VTI Geometry
Scope and Sequence Rationale

There are two basic premises which drive concept development in Geometry, and these two essentials shape the logical scope and sequence of geometric content.

First, it is generally understood that **Geometry is the study of spatial relations.** In the same way that Algebra is the study of numerical relations (equations and inequalities), and Calculus is concerned primarily with rates of change, Geometry is a comprehensive exploration of "shapes" (as sets of points), the measurements associated with those shapes, and the relationships that can be established between those shapes. As such, no treatment of Geometry should ever investigate those relationships only individually, or in isolation. This is especially noticeable with traditional textbooks, which generally use a format which addresses them in different "chapters". In the VideoText Interactive Geometry course, **concepts are discussed from a "Unit" perspective, pursuing and connecting, in an exhaustive way, all of the outcomes associated with various possibilities for a specific relationship.** Of course, as much as is possible, students need to "see" those relationships, and experience the "motion", or "transformation", necessary to clearly illustrate the concept. It really is impossible to put a value on the benefits of visualization, in life in general, and in Geometry in particular. So, in the VideoText Interactive Geometry program, **computer-generated graphics are used extensively, along with animation and color-sequencing,** in order that students can actually see the relationships develop.

The second premise is that geometric concepts should be studied **utilizing all of the power and conviction that both inductive and deductive reasoning can bring to the table.** In other words, it is always desirable, and helpful, for students to "experiment", inductively, with a geometric relationship, in an effort to come to some general conclusion. Once that general conclusion has been arrived at, however, it is even more convincing if the student is able to "prove", deductively, that the conclusion absolutely must follow, logically, from the given information. No, formal proof is not often asked for in everyday life. On the other hand, the exercise of developing that kind of thinking is invaluable, not only in some specific job-related activities, but, more generally, in the daily problem-solving situations that confront us. The VideoText Interactive Geometry program is formatted in such a way that formal proof is a cornerstone.

Unit I, then, focuses on a complete preparation for students to begin a formal study of Geometry by "re-teaching" of all of the basic geometric concepts for which students have simply memorized the appropriate term, definition, or formula. That means we must re-establish that **Mathematics in general, and Geometry in particular, is a language,** with parts of speech and sentence structure. We must develop, in detail, the concepts associated with **building geometric shapes.** We must investigate, again in detail, the concepts dealing with the **measurement of those shapes.** Finally, we must thoroughly develop the principles of inductive and deductive reasoning, giving significant attention to the dynamics of mathematical deductive logic, which are the building blocks that students will use to **construct formal proofs.**

In Unit II, we begin the actual study of "Plane Geometry" by developing all of the necessary terms, definitions, and assumptions we will be using as a basis for studying geometric relationships. In other words, we draw on the analogy that studying any area of Mathematics is like "playing a game". We must first determine **which basic elements will be "undefined"** in our Geometry, or accepted without definition. We must then determine which basic elements can be formally defined, using those undefined terms. Finally, we must **build a list of "postulates", or conditional assumptions** which will serve as the "rules of the game", guiding us through the investigation of relationships, in our Geometry. It is important to note, at this point, that every Plane Geometry study will, in certain ways, be unique to the philosophy of the instructor, depending on the acceptance of these fundamental terms. In other words, while the prevailing context will always be that of classical Euclidean Geometry, the lists of definitions and postulates may differ from person to person. The key, however, is that each study will rely on its own particular list of Essential Elements to prove the rest of the relationships to be investigated.

So, in Unit III, we use the Fundamental Terms developed in Unit II, to prove Fundamental Theorems related to points, lines, rays, segments, and angles. These theorems will be foundational to the study of Simple Closed Plane Curves, which are the primary backdrop of all studies of Plane Geometry.

At this point, since we have put in place the "rules of the game", we can begin, and, for all practical purposes, complete, a methodic investigation of the **geometric relationships associated with Triangles (Unit IV), Other Polygons (Unit V), and Circles (Unit VI).** That then allows us to conclude our study by the investigation of several applications, internal to the study of Geometry.

First, in Unit VII, we will engage in the classic geometric exploration of "Construction". This means that, with the use of only a **straight edge** (to construct lines, rays, and segments), and a **compass** (to construct circles, and arcs of circles), we will attempt to use our knowledge of geometric relationships to "build", and "operate on", various geometric shapes. Included will be the replication and division of line segments and angles, the building of polygons to desired specifications, and the generation of circles to desired specifications.

Second, in Unit VIII, we will examine, in significant detail, the relationships between the various components of triangles. This is, of course, the study of **Trigonometry,** from the Greek, meaning "tri-angle-measure". Included are the basic relationships of **sine, cosine, and tangent,** as well as applications involving the Pythagorean Theorem, the Laws of Sines and Cosines, and several other ambiguous cases.

Please understand that the organizational argument presented here is not meant to stifle the creativity of the instructor. Neither should it prohibit the instructor from utilizing a modular approach to concept development. It does, however, serve to remedy the fragmented, isolated topic, "chapter" approach to a subject which has been traditionally presented to us in "textbooks", without that element of developmental continuity. To that end, it speaks loudly to the curricular issues which all instructors face, and the attitudinal issues students deal with when they are presented with a new and different Mathematics course.