

# Geometry: A Complete Course (with Trigonometry)

## Module B - Course Notes

Written by: Thomas E. Clark

**ERRATA**  
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**VideoText *Interactive***

Geometry: A Complete Course (with Trigonometry)  
Module B - Course Notes

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Send all inquiries to:  
Videotext*Interactive*  
P.O. Box 19761  
Indianapolis, IN 46219

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# Table of Contents

## Course Notes

### Unit II - Fundamental Terms

#### Part A - Undefined Terms

LESSON 1 - In Algebra . . . . .	77
LESSON 2 - In Geometry . . . . .	78

#### Part B - The Scope of Our Geometry

LESSON 1 - Good Definitions . . . . .	80
LESSON 2 - Definitions about Points . . . . .	82
LESSON 3 - Definitions about Lines . . . . .	85
LESSON 4 - Definitions about Rays . . . . .	88
LESSON 5 - Definitions about Line Segments . . . . .	90
LESSON 6 - Definitions about Angles as Sets of Points . . . . .	93
LESSON 7 - Definitions about Measurements of Angles . . . . .	95
LESSON 8 - Definitions about Pairs of Angles . . . . .	99
LESSON 9 - Definitions about Circles . . . . .	103

#### Part C - Measurement

LESSON 1 - Need . . . . .	108
LESSON 2 - Postulate 1 - Existence of Points . . . . .	112
LESSON 3 - Postulate 2 - Uniqueness of Lines, Planes, and Spaces . . . . .	113
LESSON 4 - Postulate 3 - One, Two, and Three Dimensions . . . . .	114
LESSON 5 - Postulate 4 - Separation of Lines or Planes . . . . .	115
LESSON 6 - Postulate 5 - Intersection of Lines or Planes . . . . .	116
LESSON 7 - Postulate 6 - Ruler . . . . .	117
LESSON 8 - Postulate 7 - Protractor . . . . .	119
LESSON 9 - Postulate 8 - Circle . . . . .	121
LESSON 10 - Postulate 9 - Uniqueness of Parallel Lines . . . . .	123
LESSON 11 - Postulate 10 - Uniqueness of Perpendicular Lines . . . . .	124

# UNDEFINED TERMS IN ALGEBRA

## Five Parts of Mathematical Speech

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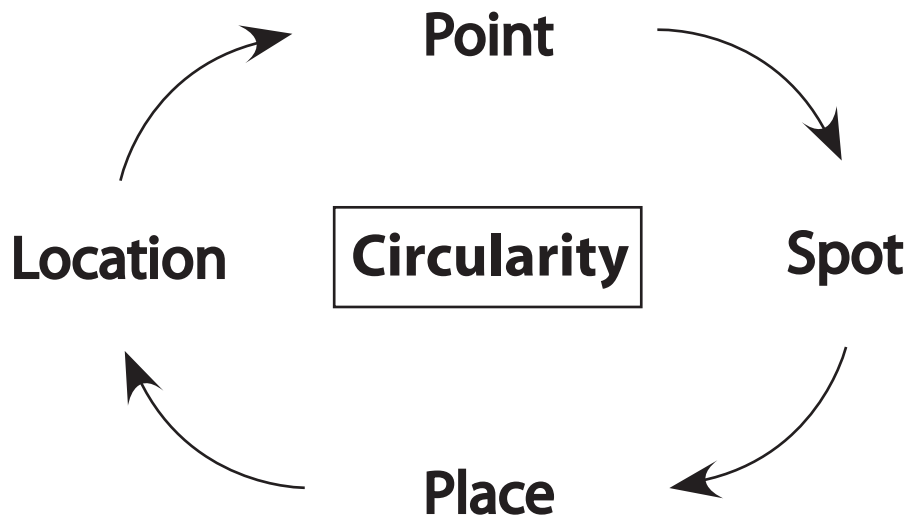
1. Number Symbols
2. Operation Symbols
3. Relation Symbols
4. Grouping Symbols
5. Placeholder Symbols

## Four Types of Mathematical Expressions

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1. Closed Phrase
2. Open Phrase
3. Closed Sentence
4. Open Sentence

# UNDEFINED TERMS IN GEOMETRY



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## Point

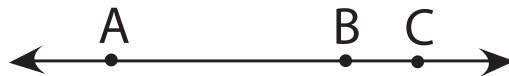
“a position in space”

- A point A

- no size
- no length
- no width
- no height

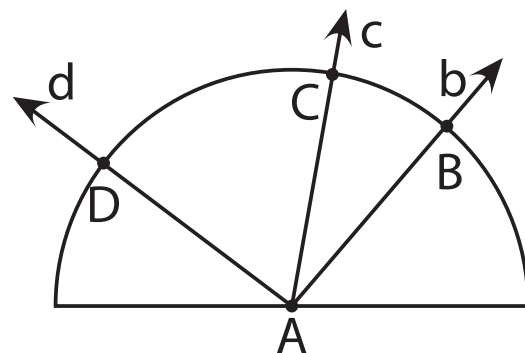
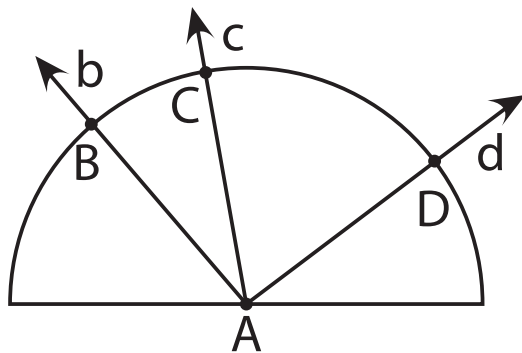
## Opposite Rays

“Two rays,  $\overrightarrow{BA}$  and  $\overrightarrow{BC}$  are considered opposite rays, if and only if, B is between A and C.”



## Betweenness of Rays

“For three coplanar rays,  $\overrightarrow{AB}$ ,  $\overrightarrow{AC}$ , and  $\overrightarrow{AD}$ , with coordinates  $b$ ,  $c$ , and  $d$  respectively,  $\overrightarrow{AC}$  is between  $\overrightarrow{AB}$ , if and only if, either  $b < c < d$ , or  $d < c < b$ .”

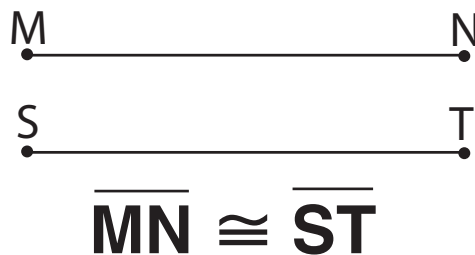


**Protractor**

# Congruent Figures

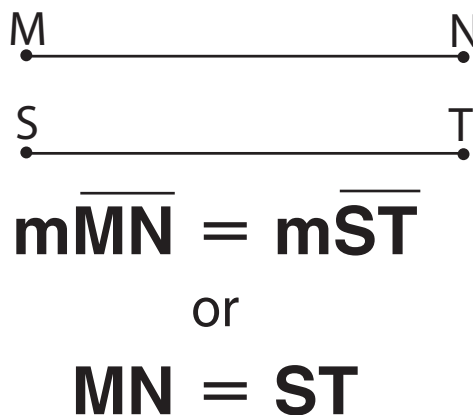
(“to agree”)

“Two geometric figures are said to be congruent, if and only if, they are exactly the same size and shape.”



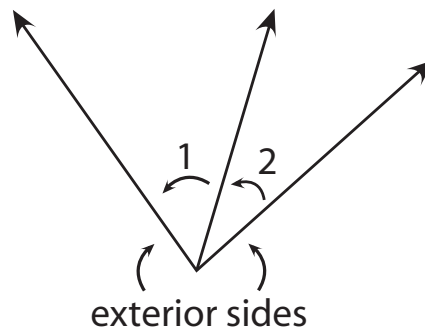
## Equal Line Segments

“Two line segments are considered equal, if and only if, the real numbers representing their lengths are equal.”



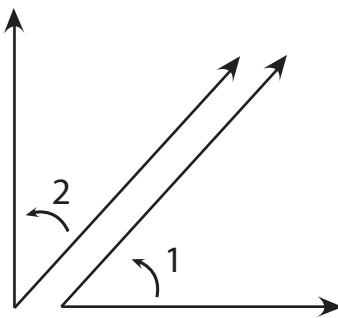
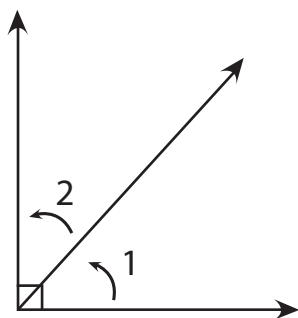
## Adjacent Angles ("lie near")

“Two angles are adjacent, if and only if, the two angles have a common vertex, and a common ray, and the ray lies between the other sides of the two angles.”



## Complementary Angles ("to complete")

“Two angles are complementary, if and only if, the sum of their measures is  $90^\circ$ ”



$\angle 1$  is the complement of  $\angle 2$

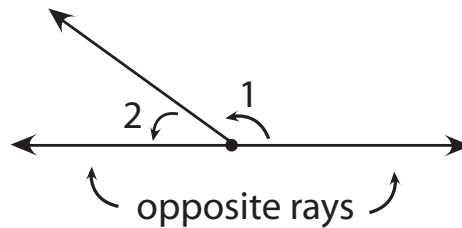
$\angle 2$  is the complement of  $\angle 1$

$$m\angle 1 + m\angle 2 = 90^\circ$$



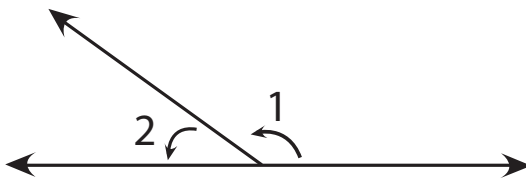
## Linear Pair of Angles

“Two angles are a linear pair, if and only if, they have a common side, and their exterior sides are opposite rays.”



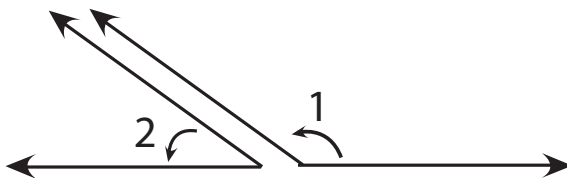
## Supplementary Angles (“to complete”)

“Two angles are supplementary, if and only if, the sum of their measures is  $180^\circ$ ”



$\angle 1$  is the supplement of  $\angle 2$

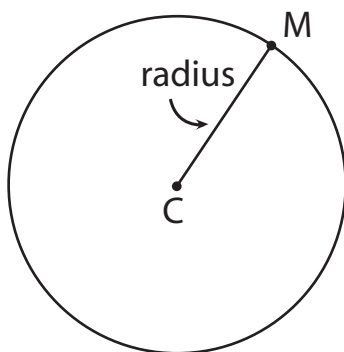
$\angle 2$  is the supplement of  $\angle 1$



$$m\angle 1 + m\angle 2 = 180^\circ$$

## Radius of a Circle ("spoke of a wheel")

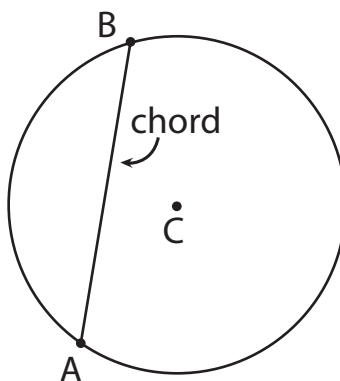
"A line segment is a radius of a circle, if and only if, the endpoints of the segment are the center of the circle, and a point on the circle."



$\overline{MC}$  is a radius of  $\odot C$

## Chord of a Circle ("cord")

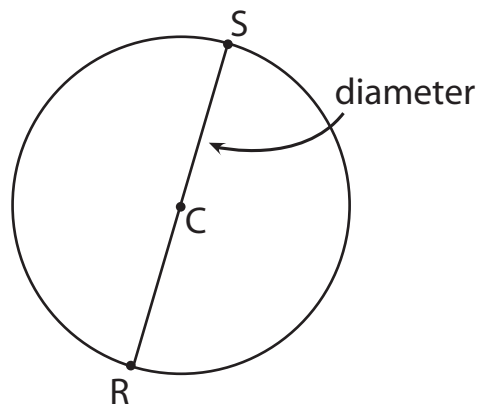
"A line segment is a chord of a circle, if and only if, the endpoints of the segment are two points on the circle."



$\overline{AB}$  is a chord of  $\odot C$

## Diameter of a Circle ("across measure")

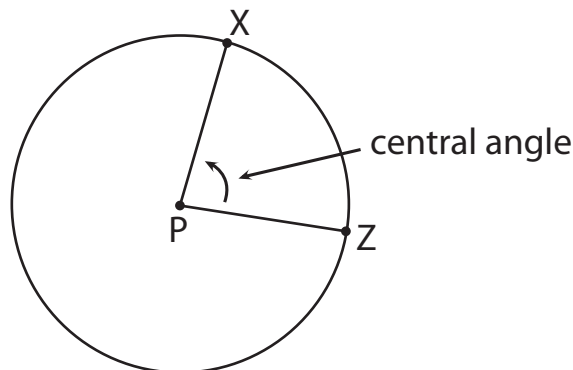
"A line segment is a diameter of a circle, if and only if, it is a chord of the circle, and it passes through the center of the circle."



$\overline{SR}$  is a diameter of  $\odot C$

## Central Angle of a Circle

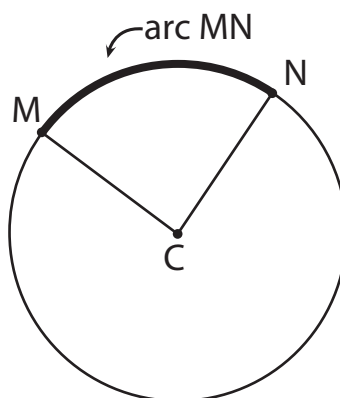
"An angle is a central angle of a circle, if and only if, the vertex of the angle is the center of the circle."



$\angle XPZ$  is a central angle of  $\odot P$

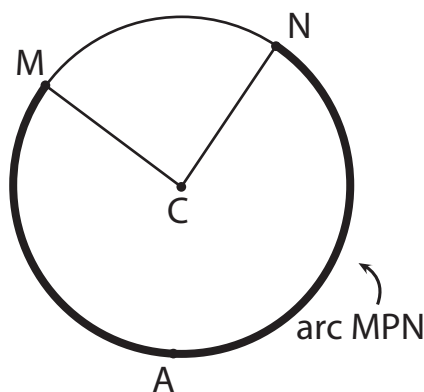
## Arc of a Circle ("bow")

"An arc of a circle is a **minor arc**, if and only if, it is the union of 2 points on the circle, **not** the endpoints of a diameter, and the set of points on the circle which lie in the **interior** of the angle formed by the radii containing the two points."



$\widehat{MN}$  (arc MN) is a minor arc of  $\odot C$

"An arc of a circle is a **major arc**, if and only if, it is the union of 2 points on the circle, **not** the endpoints of a diameter, and the set of points on the circle which lie in the **exterior** of the angle formed by the radii containing the two points."



$\widehat{MPN}$  (arc MPN) is a major arc of  $\odot C$

## Properties of Operations on Real Numbers

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**Existence**– A solution must exist

**Uniqueness**– The solution must be unique, for a particular problem

**Closure**– The solution must be in the usable set.

---

**Commutativity**– Numbers may be “moved around”, when adding or multiplying.

**Associativity**– Numbers may be “grouped differently”, when adding or multiplying

**Distributivity**– Numbers may be “handed out”, when multiplying to sums or differences.

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**Zero**–

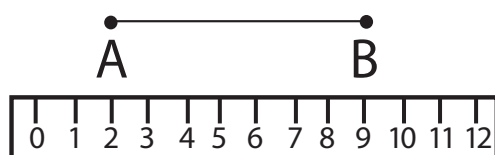
$$n + 0 = n$$
$$n - n = 0$$
$$n \cdot 0 = 0$$
$$n \div 0 \text{ is meaningless}$$

**One**–

$$n \cdot 1 = n$$
$$n \div 1 = n$$
$$n \div n = 1$$

## Postulate 6: Ruler (cont.)

**3rd Assumption**—“If you have two points on a line, for which coordinates have been assigned, then the distance between those two points, is the absolute value of the difference between their coordinates.”



$$|2 - 9| = 7$$

$$|9 - 2| = 7$$

**4th Assumption**—“If, on a line, point B lies between points A and C, then the measure of the distance from point A to point B, plus the measure of the distance from point B to point C, is equal to the measure of the distance from point A to point C.” (Segment Addition)



$$\overline{mAB} + \overline{mBC} = \overline{mAC}$$

or

$$AB + BC = AC$$