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Print Materials
for
"Algebra: A Complete Course"

Scope and Sequence Rationale

(2 pages)

Scope and Sequence Rationale

There are two basic premises which drive concept development in Algebra, and these two essentials shape the logical scope and sequence of algebraic content.

First, it is generally understood that the study of Algebra is the study of relations. In the same way that Geometry focuses on spatial concepts, and Calculus is concerned with rates of change, Algebra is a comprehensive exploration of mathematical relationships, including both equations and inequalities. As such, **no treatment of Algebra should ever separate equations from inequalities**, especially when it utilizes a format which addresses them in different “chapters.” In fact, a true adherence to the National Council of Teachers of mathematics (NCTM) standards, requires us to deal specifically with functions, and we know that the set of functions is a subset of the set of relations, without regard initially to the differences between equations and inequalities. Therefore, in this course, **equations and inequalities are studied together**, and distinctions are made only when necessary, to clarify functional differences. As an aside, documentation exists to show that students generally have little or no trouble working with all types of relations at the same time and, in fact, understand the logic of studying them together.

The second premise is that the concepts of Algebra develop by degrees. This means, of course, that **relations of first-degree should be mastered first**. In fact, as instructors, we all understand that relations of any degree other than one must be “reduced” to relations of first-degree, or “factored” into linear or first-degree factors, before they can be resolved. The impact of this understanding on the scope and sequence of Algebra content, is to organize the various types of relations, by degree. In this course, first-degree relations are examined exhaustively before higher-order relations are encountered. Unit II deals with first-degree relations with one variable. Unit III then addresses first-degree relations with two variables. Unit IV considers first-degree relations with three or more variables. **The idea here is to help students master first-degree relations, before moving on to relations of other degrees (or orders).** This is not only more mathematically correct than the traditional treatment, but it allows students to reinforce more efficiently, one-variable concepts by immediately moving to two-variable concepts, and then to concepts involving three or more variables. And we all know that a system of relations with three variables is resolved using the same approach as a system with only two variables.

Moving on to Unit V, students quickly review exponent notation, including the various properties of powers and operations with powers, and investigate relations with integral degrees of 2 or higher. Unit VI continues this exploration with a focus on algebraic fractions, in which negative, integral exponents make a prominent appearance. In Unit VII, fractional exponents are introduced, which obviously pave the way for a study of radicals and roots. This, of course, is the seed from which rational-degree relations develop, or, as they are more commonly called, relations with radicals in them.

Then, after a review of second-degree relations with one variable (Unit VIII – The Quadratic Relations) and two variables (Unit IX – The Conic Sections), the study of Algebra is completed by examining the only type of exponent not yet investigated – the variable, or placeholder. This is the start of a study of literal-degree relations, and is the basis for the development of exponential and logarithmic functions. It is only after considering all possible degrees, that we can say we have studied a complete course in Algebra. In that context, it is quite artificial to define, for everyone, what “Algebra 1” is, or “Algebra 2”, or even “Pre-Algebra”. **The logical scope of Algebra covers relations of all degrees, including numeric and literal, while the sequence of concepts begins with a mastery of first-degree relations and grows systematically to include increasingly more sophisticated degrees.**

One more organizational quality is noteworthy here. **The normal flow of each unit is based on the logical introduction of any new mathematical symbolism.** First, the new “thing” is defined and described in detail. Then, operations involving the new “thing” are explored. Finally, relations involving this new “thing” are examined, and strategies are developed to resolve them. This cycle is introduced and explained in Unit I, and is evidenced in each successive unit. For example, in Unit V, polynomials are introduced. This is new mathematical symbolism for the student, and it must be defined carefully. Then, operations with polynomials must be examined. All of this culminates, of course, in learning to solve relations with polynomials. **This logical cycle of exploration in mathematics is helpful to students**, providing them with some anticipation of the levels of exploration necessary to develop algebraic concepts.

Please understand that the organizational argument presented here is not meant to stifle the creativity of the instructor. Neither should it prohibit the instructor from utilizing a modular approach to concept development. It does, however, serve to remedy the fragmented, isolated topic, “chapter” approach, to a subject which has been traditionally presented to us in “textbooks”, without that element of developmental continuity. To that end, it speaks loudly to the curricular issues which all instructors face, and the attitudinal issues students deal with when they are presented with the fact that “everyone must pass Algebra”.