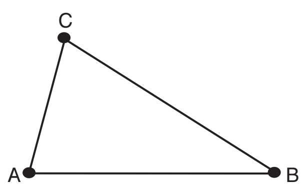
COURSE NOTE 163

TRIANGLE

"A simple closed plane curve, made up of three straight line segments."



Triangle ABC ($\triangle ABC$)

$\overline{AB}$, $\overline{BC}$, and $\overline{CA}$ are sides

A, B, and C are vertices

$\overline{AB}$ is "included by" $\angle A$ and $\angle B$

$\overline{AB}$ is "opposite" $\angle C$

$\overline{BC}$ is "included by" $\angle B$ and $\angle C$

$\overline{BC}$ is "opposite" $\angle A$

$\overline{CA}$ is "included by" $\angle C$ and $\angle A$

$\overline{CA}$ is "opposite" $\angle B$

$\angle A$ is "included by" $\overline{AB}$ and $\overline{AC}$

$\angle A$ is "opposite" $\overline{BC}$

$\angle B$ is "included by" $\overline{BA}$ and $\overline{BC}$

$\angle B$ is "opposite" $\overline{AC}$

$\angle C$ is "included by" $\overline{CA}$ and $\overline{CB}$

$\angle C$ is "opposite" $\overline{AB}$

COURSE NOTE 175

MEANS-EXTREMES PROPERTY

of a proportion

If $$\frac{a}{b} = \frac{c}{d},$$

then $a \quad d = b \quad c$

Justification:

Given: $$\frac{a}{b} = \frac{c}{d}$$

M_{bd} $$\frac{a}{\cancel{b}}(\cancel{b}\,d) = \frac{c}{\cancel{d}}(b\,\cancel{d})$$

$$a\,d = b\,c$$

Note: If $$\frac{a}{b} = \frac{b}{d},$$

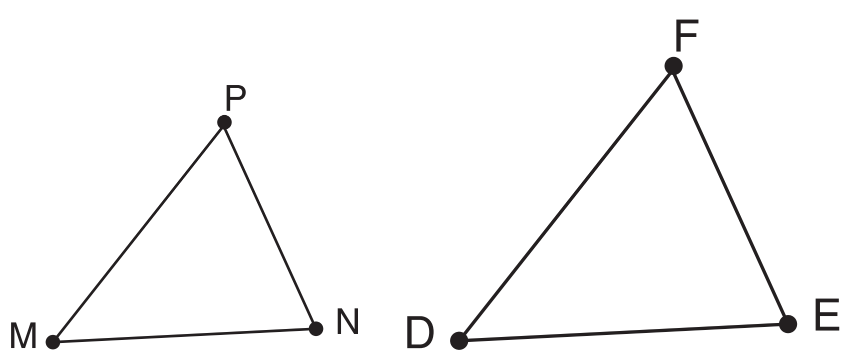
then b is called the "geometic mean" between a and d

COURSE NOTE 183

POSTULATE 12:
Triangle Similarity

"If the three sides of one triangle are in proportion to the three corresponding sides of another triangle, then the two triangles are similar."

(S.S.S. Similarity Assumption)



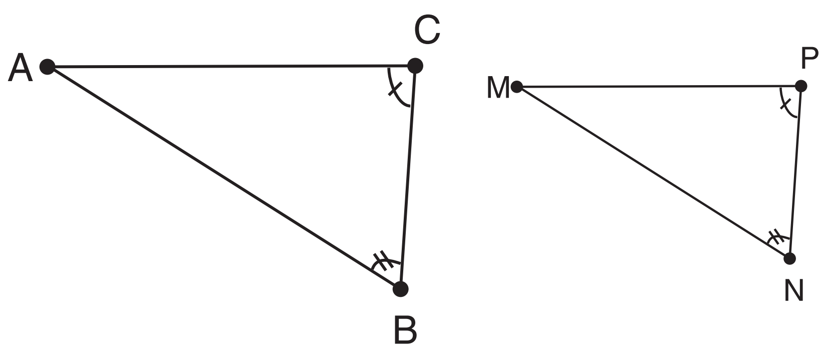
$$\text{If}\quad \frac{MN}{DE} = \frac{NP}{EF} = \frac{NP}{EF},$$

$$\text{then}\ \triangle MNP \sim \triangle DEF$$

COURSE NOTE 185

POSTULATE COROLLARY 12a: Triangle Similarity

"If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar."



If $\angle B \cong \angle N$, and $\angle C \cong \angle P$ then $\triangle ABC \sim \triangle MNP$

COURSE NOTE 189

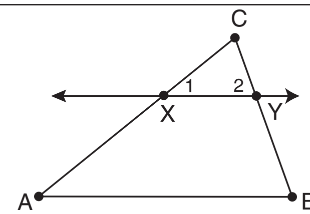
Theorem 28 (Side Splitter)

1) "If a line is parallel to one side of a triangle, and intersects the other two sides in different points, then it divides the two sides proportionally."

3) Given: $\triangle ABC$ with $\overleftrightarrow{XY} \parallel \overline{AB}$

4) Prove: $\dfrac{AX}{XC} = \dfrac{BY}{YC}$

2)

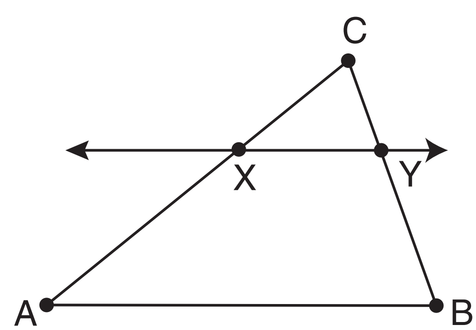


5) Analysis: Postulate 11; Postulate Corollary 12a; Special Properties of Proportions

6) STATEMENT	**REASON**
1. $\triangle ABC$ with $\overleftrightarrow{XY} \parallel \overline{AB}$	1. Given
2. $\angle 1 \cong \angle A$	2. Postulate 11 - "If two parallel lines are cut by a transversal, then corresponding angles are congruent."
3. $\angle 2 \cong \angle B$	3. Postulate 11 - "If two parallel lines are cut by a transversal, then corresponding angles are congruent."
4. $\triangle ABC \sim \triangle XYZ$	4. Postulate Corollary 12a - If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar."
5. $\dfrac{AC}{XC} = \dfrac{BC}{YC}$	5. Definition of Similarity
6. $AX + XC = AC$	6. Postulate 6 (Ruler) - Segment Addition Assumption
7. $AX = AC - XC$	7. Addition Property of Equality
8. $BY + YC = BC$	8. Postulate 6 (Ruler) - Segment Addition Assumption
9. $BY = BC - YC$	9. Addition Property of Equality
10. $\dfrac{AC - XC}{XC} = \dfrac{BC - YC}{YC}$	10. Denom.-Subt.Prop. of Proportions
11. $\dfrac{AX}{XC} = \dfrac{BY}{YC}$	11. Subsitution (Q.E.D.)

COROLLARY 28a

"If a line intersects two sides of a triangle in different points in such a way that the two sides are divided proportionally, then the line is parallel to the third side of the triangle."



$$\text{If } \frac{CX}{XA} = \frac{CY}{YB},$$

$$\text{then } \overleftrightarrow{YX} \parallel \overline{AB}$$

COURSE NOTE 195

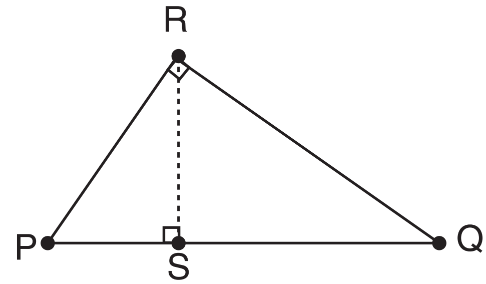
Theorem 30

1) "If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle, and to each other."

3) Given: $\triangle PRQ$, with $\angle PRQ$ a right angle, and $\overline{RS}$ as the altitude to $\overline{PQ}$

4) Prove: $\triangle PRS \sim \triangle PQR$
$\triangle RQS \sim \triangle PQR$
$\triangle PRS \sim \triangle RQS$

2)



5) Analysis: Postulate Corollary 12b
Postulate Corollary 12c

6) STATEMENT	REASON
1. $\triangle PRQ$, with $\angle PRQ$ a rt. angle, and $\overline{RS}$ as the altitude to $\overline{PQ}$	1. Given
2. $\overline{RS} \perp \overline{PQ}$	2. Definition of an Altitude
3. $\angle PSR$ and $\angle RSQ$ are rt. angles	3. Definition of Perpendicular Lines
4. $\angle P \cong \angle P$	4. Reflexive Property of Congruence
5. $\triangle PRS \sim \triangle PQR$	5. Postulate 12b - "If one acute angle of one right triangle is congruent to one acute angle of another right triangle, then the two right triangles are similar."
6. $\angle Q \cong \angle Q$	6. Reflexive Property of Congruence
7. $\triangle RQS \sim \triangle PQR$	7. Postulate Corollary 12b
8. $\triangle PRS \sim \triangle RQS$	8. Postulate Corollary 12c - "If two triangles are similar to a third triangle, then the two triangles are similar to each other". (Q.E.D.)

COURSE NOTE 221

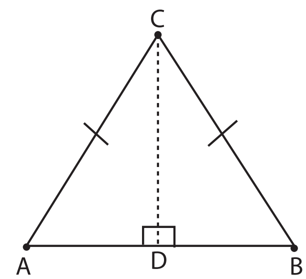
THEOREM 33

1) "If two sides of a triangle are congruent, then the angles opposite those sides are congruent."

3) Given: $\triangle ABC$, $\overline{AC} \cong \overline{BC}$

4) Prove: $\angle A \cong \angle B$

2)



5) Analysis: H.L. Postulate Corollary, Definition of Congruent Triangles

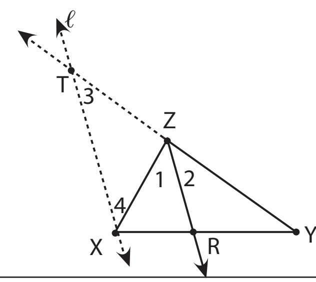
6) STATEMENT	REASON
1. $\overline{AC} \cong \overline{BC}$	1. Given
2. Draw altitude $\overline{CD}$ to $\overline{AB}$	2. Definition of an altitude
3. $\angle ADC$ and $\angle BDC$ are right angles	3. Definition of an altitude
4. $\triangle ADC$ and $\triangle BDC$ are rt. triangles	4. Definition of a right triangle
5. $\overline{DC} \cong \overline{DC}$	5. Reflexivity of congruence
6. $\triangle ADC \cong \triangle BDC$	6. HL Postulate Corollary
7. $\angle A \cong \angle B$	7. C.P.C.T.C. (Q.E.D.)

COURSE NOTE 225

THEOREM 35

1) "If a ray bisects one angle of a triangle, then it divides the side opposite that angle into segments, which are in the same proportion as the other two sides."

2)



3) Given: $\triangle XYZ$, $\angle 1 \cong \angle 2$

4) Prove: $\dfrac{XR}{YR} = \dfrac{XZ}{YZ}$

5) Analysis: Auxiliary Lines, Theorem 28 (Side-Splitter), Theorem 34

6) STATEMENT	**REASON**
1. $\angle 1 \cong \angle 2$	1. Given
2. Draw $\overleftrightarrow{YZ}$	2. Postulate 2
3. Draw line ℓ parallel to $\overrightarrow{ZR}$, through point X	3. Postulate 9
4. $\overleftrightarrow{YZ}$ intersects ℓ at T	4. Postulate 5
5. $\dfrac{XR}{YR} = \dfrac{TZ}{YZ}$	5. Theorem 28 - Side-Splitter Theorem
6. $\angle 1 \cong \angle 4$	6. Theorem 16
7. $\angle 2 \cong \angle 3$	7. Postulate 11
8. $\angle 3 \cong \angle 4$	8. Substitution for congruence
9. $\overline{TZ} \cong \overline{XZ}$	9. Theorem 34 - "If two angles of a triangle are congruent, then the sides opposite them are congruent."
10. TZ = XZ	10. Definition of congruence
11. $\dfrac{XR}{YR} = \dfrac{XZ}{YZ}$	11. Substitution for equality (Q.E.D.)

COURSE NOTE 226

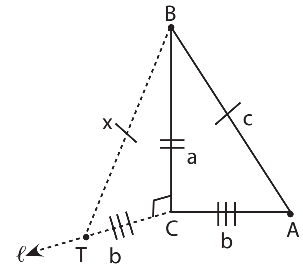
THEOREM 36

1) "If an a given triangle, the sum of the squares of the measures of two of the sides, is equal to the square of the measure of the third side, then the given triangle is a right triangle." (Converse of the Pythagorean Theorem)."

3) Given: $\triangle ACB$, with $a^2 + b^2 = c^2$

4) Prove: $\triangle ACB$ is a right triangle

2)



5) Analysis: Auxiliary lines, Definition of Congruent Triangles, The Pythagorean Theorem

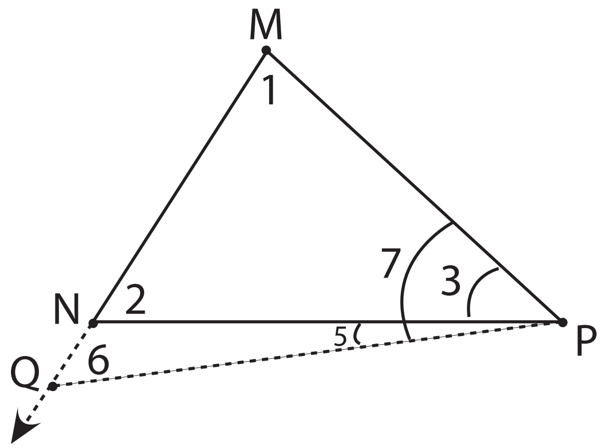
6) STATEMENT	REASON
1. $\triangle ACB$	1. Given
2. Draw a line ℓ through point C, such that $\ell \perp \overline{BC}$	2. Theorem 6
3. Find a point T on ℓ, such that TC = AC	3. Postulate 6 - (Ruler)
4. $\angle TCB$ is a right angle	4. Definition of Perpendicular Lines
5. Draw $\overline{TB}$	5. Postulate 2
6. $\triangle TCB$ is a right triangle	6. Definition of a Right Triangle
7. $a^2 + b^2 = x^2$	7. Theorem 31 - (Pythagorean Theorem)
8. $a^2 + b^2 = c^2$	8. Given
9. $x^2 = c^2$	9. Substitution
10. $x = c$	10. Multiplication of Equality
11. $\overline{TB} \cong \overline{AB}$	11. Definition of Congruent Segments
12. $\overline{CB} \cong \overline{CB}$	12. Reflexive Property of Congruence
13. $\triangle TCB \cong \triangle ACB$	13. Postulate 13 - S.S.S. Assumption
14. $\angle TCB \cong \angle ACB$	14. C.P.C.T.C.
15. $\angle ACB$ is a right angle	15. Substitution
16. $\triangle ACB$ is a right triangle	16. Definition of a Right Triangle (Q.E.D.)

COURSE NOTE 230

COROLLARY 38a

"In a given triangle, if the measure of one side is greater than the measure of a second side, then the measure of the angle opposite the longer side, is greater than the measure of the angle opposite the shorter side."

Proof Analysis



Given: MP > MN
Prove: m∠N > m∠P

Extend $\overline{MN}$ so that MQ = MP
Draw $\overline{QP}$
△MQP is Isosceles

m∠6 = m∠7
m∠7 = m∠3 + m∠5
m∠6 = m∠3 + m∠5

m∠2 = m∠6 + m∠5
m∠2 = (m∠3 + m∠5) + m∠5
m∠2 > m∠3
m∠N > m∠P

COURSE NOTE 232

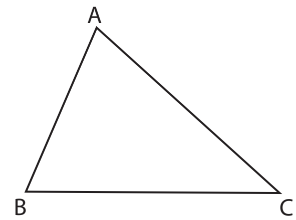
THEOREM 39

1) "In a given triangle, if two angles are not congruent, then the sides opposite those angles are not congruent."

3) Given: $\triangle ABC$, with $\angle B \ncong \angle C$

4) Prove: $\overline{AB} \ncong \overline{AC}$

2)



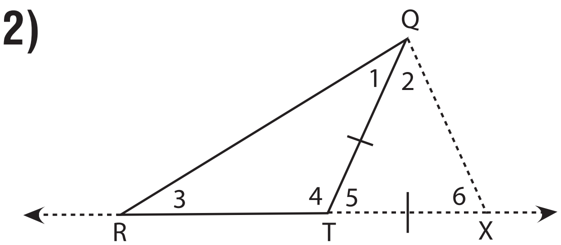
5) Analysis: Indirect Proof, Theorem 33

6) STATEMENT	**REASON**
1. $\triangle ABC$, with $\angle B \ncong \angle C$	1. Given
2. Assume $\overline{AB} \cong \overline{AC}$	2. Indirect Proof Assumption
3. $\angle B \cong \angle C$	3. Theorem 33 - "If two sides of a triangle are congruent, then the angles opposite those sides are congruent."
4. $\overline{AB} \ncong \overline{AC}$	4. R.A.A.

THEOREM 40

1) "In a given triangle, the sum of the measures of two sides, is greater than the measure of the third side."

2)



3) Given: $\triangle RTQ$

4) Prove: RT + TQ > RQ

5) Analysis: Auxiliary lines, Theorem 33, Corollary 38a

6)

STATEMENT	REASON
1. $\triangle RTQ$	1. Given
2. Draw $\overleftrightarrow{RT}$	2. Postulate 2
3. Find X on $\overleftrightarrow{RT}$, so that TX = TQ	3. Postulate 6 (Ruler)
4. Draw $\overline{QX}$	4. Postulate 2
5. $\overline{TX} \cong \overline{TQ}$	5. Definition of Congruent Segments
6. $\angle 2 \cong \angle 6$	6. Theorem 33 - "If two sides of a triangle are congruent, then the angles opposite those sides are congruent."
7. $m\angle 2 = m\angle 6$	7. Definition of Congruent Angles
8. $m\angle RQX = m\angle 1 + m\angle 2$	8. Postulate 7 (Protractor)
9. $m\angle RQX > m\angle 2$	9. Definition of ">"
10. $m\angle RQX > m\angle 6$	10. Substitution
11. RX > RQ	11. Corollary 39a - "The measure of the angle opposite the longer side of a triangle is greater than the measure of the angle opposite the shorter side."
12. RX = RT + TX	12. Postulate 6 (Ruler)
13. RT + TX > RQ	13. Substitution
14. RT + TQ > RQ	14. Substitution