## VideoText Interactive

## HomeSchool and Independent Study Sampler

Print Materials<br>for "Algebra: A Complete Course"

## Unit II, Part A, Lesson 4 "Combinations"

Course Notes (2 pages) Student WorkText (5 pages) Solutions Manual (2 pages) Quizzes - Forms A and B (4 pages) Quiz Solutions (4 pages)
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## COURSE NOTE 51

## COMBINATIONS

$$
3 w+4=10
$$

Add ${ }^{-} 4 \quad 3 w+4+{ }^{-} 4=10+{ }^{-} 4$
$3 w=6$
Multiply $\begin{aligned} \frac{1}{3} \quad \frac{1}{3}(3 w) & =\frac{1}{3}(6) \\ 1 w & =\frac{6}{3} \text { or } 2\end{aligned}$
Check $3(2)+4=10$

$$
6+4=10
$$

$$
10=10 \quad \text { True }
$$

$S=\{2\}$


Or ...

$$
\begin{array}{lll} 
& 3 w+4 & =10 \\
\text { Multiply } \frac{1}{3} & \frac{1}{3}(3 w+4) & =\frac{1}{3}(10) \\
& w+\frac{4}{\mathbf{3}} & -\frac{10}{3} \\
\text { Add } \frac{-4}{3} & w+\frac{4}{3}+\frac{-4}{3} & -\frac{10}{3}+\frac{-4}{3} \\
& w & =\frac{6}{3} \text { or } 2 \checkmark
\end{array}
$$

COURSE NOTE 52

$$
-6 x-2 \geq 40
$$

Add 2

$$
{ }^{-} 6 x-2+{ }^{+} 2 \quad 2 \quad 40+{ }^{+} 2
$$

$$
\text { Multiply } \frac{1}{-6} \quad \frac{1}{-6}(-6 x) \quad \leq \frac{1}{-6}(42)
$$

Check

$$
\begin{aligned}
& 1 x \leq \frac{42}{-6} \text { or }{ }^{-7} \\
&-6(-10)-2 \geq 40 \\
& 60-2 \geq 40 \\
& 58 \geq 40 \\
& S=\{x \mid x \leq 7\} \\
&
\end{aligned}
$$

## Part A - Basic Equations and Inequalities

## LESSON 4 Combinations

Objective: To be able to solve simple equations or inequalities by making 0 's and 1 's appropriately.

## Important Terms:

'The Opposite Of'" - a real number which has the same absolute value as a given number, but the opposite sign, so that the sum of the two numbers is 0 . For example. the opposite of +3 is -3 , because $+3+(-3)=0$.

Reciprocal - a real number (not equal to 0 ) which has the same sign as a given number but which, in fraction form, has the numerator and denominator interchanged, so that the product of the two numbers is 1 . For example, the reciprocal of $\frac{+2}{3}$ is $\frac{3}{+2}$ because $\frac{+2}{3} \cdots+\frac{3}{2}=\frac{0}{6}$ or 1 .

Example 1: Find the solution set for the following open sentence by making the appropriate 0's and 1's.

$$
3 x-8=34
$$

Solution: In this equation, we are trying to find appropriate values for " 1 " of the placeholder. That means we want only lx, so we must make a " 1 " out of the 3 and a " 0 " out of the -8 .

## Example 1 cont'd:

Method $\mathbf{1}$
We make the " 1 " first by multiplying by $\frac{1}{3}$ (the reciprocal of $\mathbf{3}$ or $\frac{\mathbf{3}}{\mathbf{1}}$ ).

$$
3 x-8=34
$$

Mult. $\frac{1}{3} \quad \frac{1}{3}(3 x-8)=\frac{1}{3}(34)$

$$
1 x-\frac{8}{3}=\frac{34}{3}
$$

Now make a zero.

$$
\text { Add } \begin{aligned}
\frac{8}{3} \quad 1 x-\frac{8}{3}+\frac{8}{3} & =\frac{3}{3}+\frac{8}{3} \\
1 x+0 & =\frac{42}{3} \\
1 x & =14
\end{aligned}
$$

We check this solution by substitution in the original equation.

$$
\begin{aligned}
3(14)-8 & =34 \\
42-8 & =34 \\
34 & =34 \quad \text { It checks. }
\end{aligned}
$$

The solution set is $S=\{14)$.

## Method 2

We make the " 0 " first by adding +8 (the opposite of -8 ).

$$
\begin{aligned}
& 3 x-8=34 \\
& \text { Add }+8 \quad 3 x-8+(+8)=34+(+8) \\
& 3 \mathrm{x}+\mathrm{O}=42 \\
& 3 x=42
\end{aligned}
$$

Now make a 1 . Multiply by $\frac{\mathbf{1}}{\mathbf{3}}$ (the reciprocal of $\mathbf{3}$ or $\frac{\mathbf{3}}{1}$ ).
Mult. $\frac{1}{3} \quad \frac{1}{3}(3 x)=\frac{1}{3}(42)$

$$
\begin{aligned}
& \frac{3}{3} x=\frac{42}{3} \\
& 1 x=14
\end{aligned}
$$

## Example 1 cont'd:

We already know this is the correct answer.
Notice that it makes no difference whether we make the 1 or 0 first. Upon closer examination, however, you might prefer to make the 0 first as that may possibly eliminate some of the fractions which may occur in the solution process.

Example 2: Find the solution set for each of the following open sentences by making the appropriate 0's and 1's.
a. $-3 x+1<-26$
b. $\frac{3 n}{4}-6 \geq 3$

Solution: a. We want to make a " 0 " out of the +1 , so we add -1 (its opposite).

$$
-3 x+1<-26
$$

Add - $\mathbf{1} \quad-3 x+1+(-1)<-26+(-1)$

$$
-3 x<-27
$$

We want to make a " 1 " out of the -3 , so we multiply by $\frac{1}{-3}$ (its reciprocal).
Mult. $\frac{1}{-3} \quad \frac{1}{-3}(-3 x)>\frac{1}{-3}(-27)$

$$
\frac{-3}{-3} x>\frac{-27}{-3}
$$

$$
1 x>9
$$

Notice we reverse the relation symbol when we multiply an inequality by a negative number.

We can partially check this range of solutions by a sample substitution in the original inequality. $11>9$, so we will try that sample.

$$
\begin{aligned}
-3(11)+1 & <-26 \\
-33+1 & <-26 \\
-32 & <-26 \text { Itchecks. }
\end{aligned}
$$

The solution set is as follows:


## Example 2 cont'd:

b. We want to make a " 0 " out of the -6 so we add +6 (its opposite).

$$
\frac{3 n}{4}-6 \geq 3
$$

Add $+6 \quad \frac{3 n}{4}-6+(+6) \geq 3+(+6)$

$$
\begin{array}{rrr}
\frac{3 n}{4} & +\mathrm{o} 2 & 9 \\
\frac{3 n}{4} & 2 & 9
\end{array}
$$

We want to make a " 1 " out of the $\frac{3}{4}$, so we multiply by $\frac{4}{3}$ (its reciprocal).
Mult. $\frac{4}{3} \quad \frac{4}{3}\left(\frac{3 n}{4}\right) \geq \frac{4}{3}(9)$

$$
\begin{aligned}
\frac{12}{12} n & \geq \frac{36}{3} \\
1 n & \geq 12
\end{aligned}
$$

Again we partially check this range of solutions by a sample substitution in the original inequality. $16 \geq 12$, so we will try that sample.

$$
\begin{array}{rl}
\frac{3(16)}{4}-6 & \geq 3 \\
\frac{48}{4}-6 & \geq 3 \\
12-6 & 23 \\
6 & \geq 3
\end{array} \text { It checks. }
$$

The solution set is as follows:


## Lesson 4 - Exercises:

Find the solution set for each of the following open sentences by making the appropriate 0 's and l's.

1. $2 n-1=5$
2. $\frac{x}{2}-6=14$
3. $\frac{r}{4}+8=7$
4. $3 t+5=29$
5. $3 t+8>20$
6. $4 x-12<16$
7. $\frac{n}{8}+16>15$
8. $\frac{7 x}{9}-3 \geq 4$
9. $5 c+7<18$
10. $2 w+7 \leq 1$
11. $-6 z-7 \geq 11$
12. $5 x+4 \leq-6$
13. $\frac{2}{3} x-5<7$
14. $\frac{3}{4} y-2<-8$
15. $4 x+13 \geq 5$
16. $2 z-1>7$
17. $-5 m-10<25$
18. $9 y+4>-14$
19. $-\frac{2 m}{-3}-5 \geq 1$

## p. 87 Lesson 4 - Combinations

1. $\begin{aligned} 2 n-1 & =5 \\ 2 n-1+1 & =5+1 \\ 2 n+0 & =6 \\ \frac{1}{2}(2 n) & =\frac{1}{2} \cdot 6 \\ \frac{2}{2} n & =\frac{6}{2} \\ 1 \cdot n & =3 \\ n & =3 \\ S & =\{3\}\end{aligned}$

$$
S=\{3\}
$$

2. $\frac{x}{2}-6=14$
$\frac{x}{2}-6+6=14+6$
$\frac{4}{2}+0=20$
$\frac{x}{2}=20$
$\begin{aligned} \frac{1}{2} \cdot x & =20 \\ \frac{2}{1} \cdot \frac{1}{2} \cdot x & =\frac{2}{1} \cdot 20 \\ \frac{2}{2} \cdot x & =40 \\ 1 \cdot x & =40 \\ x & =40\end{aligned}$
$S=\{40\}$
3. $4 x-12<16$
$4 x-12+12<16+12$
$4 x+0<28$
$4 x<28$
$\frac{1}{4}(4 x)<\frac{1}{4} \cdot 28$
$\frac{4}{4} \cdot x<\frac{28}{4}$
4. $x<7$
$x<7$
$S=\{x \mid x<7\}$
5. $\frac{7}{8}+16>15$
$\frac{n}{8}+16+-16>15+-16$
$\frac{n}{8}+0>-1$
$\frac{n}{8}>-1$
$\frac{1}{8} \cdot n>{ }^{-1}$
$\frac{8}{1} \cdot \frac{1}{8} \quad n>\frac{8}{1} \cdot(-1)$
$\frac{8}{8} n>-8$
$1 \cdot n>-8$ $n>-8$
$S=\{n \mid n>-8\}$
6. $\frac{72}{4}-3 \geq 4$
$\frac{7 x}{9}-3+3 \geq 4+3$
$\frac{7 x}{9}+0 \geq 7$
$\frac{7 x}{9} \geq 7$
$\frac{9}{7} \cdot \frac{7}{9} \cdot x \geq \frac{9}{7} \cdot 7$
7. $x \geq 9$
$x \geq 9$
$S=\{x \mid x \geq 9\}$
$S=\{t \mid t>4\}$
8. $2 w+7 \leq 1$ $2 w+7+-7 \leq 1+-7$
$2 w+0 \leq-6$
$2 w \leq-6$
$\frac{1}{2} \cdot 2 w \leq \frac{1}{2}(-6)$
$\frac{2}{2} \cdot w<\frac{-6}{2}$
$1 w \leq-3$ $w \leq-3$

$$
S=\{w \mid w \leq-3\}
$$

11. $-6 z-7 \geq 11$
$-6 z-7+7 \geq 11+7$
$-6 z+0 \geq 18$
$-6 z \geq 18$
$\frac{1}{-6}(-6) \leq \frac{1}{-6}(18)$
$\frac{-6}{-6} \cdot z \leq \frac{18}{-6}$
$1 \cdot z \leq-3$
$z \leq-3$
$S=\{z \mid z \leq-3\}$
12. $5 x+4 \leq-6$ $5 x+4+-4 \leq-6-4$ $5 x+0 \leq-10$
$5 x \leq-10$
$\frac{1}{5}(5 x) \leq \frac{1}{5}(10)$
$\frac{5}{5} x \leq \frac{-10}{5}$
13. $x \leq-2$ $x \leq-2$
$S=\{x \mid x \leq-2\}$
14. $\frac{2}{3} x-5<7$
$\frac{2}{3} x-5+5<7+5$
$\frac{2}{3} x+0<12$
$\frac{2}{3} x<12$
$\frac{3}{2} \cdot \frac{2}{3} \cdot x<\frac{3}{2} \cdot 12$
$\frac{6}{6} \cdot x<\frac{36}{2}$
15. $x<18$
$x<18$
$S=\{x \mid x<18\}$
16. $\frac{3}{4} y-2<-8$
$\frac{3}{4} y-2+2<-8+2$
$\frac{3}{4} y+0<-6$
$\frac{3}{4} y<-6$
$\frac{4}{3} \cdot \frac{3}{4} y<\frac{4}{3}(-6)$
$\frac{12}{12} \cdot y<\frac{-24}{3}$
$1 \cdot y<-8$ $y<-8$
$S=\{y \mid y<-8\}$
17. $4 x+13 \geq 5$
$4 x+13+{ }^{-} 13 \geq 5+{ }^{-} 13$
$\begin{aligned} 4 x+0 & \geq-8 \\ 4 x & \geq-8\end{aligned}$
$\begin{aligned} \frac{1}{4} \cdot 4 x & \geq \frac{1}{4}(-8) \\ \frac{4}{4} \cdot x & \geq \frac{-8}{4} \\ 1 \cdot x & \geq-2 \\ x & \geq-2\end{aligned}$
18. $\begin{aligned} & 2 z-1>7 \\ & 2 z-1+1>7+1 \\ & 2 z+0>8 \\ & 2 z>8 \\ & \frac{1}{2} \cdot(2 z)>\frac{1}{2} \cdot 8 \\ & \frac{2}{2} \cdot z>\frac{8}{2} \\ & 1 \cdot z>4 \\ & z>4 \\ & S=\{z \mid z>4\} \\ &2\}\end{aligned}$

$$
S=\{x \mid x \geq-2\}
$$

17. $-5 m-10<25$
$-5 m-10+10<25+10$

$$
-5 m+0<35
$$

$$
{ }^{-} 5 m<35
$$

$$
\frac{1}{-5}(-5)>\frac{1}{-5}(35)
$$

$$
\frac{-5}{-5} \cdot m>\frac{35}{-5}
$$

$$
1 \cdot m>-7
$$

$$
m>^{-7}
$$

$$
S=\{m \mid m>-7\} \quad S=\{y \mid y>-2\}
$$

19. $\quad \frac{2 m}{-3}-5 \geq 1$
$\frac{2 m}{-3}-5+5 \geq 1+5$
$\frac{2 m}{-3}+0 \geq 6$

$$
\begin{aligned}
\frac{2 m}{-3} & \geq 6 \\
\frac{2}{-3} \cdot m & \geq 6 \\
\frac{-3}{2} \cdot \frac{2}{-3} \cdot m & \leq \frac{-3}{2} \cdot 6 \\
\frac{-6}{-6} \cdot m & \leq \frac{-18}{2} \\
1 \cdot m & \leq-9
\end{aligned}
$$

$m \leq-9$
$S=\{m \mid m \leq-9\}$
$\qquad$
Class $\qquad$ Date $\qquad$ Score $\qquad$

# Unit II - First Degree Relations with One Placeholder Part A - Basic Equations and Inequalities <br> Lesson 1 - Solution Statements and Solution Sets Lesson 2-First Type - Making Zeros Lesson 3 - Second Type - Making Ones Lesson 4 - Combinations 

For each of the following solution statements, give the solution set, using the proper set notation roster or rule, and using a number line.

$$
\text { 1. } w=-3
$$

$$
S=\{
$$

\}

2. $x=2$
$S=\{$
\}

3. $a>1$
$S=\{$
\}

4. $m \leq^{-} 2$
$S=\{$

$\qquad$

## Unit II, Part A, Lessons 1, 2, 3 and 4, Quiz Form A - Continued -

Find the solution(s) for each of the following open sentences. Express your answer using set notation.
5. $x+7=15$
6. $c-7=12$
7. $w-4>^{-7}$
8. $4 x=12$
9. $-3 n={ }^{-} 15$
10. $3 n<30$
11. $-4 y \geq 12$
12. $3 x-1=17$
13. $\frac{4}{3} m+1<25$
$\qquad$
$\qquad$
$\qquad$ Score $\qquad$

# Unit II - First Degree Relations with One Placeholder Part A - Basic Equations and Inequalities <br> Lesson 1 - Solution Statements and Solution Sets <br> Lesson 2-First Type - Making Zeros <br> Lesson 3-Second Type-Making Ones <br> Lesson 4 - Combinations 

For each of the following solution statements, give the solution set, using the proper set notation roster or rule, and using a number line.
1.

$$
y \geq-4
$$

$S=\{$
\}

2. $b<0$
$S=\{$
\}

3. $x=3$
$S=\{$
\}

4. $a==^{-} 1$
$S=\{$
\}


## Name

## Unit II, Part A, Lessons 1, 2, 3 and 4, Quiz Form B - Continued -

Find the solution(s) for each of the following open sentences. Express your answer using set notation.
5. $5 a=30$
6. $x+9=15$
7. $-3 m<15$
8. $m-9=30$
9. $-4 y=-28$
10. $5 x+2=32$
11. $f-5<-8$
12. $5 x>60$
13. $\frac{5}{2} n+1 \geq 26$
$\qquad$
Class $\qquad$ Date $\qquad$ Score $\qquad$

# Unit II - First Degree Relations with One Placeholder Part A - Basic Equations and Inequalities Lesson 1 - Solution Statements and Solution Sets Lesson 2- First Type - Making Zeros Lesson 3-Second Type-Making Ones Lesson 4 - Combinations 

For each of the following solution statements, give the solution set, using the proper set notation roster or rule, and using a number line.

1. $w=-3$
$S=\{-3$
\}

2. $x=2$
$S=\{$
2
\}

3. $a>1$
$S=\{a \mid a>1\}$

4. $m \leq-2 \quad S=\{m ; m \leq-2\}$

$\qquad$

## Unit II, Part A, Lessons 1, 2, 3 and 4, Quiz Form A - Continued -

Find the solutions) for each of the following open sentences. Express your answer using set notation.
5. $x+7=15$

$$
\begin{aligned}
x+7+-7 & =15+-7 \\
x+0 & =8 \\
x & =8 \\
5 & =\{8\}
\end{aligned}
$$

6. $c-7=12$
$c-7+7=12+7$
$c+0=19$
$c=19$

$$
S=\{19\}
$$

7. $w-4>{ }^{-7}$
$w-4+4>-7+4$
$\omega+0>-3$
$\omega>-3$
$S=\{\omega \mid \omega>-3\}$
8. $4 x=12$

$$
\frac{1}{4}(4 x)=\frac{1}{4}(12)
$$

$$
\frac{4}{4} x=\frac{12}{4}
$$

$$
1 x=3
$$

$$
x=3
$$

$$
S=\{3\}
$$

9. $-3 n={ }^{-} 15$
$\frac{1}{-3}(-3 n)=\frac{1}{-3}(-15)$
$\frac{-3}{-3} n=\frac{-15}{-3}$
$1 n=5$

$$
n=5
$$

$$
S=\{5\}
$$

10. $3 n<30$
$\frac{1}{3}(3 n)<\frac{1}{3}(30)$
$\frac{3}{3} n<\frac{30}{3}$
$1 n<10$
$n<10$
$S=\{n \mid n<10\}$
11. $-4 y \geq 12$

$$
\begin{gathered}
\frac{1}{-4}(-4 y) \leq \frac{1}{-4}(12) \\
\frac{-4}{-4} y \leq \frac{12}{-4} \\
1 y \leq-3 \\
y \leq-3 \\
S=\{y \mid y \leq-3\}
\end{gathered}
$$

12. $3 x-1=17$

$$
3 x-1+1=17+1
$$

$$
3 x+0=18
$$

$$
3 x=18
$$

$$
\frac{1}{3}(3 x)=\frac{1}{3}(18)
$$

$$
\frac{3}{3} x=\frac{18}{3}
$$

$$
1 x=6
$$

$$
x=6
$$

$$
s=\{6\}
$$

13. $\frac{4}{3} m+1<25$

$$
\frac{4}{3} m+1+-1<25+-1
$$

$$
\frac{4}{3} m+0<24
$$

$$
\frac{4}{3} m<24
$$

$$
\frac{3}{4}\left(\frac{4}{3} m\right)<\frac{3}{4}(24)
$$

$$
\frac{12}{12} m<\frac{72}{4}
$$

$$
1 m<18
$$

$$
m<18
$$

$$
S=\{m \mid m<18\}
$$

# Quiz Form B 

Name $\qquad$
Class $\qquad$ Score $\qquad$

# Unit II - First Degree Relations with One Placeholder Part A - Basic Equations and Inequalities <br> Lesson 1 - Solution Statements and Solution Sets <br> Lesson 2- First Type - Making Zeros <br> Lesson 3-Second Type-Making Ones <br> Lesson 4 - Combinations 

For each of the following solution statements, give the solution set, using the proper set notation roster or rule, and using a number line.

$$
\text { 1. } y \geq-4 \quad S=\{y \mid y \geq-4\}
$$


2. $b<0$
$S=\{b \mid b<0$
\}

3. $x=3$
$S= \begin{cases}1 & 3\end{cases}$

4. $a={ }^{-} 1$
$S=\{-1$
\}

$\qquad$
Unit II, Part A, Lessons 1, 2, 3 and 4, Quiz Form B - Continued -

Find the solutions) for each of the following open sentences. Express your answer using set notation.
5. $5 a=30$

$$
\begin{aligned}
\frac{1}{5}(5 a) & =\frac{1}{5}(30) \\
\frac{5}{5} a & =\frac{30}{5} \\
1 a & =6 \\
a & =6 \\
5 & =\{6\}
\end{aligned}
$$

8. $m-9=30$

$$
\begin{aligned}
m-9+9 & =30+9 \\
m+0 & =39 \\
m & =39 \\
5 & =\{39\}
\end{aligned}
$$

11. $f-5<-8$

$$
\begin{gathered}
f-5+5<-8+5 \\
f+0<-3 \\
f<-3 \\
s=\{f \mid f<-3\}
\end{gathered}
$$

6. 

$$
\begin{aligned}
x+9 & =15 \\
x+9+-9 & =15+-9 \\
x+0 & =6 \\
x & =6 \\
5 & =\{6\}
\end{aligned}
$$

9. 

$$
\begin{aligned}
-4 y & =-28 \\
\frac{1}{-4}(-4 y) & =\frac{1}{-4}(-28) \\
\frac{-4}{-4} y & =\frac{-28}{-4} \\
1 y & =7 \\
y & =7 \\
S & =\{7\}
\end{aligned}
$$

12. $5 x>60$

$$
\begin{gathered}
\frac{1}{5}(5 x)>\frac{1}{5}(60) \\
\frac{5}{5} x>\frac{60}{5} \\
1 x>12 \\
x>12 \\
S=\{x \mid x>12\}
\end{gathered}
$$

7. $-3 m<15$

$$
\begin{aligned}
& \frac{1}{-3}(-3 m)>\frac{1}{-3}(15) \\
& \frac{-3}{-3} m>\frac{15}{-3} \\
& 1 m>-5 \\
& m>-5 \\
& S=\{m \mid m>-5\}
\end{aligned}
$$

10. $5 x+2=32$

$$
\begin{aligned}
5 x+2+-2 & =32+-2 \\
5 x+0 & =30 \\
5 x & =30 \\
\frac{1}{5}(5 x) & =\frac{1}{5}(30) \\
\frac{5}{5} x & =\frac{30}{5} \\
1 x & =6 \\
x & =6 \\
5 & =\{6\}
\end{aligned}
$$

13. $\frac{5}{2} n+1 \geq 26$

$$
\begin{aligned}
\frac{5}{2} n+1+-1 & \geq 26+-1 \\
\frac{5}{2} n+0 & \geq 25 \\
\frac{5}{2} n & \geq 25 \\
\frac{2}{5}\left(\frac{5}{2} n\right) & \geq \frac{2}{5}(25) \\
\frac{10}{10} n & \geq \frac{50}{5} \\
1 n & \geq 10 \\
n & \geq 10 \\
S=\{n \mid n & \geq 10\}
\end{aligned}
$$

