Geometry: A Complete Course

(with Trigonometry)

Module C - Student WorkText

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ERRATA
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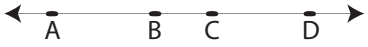
VideoText *Interactive*

6. In the figure below, $\overline{AC}$ and $\overline{BD}$ share the common segment $\overline{BC}$.



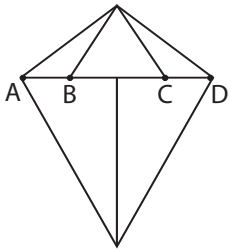
Prove the following conditional statements, using the two-column format.

a) If $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

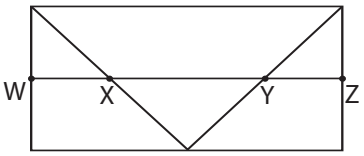
b) If $\overline{AC} \cong \overline{BD}$, then $\overline{AB} \cong \overline{CD}$.

7. Use the results of Exercise 6,and the information given in each of the following, to draw a conclusion concerning each diagram. (Note: You are not being asked to formally prove anything.)

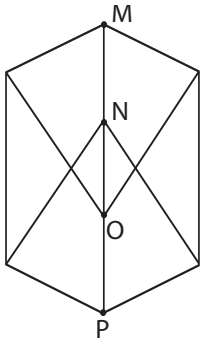
a) $\overline{AB} \cong \overline{CD}$



b) $\overline{WY} \cong \overline{XZ}$

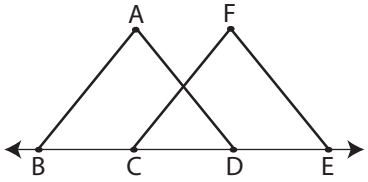


c) $\overline{MN} \cong \overline{OP}$



8. Given: $\overline{AB} \cong \overline{BD}$

$\overline{BC} \cong \overline{DE}$



Prove: $\overline{AB} \cong \overline{CE}$

(hint: Use the results of exercise 6)

Thinking through Theorem 1:

9. Are the six points shown in the figure in Exercise 8 all necessarily in the same plane? Explain why or why not.

10. Does the relationship of the triangles in Exercise 8 have any effect on the proof in Exercise 8? Explain.

Lesson 1 — Exercises:

1. Prove Theorem 3 – "If you have a given ray, then there is exactly one point at a given distance from the endpoint of the ray." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)

a) State the theorem.

b) Draw and label a diagram to accurately show the conditions of the theorem.

c) List the given information.

d) Write the statement we wish to prove.

e) Demonstrate the direct proof using the two-column format.

2. A ray has how many endpoints?

3. What is the endpoint of $\overrightarrow{QR}$?

4. Do $\overrightarrow{BA}$ and $\overrightarrow{AB}$ name the same ray? Why or why not?

5. Must $\overrightarrow{QT}$ and $\overrightarrow{QS}$ be opposite rays? Why or why not?

6. Must $\overrightarrow{QT}$ and $\overrightarrow{QS}$ be opposite rays if points Q, T, and S are collinear? Why or why not?

7. Draw a diagram that illustrates the information given.

a) Point Q is on $\overleftrightarrow{AB}$, but Q is not on $\overrightarrow{AB}$.

b) $\overrightarrow{AB}$ and $\overrightarrow{AX}$ are opposite rays.

c) $\overrightarrow{RS}$ and $\overrightarrow{RT}$ are the same ray.

d) $\overrightarrow{PQ}$ and $\overrightarrow{PT}$ are not collinear.

8. For each of the following, tell whether $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are opposite rays. Answer "yes", "no", or "not enough information".

a) The coordinate of A is 7, the coordinate of B is 8, and the coordinate of C is –14.

b) A, B, and C are not collinear.

9. Point B is between points A and C. Point D is between B and C. If BC = x, AD = y, and BA = z, find the lengths of BD and DC, in terms of x, y, and z.

10. The length of $\overline{AB}$ is 6. The coordinate of point B is –6.

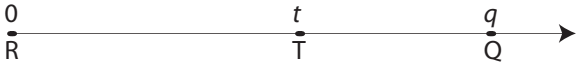
a) What are the possible coordinates of point A?

b) What are the possible coordinates of the midpoint of $\overline{AB}$?

c) Find the length of the line segment whose endpoints are the possible midpoints of $\overline{AB}$.

11. Given: The coordinates of R and T are 0 and *t*, respectively; $t > 0$; T is between R and Q. (The diagram is already drawn.)

Prove: The coordinate of Q is positive.



Complete the proof:

STATEMENT	REASON
1. T is between R and Q; the coordinates of R and T are zero and *t*, respectively.	1. _____
2. Q has a coordinate, call it *q*.	2. Postulate 6 (Ruler) – First Assumption
3. RT + TQ = RQ	3. _____
4. $RT = \lvert 0 - t \rvert = \lvert -t \rvert = \lvert t \rvert$ $TQ = \lvert t - q \rvert$ $RQ = \lvert 0 - q \rvert = \lvert -q \rvert = \lvert q \rvert$	4. Postulate 6 (Ruler) – Third Assumption
5. $\lvert t \rvert + \lvert t - q \rvert = \lvert q \rvert$	5. _____
6. $\lvert t - q \rvert < \lvert q \rvert$	6. Definition of >.
7. $t - q < \lvert q \rvert$ and $t - q > -\lvert q \rvert$ (consider $t - q < \lvert q \rvert$) (See step 28)	7. Absolute Value Property: If $\lvert x \rvert < k$, then $x < k$ and $x > -k$ (VideoText Algebra-II-D4)
8. $t - q + q < \lvert q \rvert + q$	8. _____
9. $t + 0 < \lvert q \rvert + q$	9. _____

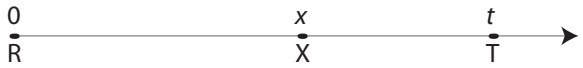
32. $-t+0<\lvert q\rvert+-q$	32. _____
33. $-t<\lvert q\rvert+-q$	33. _____
34. $\lvert q\rvert=q$	34. Definition of Absolute Value
35. $-t<q+-q$	35. _____
36. $-t<0$	36. _____
37. $(-1)(-t)>(-1)(0)$	37. _____
38. $(-1)(-t)>0$	38. _____
39. $t>0$	39. _____

(Note: $t>0$ was given. So $t-q>-\lvert q\rvert$ has no bearing on the proof. And we have already considered $t>0$)

12. Using the concepts in Exercise 11 as a guide, write a proof for the following:

Given: The coordinates of R and T are 0 and *t*, respectively; $t>0$; X is between R and T.

Prove: The coordinate of X is positive.



Lesson 2 — Exercises:

1. Prove Theorem 4 – "If you have a given line segment, then that segment has exactly one midpoint." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)

a) State the theorem.

b) Draw and label a diagram to accurately show the conditions of the theorem.

c) List the given information.

d) Write the statement we wish to prove.

e) Demonstrate the indirect proof using the two-column format.

2. For Q to be the midpoint of $\overline{MN}$, what conditions must be satisfied?

3. If $\overline{AB} \cong \overline{CD}$, how is AB related to CD?

4. The midpoint of $\overline{PQ}$ is point N. $\overline{RS}$ bisects $\overline{PQ}$. What is the intersection of $\overline{PQ}$ and $\overline{RS}$?

5. If X, Y, and Z are three collinear points, and XZ = YZ, what statement can be made about point Z?

6. If line ℓ bisects $\overline{AM}$, what point of $\overline{AM}$ does ℓ contain?

7. If $\overline{CD}$ is not congruent to $\overline{EF}$, how is CD related to EF?

8. If M and N have coordinates $\sqrt{15}$ and $\sqrt{183}$ respectively, does $\overline{MN}$ have a midpoint?

For Exercises 9 through 11, use the information provided and demonstrate the indicated proof. Also name the property which describes the relationship.

9. Given: $\overline{AB}$ is a line segment

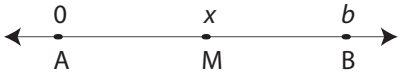
Prove: $\overline{AB} \cong \overline{AB}$

5. $AB = \lvert 0 - b \rvert = \lvert -b \rvert = b$, and $AM = \left\lvert 0 - \frac{b}{2} \right\rvert = \left\lvert -\frac{b}{2} \right\rvert = \frac{b}{2}$, and $MB = \left\lvert \frac{b}{2} - b \right\rvert = \left\lvert -\frac{b}{2} \right\rvert = \frac{b}{2}$	5. _____
6. $AM = MB$	6. _____
7. $\overline{AM} \cong \overline{MB}$	7. _____
8. $AM + MB = \frac{b}{2} + \frac{b}{2}$	8. _____
9. $AM + MB = \left(\frac{1}{2} + \frac{1}{2} \right) \cdot b$	9. _____
10. $AM + MB = 1 \cdot b$	10. _____
11. $AM + MB = b$	11. _____
12. $AM + MB = AB$	12. _____
13. M is between point A and point B	13. _____
14. M is the midpoint of AB	14. _____

14. Complete the following proof, by supplying the missing reasons.

Given: The coordinates of point A and point B on AB are 0 and b, respectively; b > 0; point M is the midpoint of AB.

Prove: The coordinate of M is $\frac{b}{2}$



Statement	Reason
1. M is the midpoint of AB	1. _____
2. M corresponds to some real number x	2. _____
3. $b > 0$	3. _____
4. $b - x > 0 - x$	4. _____

Unit III — Fundamental Theorems

Part E — Theorems About Angles–Part 1 (One Angle)

Lesson 1 — *Theorem 7: "If, in a half-plane, there is a ray in the edge of the half-plane, then there is exactly one other ray through the endpoint of the given ray, such that the angle formed by the two rays has a given measure."*

Objective: To understand this theorem as an application of previously accepted postulates and demonstrate its proof directly.

Lesson 1 — Exercises:

1. Prove Theorem 7 – "If, in a half-plane, there is a ray in the edge of the half-plane, then there is exactly one other ray through the endpoint of the given ray, such that the angle formed by the two rays has a given measure." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)

a) State the theorem.

b) Draw and label a diagram to accurately show the conditions of the theorem.

c) List the given information.

d) Write the statement we wish to prove.

e) Demonstrate the direct proof using the two-column format.

Unit III — Fundamental Theorems

Part F — Theorems About Angles–Part 2 (Two Angles)

Lesson 3 — *Theorem 11: "If you have right angles, then those right angles are congruent."*

Objective: To understand this theorem as an application of previously accepted postulates and demonstrate its proof directly.

Lesson 3 — Exercises:

1. Prove Theorem 11 – "If you have right angles, then those right angles are congruent." (Note: This is the same theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check.)

a) State the theorem.

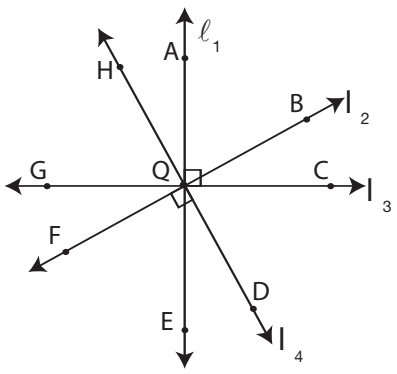
b) Draw and label a diagram to accurately show the conditions of the theorem.

c) List the given information.

d) Write the statement we wish to prove.

e) Demonstrate the direct proof using the two-column format.

2. Name all the right angles in the figure below.

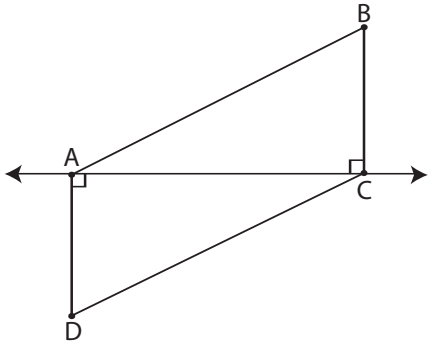


8. Given: $\overleftrightarrow{AC} \perp \overline{AD}$

$\overleftrightarrow{AC} \perp \overline{CB}$

$\angle 1 \cong \angle 2$

Prove: $\angle DAB \cong \angle BCD$

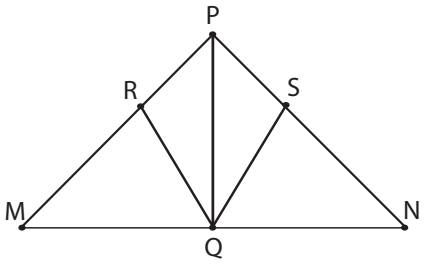


9. Given: $\overline{PQ} \perp \overline{MN}$

$\overline{RQ}$ bisects $\angle MQP$

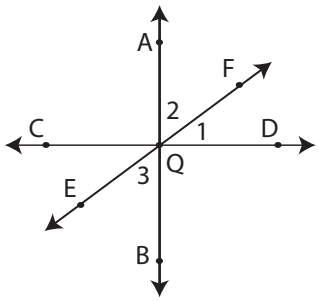
$\overline{SQ}$ bisects $\angle NQP$

Prove: $\angle MQS \cong \angle NQR$



10. Given: $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$ at point Q

Prove: $\angle 1$ and $\angle 3$ are complementary



Unit III — Fundamental Theorems

Part G — Theorems About Angles–Part 3 (More Than Two Angles)

Lesson 2 — *Theorem 14: "If two angles are supplementary to the same angle or congruent angles, then they are congruent to each other."*

Objective: To understand this theorem as an application of previously accepted postulates and demonstrate its proof directly.

Lesson 2 — Exercises:

1. Prove Theorem 14, Part 2 – "If two angles are supplementary to the same angle, then they are congruent to each other." (Note: This is the second part of the theorem we proved in the lesson. We are using it as an exercise to make sure you understand its proof. Use your course notes to check, but understand that the course notes refer to the first case.)

a) State the theorem.

b) Draw and label a diagram to accurately show the conditions of the theorem.

c) List the given information.

d) Write the statement we wish to prove.

e) Demonstrate the direct proof using the two-column format.

2. Prove Theorem 14, Part 1 – "If two angles are supplementary to the congruent angles, then they are congruent to each other." (Note: This is the first part of the theorem we proved in the lesson.)

a) State the theorem.

b) Draw and label a diagram to accurately show the conditions of the theorem.

c) List the given information.

d) Write the statement we wish to prove.

e) Demonstrate the direct proof using the two-column format.

3. Find the measure of the supplement of each of the following angles:

a) $m\angle ABC = 42$

b) $m\angle MNO = x - 3$

c) $m\angle RST = x^2$

4. Two supplementary angles are congruent. Find their measures.

5. If $\angle ACB$ and $\angle EDF$ are supplementary, find the value of x, $m\angle ACB$, and $m\angle EDF$ in each of the following:

a) $m\angle ACB = 2x$; $m\angle EDF = x - 15$

b) $m\angle ACB = x + 16$; $m\angle EDF = 2x - 16$

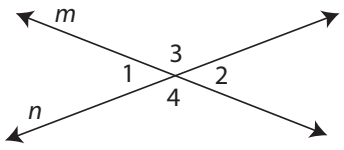
c) $m\angle ACB = x^2$; $m\angle EDF = 12x - 9$

6. Find the measure of an angle that is twice as large as its supplement.

7. If $\angle A \cong \angle B$, $m\angle A + m\angle C = 180$, and $m\angle B + m\angle D = 180$, then $\angle C \cong \angle D$. Why?

2. Prove Theorem 15, Part 2 – "If two lines intersect, then the vertical angles formed are congruent." (Note: This is the same theorem we proved in the lesson, but you are being asked to prove the other 2 vertical angles congruent – angles 2 and 4.)
   a) State the theorem.
   b) Draw and label a diagram to accurately show the conditions of the theorem.
   c) List the given information.
   d) Write the statement we wish to prove.
   e) Demonstrate the direct proof using the two-column format.

For Exercises 3 through 7, use the diagram to the right:



3. If $m\angle 3 = 110$, find:
   a) $m\angle 4$
   b) $m\angle 1$
   c) $m\angle 2$

4. If $m\angle 1 = \frac{1}{2} m\angle 3$, find:
   a) $m\angle 1$
   b) $m\angle 2$
   c) $m\angle 3$
   d) $m\angle 4$

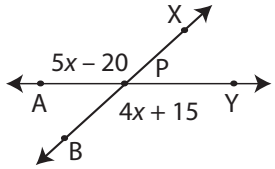
5. If $\angle 3$ and $\angle 4$ are congruent and supplementary, what can you conclude about line *m* and line *n*?

6. If $\angle 3$ and $\angle 4$ are congruent and complementary, what is $m\angle 4$?

7. Suppose $m\angle 3$ is increased by 10. What effect would this have on:
   a) $m\angle 4$
   b) $m\angle 1$
   c) $m\angle 2$

In Exercises 8 through 11, find the value of x and the measure of each simple angle in the diagram, for which there is no given measure.

8.



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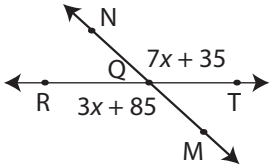

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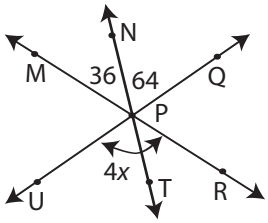

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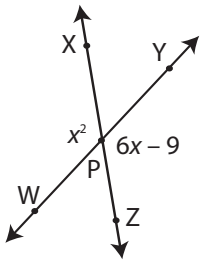
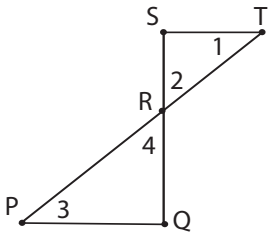


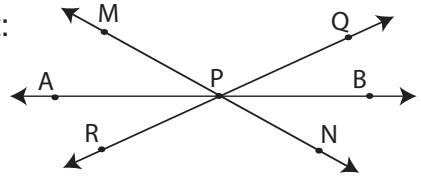
12. Given: $\angle 1 \cong \angle 2$

$\angle 3 \cong \angle 4$

Prove: $\angle 1 \cong \angle 3$



For Exercises 13 through 15, use the diagram to the right:



13. Given: $\angle APR \cong \angle NPB$

Prove: $\angle MPA \cong \angle APR$

14. Given: $\angle APR \cong \angle NPB$

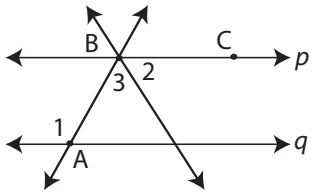
Prove: $\angle MPA \cong \angle QPB$

15. Given: $\overrightarrow{PA}$ bisects $\angle MPR$

Prove: $\overrightarrow{PB}$ bisects $\angle NPQ$

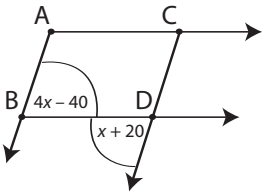
5. $p \parallel q$, $m\angle 1 = 100$, $m\angle 2 = 55$

Find $m\angle 3$



6. $\overrightarrow{AB} \parallel \overrightarrow{CD}$

Find m $\angle ABD$

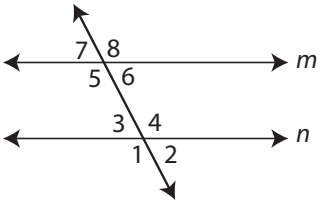


7. $m \parallel n$, $m\angle 6 = 75$

a) Find $m\angle 3$ **d)** Find $m\angle 4$

b) Find $m\angle 7$ **e)** Find $m\angle 1$

c) Find $m\angle 2$ **f)** Find $m\angle 8$



8. $\ell \parallel m$. Find x.

a) $m\angle 1 = 3x + 7$

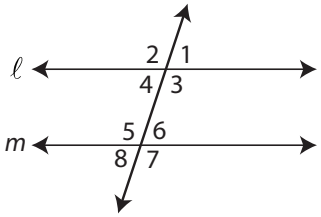
$m\angle 6 = 5x - 3$

b) $m\angle 4 = 8x + 12$

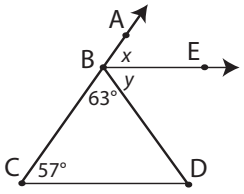
$m\angle 6 = 2x + 54$

c) $m\angle 4 = x^2 + 5x$

$m\angle 8 = 9x + 12$



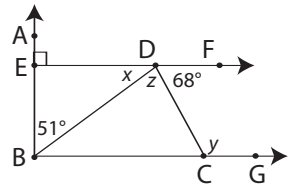
9. $\overrightarrow{BE} \parallel \overline{CD}$. Find x and y.



10. $\overrightarrow{ED} \parallel \overrightarrow{BC}$

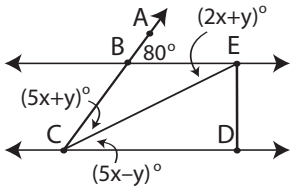
$\overline{AB} \perp \overrightarrow{ED}$

Find x, y and z.



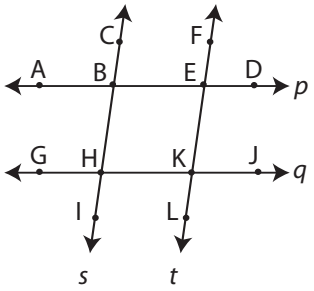
11. $\overrightarrow{BE} \parallel \overrightarrow{CD}$

Find x, y, $m\angle ACE$ and $m\angle BEC$



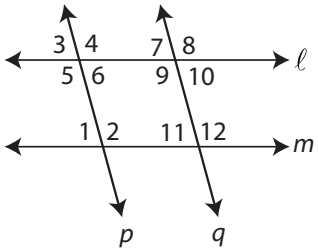
12. Given: $p \parallel q$, $s \parallel t$

Prove: $\angle GHC \cong \angle AEF$



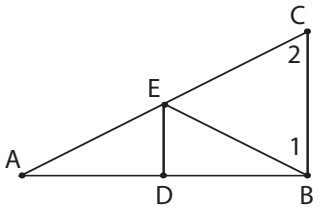
13. Given: $\ell \parallel m$, $p \parallel q$

Prove: $\angle 2 \cong \angle 9$



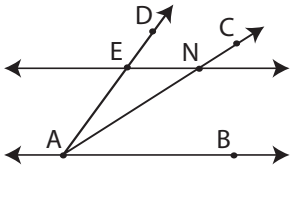
14. Given: $m\angle 1 = m\angle 2$
$\overline{DE} \parallel \overline{BC}$

Prove: $\overline{ED}$ bisects $\angle AEB$



15. Given: $\overrightarrow{AC}$ bisects $\angle DAB$
$\overleftrightarrow{EN} \parallel \overleftrightarrow{AB}$

Prove: $\angle DAN \cong \angle ENA$



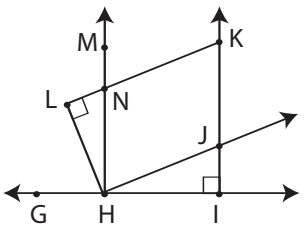
10. Given: $\overline{LK} \parallel \overrightarrow{HJ}$

$\overline{IK} \perp \overleftrightarrow{GI}$

$\overline{LH} \perp \overline{LK}$

$\overrightarrow{HM} \parallel \overrightarrow{IK}$

$m\angle LKI = 72°$



Find:

a) $m\angle MHJ$

b) $m\angle MHI$

c) $m\angle HJI$

d) $m\angle JHI$

e) $m\angle LHJ$

f) $m\angle LHM$

g) $m\angle GHL$

h) $m\angle HJK$

i) $m\angle MNK$

j) $m\angle LNM$

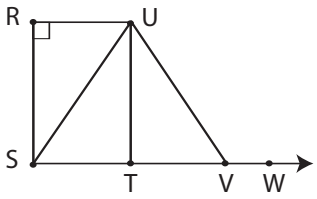
11. Given: $\overline{RU} \parallel \overline{SV}$

$\overline{RS} \parallel \overline{UT}$

$\overline{RS} \perp \overline{RU}$

$m\angle USW = 55°$

$\overline{TU}$ bisects $\angle SUV$

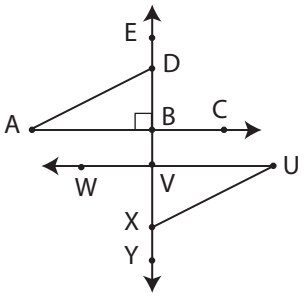


Prove: $m\angle TUV = 35°$

12. Given: $\overrightarrow{AC} \parallel \overrightarrow{UW}$

$\overleftrightarrow{EY} \perp \overrightarrow{AC}$

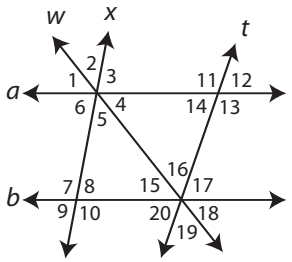
$\overline{AD} \parallel \overline{UX}$



Prove: $m\angle YDA + m\angle ABE =$

$m\angle EXU + m\angle UVY$

Using the figure to the right, determine whether the information given in Exercises 15 through 18 allows you to conclude that line *a* is parallel to line *b* using Theorem 21.



15. m∠17 + m∠13 = 180

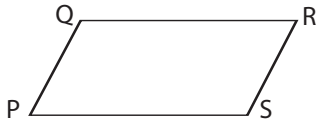
16. m∠5 + m∠6 + m∠8 = 180

17. m∠14 + m∠15 + m∠16 = 180

18. m∠1 + m∠16 + m∠20 = 180

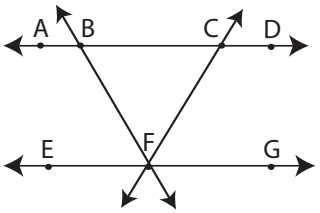
19. Given: ∠R and ∠S are supplementary angles
∠Q ≅ ∠S

Prove: $\overline{QP} \parallel \overline{RS}$



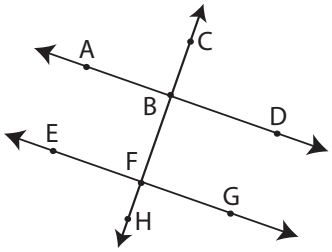
20. Given: ∠BFE ≅ ∠BFC
∠BCF ≅ ∠BFE
$\overrightarrow{FC}$ bisects ∠BFG

Prove: $\overleftrightarrow{AD} \parallel \overleftrightarrow{EG}$



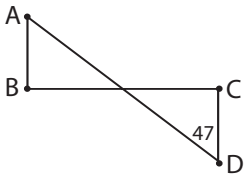
21. Given: ∠ABC ≅ ∠CBD
∠ABC ≅ ∠EFH

Prove: $\overleftrightarrow{AD} \parallel \overleftrightarrow{EG}$



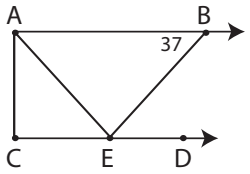
8. In the figure to the right, m∠D is given, and $\overline{AB} \perp \overline{BC} \perp \overline{CD}$.

Find m∠A. Justify your answer.



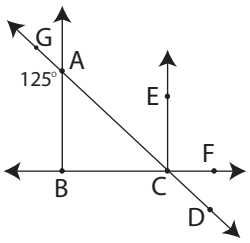
9. In the figure to the right, $\overrightarrow{AB} \perp \overline{AC} \perp \overrightarrow{CD}$.

Find m∠BEC and m∠BED. Justify your answers.



10. In the figure to the right, $\overrightarrow{BA} \perp \overleftrightarrow{BF} \perp \overrightarrow{CE}$.

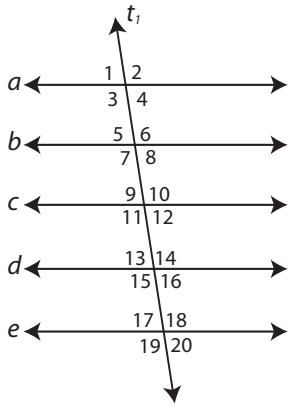
Find m∠BAC and m∠DCE. Justify your answers.



11. Suppose a set of points consists of a plane and a line not in the plane. State the minimum number of points that must be in the set.

12. Suppose two different planes both contain point A and point B. Describe the relationship between the planes.

In Exercises 2 through 7, use the figure at the right and the given information to apply Theorem 23, naming two lines which are parallel to each other because these two lines are parallel to a third line.



2. $a \parallel b$; $\angle 5 \cong \angle 13$

3. $\angle 17 \cong \angle 12$; $\angle 12$ and $\angle 14$ are supplementary angles

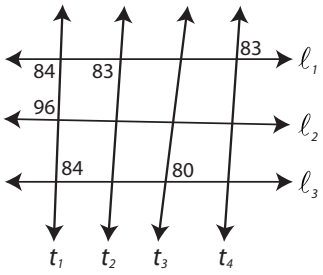
4. $\angle 19 \cong \angle 2$; $\angle 2 \cong \angle 10$

5. $\angle 5$ and $\angle 15$ are supplementary angles; $\angle 5 \cong \angle 9$

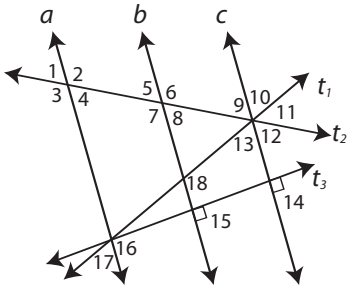
6. $\angle 4 \cong \angle 17$; $\angle 7$ and $\angle 17$ are supplementary angles

7. $\angle 1 \cong \angle 16$; $\angle 16$ and $\angle 18$ are supplementary angles

8. Of the seven lines making up the figure below, which lines must be parallel? Give a reason for each answer, using theorems or corollaries we have proved in our Geometry.



Referring to the figure to the right, determine whether the information given in Exercises 9 through 12 allows you to conclude that line *a* is parallel to line *b* using Theorem 23. Say yes or no, and give reasons for your answers.



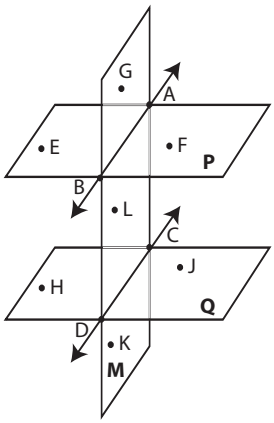
9. $\angle 13$ and $\angle 18$ are supplementary angles; $\angle 1 \cong \angle 9$

10. $b \perp t_3$; $c \perp t_3$; $\angle 1 \cong \angle 8$

11. $m\angle 18 = m\angle 11 + m\angle 12$;
$\angle 3$ and $\angle 8$ are supplementary angles

12. $m\angle 11 + m\angle 12 + m\angle 17 = 180$; $m\angle 7 = m\angle 10 + m\angle 11$

4. Recall from Unit II, Part B, Lesson 6, Exercise 4 (page 143), that an angle formed by two intersecting planes is a dihedral angle, and is generally named by using a point on each face and two points on the edge. In the figure at the right, two planes, P and Q, are cut by a third plane, M. Planes P and M intersect in $\overleftrightarrow{AB}$. Planes Q and M intersect in $\overleftrightarrow{CD}$. Points E and F are on plane P. Points J and H are on plane Q. Points G, L, and K are on plane M.



a) Name two pairs of alternate interior dihedral angles.

b) Name two pairs of alternate exterior dihedral angles.

c) Name two pairs of corresponding dihedral angles.

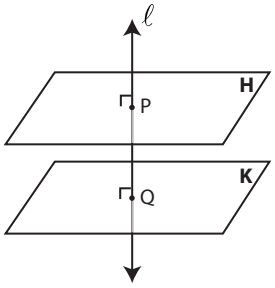
d) Name two pairs of vertical dihedral angles.

e) Plane P is parallel to plane Q.

1) Do the alternate interior dihedral angles have equal measures?

2) Do the corresponding dihedral angles have equal measures?

5. Prove: If two planes are perpendicular to the same line, then the two planes are parallel.



Given: Plane H $\perp$ ℓ at point P

Plane K $\perp$ ℓ at point Q

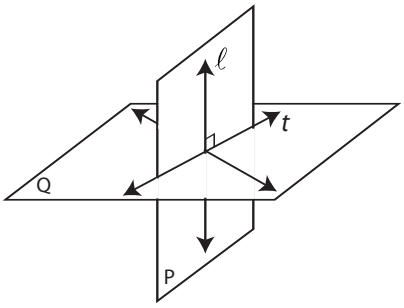
Prove: Plane H $\parallel$ Plane K

Complete the following proof: (Hint: Draw the plane indicated in Statement 3)

STATEMENT	REASON
1. Plane H $\perp$ ℓ at point P Plane K $\perp$ ℓ at point Q	1. Given
2. Line ℓ contains point P and point Q	2. _____

3. There exists a plane which passes through line ℓ and contains points P and Q. Through line ℓ pass plane M intersecting plane H in $\overleftrightarrow{PA}$ and intersecting plane K in $\overleftrightarrow{QB}$.	3. Postulate 5 – _____ _____
4. Line $\ell \perp \overleftrightarrow{PA}$ Line $\ell \perp \overleftrightarrow{QB}$	4. Definition of a plane perpendicular to a line- "A plane is perpendicular to a given line, if and only if, every line that intersects the given line in the plane, is perpendicular to the given line."
5. $\angle APQ$ is a right angle $\angle BQP$ is a right angle	5. _____
6. $m\angle APQ = 90$ $m\angle BQP = 90$	6. _____
7. $m\angle APQ + m\angle BQP = 90 + 90$	7. _____
8. $m\angle APQ + m\angle BQP = 180$	8. _____
9. $\angle APQ$ and $\angle BQP$ are supplementary	9. _____
10. $\overleftrightarrow{PA} \parallel \overleftrightarrow{QB}$	10. _____
11. $\overleftrightarrow{PA}$ and $\overleftrightarrow{QB}$ do not intersect	11. Definition of Parallel Lines - Lines which are coplanar and do not intersect.
12. Plane H and plane K do not intersect	12. Any given line in plane H will not intersect the corresponding line in plane K, any given line being one of the infinitely many in plane H, and having the same relationship as $\overleftrightarrow{PA}$ and $\overleftrightarrow{QB}$.
13. Plane H $\parallel$ Plane K	13. Definition of Parallel Planes

6. Prove that if a line is perpendicular to a given plane, then any plane containing that line is perpendicular to the given plane.



Given: Line $\ell \perp$ plane Q

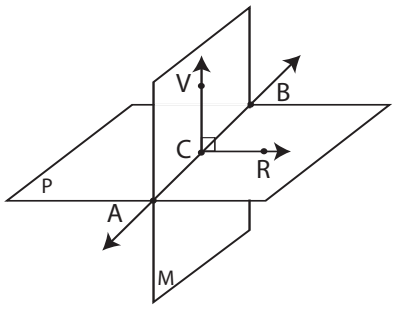
Plane P contains line ℓ

Prove: P $\perp$ Q

Complete the following proof:

STATEMENT	REASON
1. $\ell \perp$ plane Q; plane P contains ℓ	1. Given
2. Line t is in plane Q and plane P	2. _____
3. $\ell \perp t$	3. _____
4. Plane P $\perp$ plane Q	4. Definition of Perpendicular Planes – "Two intersecting planes are perpendicular, if and only if, there exists a line in one of the planes, which is perpendicular to both the other plane, and the line of intersection of the two planes."

7. Prove that if two planes intersect to form a right dihedral angle, then the planes are perpendicular.



Given: Plane P intersects plane M in $\overleftrightarrow{AB}$

$\angle$V-AB-R is a right dihedral angle

Prove: Plane P $\perp$ Plane M