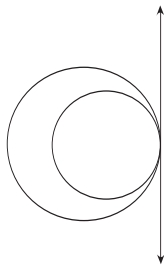

Unit VI — Circles

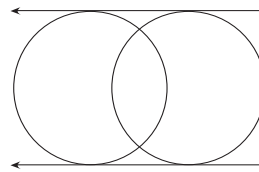
Part A — Fundamental Terms

p. 522 – Lesson 1 — Lines and Segments

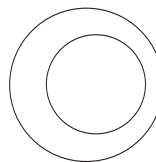
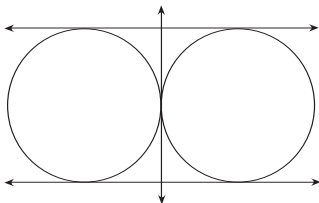
1. \overline{QA} , \overline{QK} , \overline{AC} , \overline{FE}
2. \overline{QK} , \overline{IB}
3. \overline{PQ} , \overline{PA} , \overline{PK} , \overline{OA} , \overline{OI} , \overline{OB}
4. Tangent
5. Chord
6. Secant
7. Tangent Segment
8. Chord
9. Secant Segment
10. QK
11. OI; OB
12. Chord; Chord
13. Point R is outside circle T.
14. 18 cm
15. 2x inches
16. 8.23 cm; $\frac{\pi}{2}$
17. None
18. a) One radius
b) Infinitely many chords
c) None
19. a) One chord
b) One secant
c) No tangents
20. a) Point Q is in the interior of circle T.
b) Point Q is on circle T.
c) Point Q is in the exterior of circle T.
21. a) $0 < PQ \leq 14$
b) $0 < PQ < 14$
c) $PQ > 0$
22. Any two tangent circles whose interiors intersect.



23. Any two circles that intersect in two points.



24. Any two tangent circles whose interiors do not intersect.
25. Any two circles such that one is contained entirely within the interior of the other.



Unit VI — Circles

Part A — Fundamental Terms

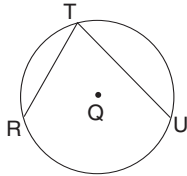
p. 524 – Lesson 2 — Arcs and Angles

- \widehat{AB} , \widehat{BC} , \widehat{CD} , \widehat{DE} , \widehat{EA} (Answers may vary. Here are other answers: \widehat{CE} , \widehat{DA} , \widehat{EB})
- \widehat{ABD} , \widehat{ABE} , \widehat{EAC} (Answers may vary. Here are other answers: \widehat{EAD} , \widehat{DAC} , \widehat{CAB} , \widehat{BCE})
- \widehat{ABC} , \widehat{AEC} , \widehat{BCD} , \widehat{BAD}
- $\angle AQB$, $\angle AQE$, $\angle AQD$ (Answers may vary. Here are other answers: $\angle BQE$, $\angle EQC$, $\angle EQD$, $\angle BQC$, $\angle DQC$)
- $\angle BAC$, $\angle EAC$ (Answers may vary. Here are other answers: $\angle BAE$, $\angle ABD$)

6. (Answers and diagrams may vary)

Inscribed Angle: $\angle RTU$

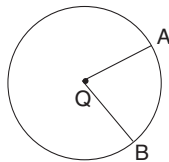
Minor Arc: \widehat{RU} (also \widehat{RT} and \widehat{UT})



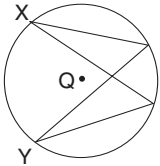
7. (Answers and diagrams may vary)

Central Angle: $\angle AQB$

Minor Arc: \widehat{AB}

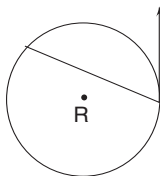


8. (Diagrams may vary)

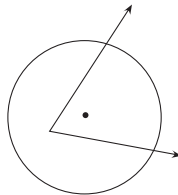


9. a) $\angle UYV$; $\angle UXV$ b) $\angle XQW$; $\angle WQV$ c) $\angle WVX$ d) $\angle YUX$; $\angle YVX$

10.



11.



12. a) $m\angle AOB = 75$

b) $m\widehat{AE} = 75$

c) $m\widehat{CD} = 20$

d) $OE = 5$

e) $m\widehat{ACE} = 285$

f) $m\angle COE = 125$

13. a) $m\angle WOZ = 34$

$$4x + (3x + 10) + x + 2x + 10 = 360$$

$$10x + 20 = 360$$

$$10x = 340$$

$$x = 34$$

b) $m\widehat{XYZ} = 4x + (2x + 10)$

$$= 6x + 10$$

$$= 6(34) + 10$$

$$= 204 + 10$$

$$= 214$$

c) $m\angle WOX = 3x + 10$

$$= 3(34) + 10$$

$$= 102 + 10$$

$$= 112$$

d) $m\angle YOZ = 2x + 10$

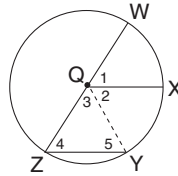
$$= 2(34) + 10$$

$$= 68 + 10$$

$$= 78$$

14. a) \widehat{ADC} (or \widehat{ACB}) b) \widehat{DAC} (or \widehat{DBC}) c) \widehat{ABD} (or \widehat{ACD}) d) \widehat{BAC} (or \widehat{BDC})

15. Given: \overline{WZ} is a diameter of $\odot Q$.
 $m\widehat{WX} = m\widehat{XY} = n$
 Prove: $m\angle Z = n$



STATEMENT	REASONS
1. \overline{WZ} is a diameter of $\odot Q$	1. Given
2. $m\widehat{WX} = m\widehat{XY} = n$	2. Given
3. $m\angle 1 = m\widehat{WX} = n$ $m\angle 2 = m\widehat{XY} = n$	3. Definition of the measure of a central angle and the measure of its intercepted arc.
4. $m\angle 1 = m\angle 2$	4. Substitution
5. $m\angle 1 + m\angle 2 + m\angle 3 = 180$	5. A straight line contains 180° .
6. $m\angle 4 + m\angle 5 + m\angle 3 = 180$	6. Theorem 25 - If you have any given triangle, then the sum of the measures of its angles is 180° .
7. $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5 + m\angle 3$	7. Substitution
8. $m\angle 1 + m\angle 2 = m\angle 4 + m\angle 5$	8. Subtraction Property of Equality
9. Draw \overline{QY}	9. Postulate 2 - For any two different points, there is exactly one line containing them.
10. $\overline{QY} \cong \overline{QZ}$	10. Radii of the same circle are congruent.
11. $\angle 4 \cong \angle 5$	11. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
12. $m\angle 4 = m\angle 5$	12. Definition of Congruent Angles
13. $m\angle 1 + m\angle 2 = m\angle 4 + m\angle 4$	13. Substitution
14. $2m\angle 1 = 2m\angle 4$	14. Properties of Algebra
15. $m\angle 1 = m\angle 4$	15. Multiplication Property of Equality
16. $n = m\angle 4$ (or $m\angle Z$)	16. Substitution

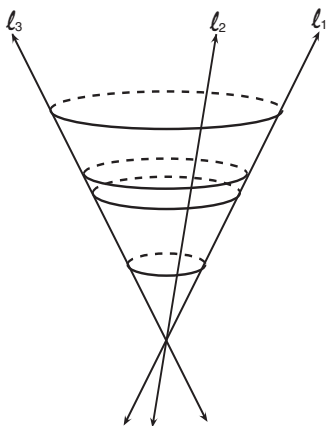
Unit VI — Circles

Part A — Fundamental Terms

p. 527 – Lesson 3 — Circle Relationships

- One external; no internal
- Two external; no internal
- No external; no internal
- No external; no internal
- Two external; one internal
- Two external; two internal
- Point W is in the interior of both circles. Point X is in the exterior of the inner circle and in the interior of the outer circle. Point Y is in the exterior of the inner circle and in the interior of the outer circle. Point Z is in the exterior of both circles.
- $CD = CE + EF + FD$
 $CD = 10 + 12 + 3$
 $CD = 25$
- 63
- 63
- 85
- 85
- 212
- 212
- 275
- 275

17. \widehat{JK} and \widehat{PN} ; \widehat{KT} and \widehat{NM}
 \widehat{TSJ} and \widehat{MRP} ; \widehat{TJK} and \widehat{MRN}
18. No. Congruent figures must have the same size (in this case, length, if stretching out straight) and shape.
19. a) must
 b) must
 c) can be but need not
 d) must
 e) can be but need not
 f) must
 g) must
 h) cannot
 i) must
20. infinitely many
21. infinitely many
22. yes; yes; no
23. yes; yes; no
- 24.



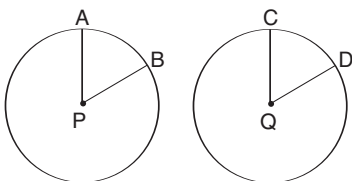
Unit VI — Circles

Part B — Angle and Arc Relationships

p. 532 – Lesson 1 — Theorem 65 and 66

1. a) Theorem 65 - "In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent."

b)



c) Given: $\odot P \cong \odot Q$; $\angle P \cong \angle Q$

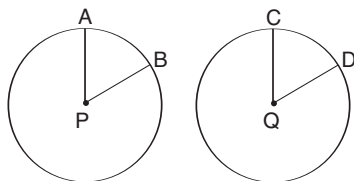
d) Prove: $\widehat{AB} \cong \widehat{CD}$

e)

STATEMENT	REASONS
1. $\odot P \cong \odot Q$	1. Given
2. $\angle P \cong \angle Q$	2. Given
3. $m\angle P = m\angle Q$	3. Definition of Congruent Angles
4. $m\angle P = m\widehat{AB}$	4. Definition of Arc Measure
5. $m\angle Q = m\widehat{CD}$	5. Definition of Arc Measure
6. $m\widehat{AB} = m\widehat{CD}$	6. Substitution
7. $\widehat{AB} \cong \widehat{CD}$	7. Definition of Congruent Arcs

2. a) Theorem 66 - "In a circle or in congruent circles, if two minor arcs are congruent, then their corresponding central angles are congruent."

b)



c) Given: $\odot P \cong \odot Q$; $\widehat{AB} \cong \widehat{CD}$

d) Prove: $\angle P \cong \angle Q$

e)

STATEMENT	REASONS
1. $\odot P \cong \odot Q$	1. Given
2. $\widehat{AB} \cong \widehat{CD}$	2. Given
3. $m\widehat{AB} = m\widehat{CD}$	3. Definition of Congruent Arcs
4. $m\angle P = m\widehat{AB}$	4. Definition of Arc Measure
5. $m\angle Q = m\widehat{CD}$	5. Definition of Arc Measure
6. $m\angle P = m\angle Q$	6. Substitution
7. $\angle P \cong \angle Q$	7. Definition of Congruent Angles

3. \widehat{AB} and \widehat{DC} are in the same circle, and $m\widehat{AB} = m\widehat{CD} = 50^\circ$. So, $\widehat{AB} \cong \widehat{CD}$ by Theorem 65 - In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent.
4. \widehat{PQ} and \widehat{RS} are in congruent circles, and $m\widehat{PQ} = m\widehat{RS} = 135^\circ$. So, $\widehat{PQ} \cong \widehat{RS}$ by Theorem 65 - In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent.
5. $m\widehat{ZW} = m\widehat{XY} = 95^\circ$; but, \widehat{ZW} and \widehat{XY} are not arcs of the same circle or of congruent circles, so \widehat{ZW} and \widehat{XY} are not congruent.

6. $3x = \frac{1}{4}x + 33$ $m\widehat{AB} = \frac{1}{4}x + 33$ $m\widehat{CD} = 3x$
 $12x = x + 132$ $m\widehat{AB} = \frac{1}{4}(12) + 33$ $m\widehat{CD} = 3(12)$
 $11x = 132$ $m\widehat{AB} = \frac{1}{4} \cdot \frac{4 \cdot 3}{1} + 33$ $m\widehat{CD} = 36$
 $x = 12$ $m\widehat{AB} = 3 + 33$
 $m\widehat{AB} = 36$

7. a) 65° b) 60° c) 65° d) 55° e) 115°

8.

STATEMENT	REASONS
1. \overline{AB} is a diameter and \overline{AD} is a chord of $\odot Q$	1. Given
2. Radius $\overline{QC} \parallel \overline{AD}$	2. Given
3. $\overline{QA} \cong \overline{QD}$	3. Radii of the same circle are congruent.
4. $\angle 1 \cong \angle 2$	4. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
5. $\angle 2 \cong \angle 3$	5. Theorem 16 - If two parallel lines are cut by a transversal, then alternate interior angles are congruent.
6. $\angle 1 \cong \angle 3$	6. Transitive Property of Congruent Angles
7. $\angle 3 \cong \angle 1$	7. Symmetric Property of Congruent Angles
8. $\angle 1 \cong \angle 4$	If $\angle 1 \cong \angle 3$, then $\angle 3 \cong \angle 1$
9. $\angle 3 \cong \angle 4$	8. Postulate 11 - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
10. $\widehat{CD} \cong \widehat{BC}$	9. Transitive Property of Congruent Angles
	10. Theorem 65 - In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent.

9.

STATEMENT	REASONS
1. \overline{AB} and \overline{CD} are diameters 2. $\angle AQC \cong \angle BQC$ 3. $\widehat{AC} \cong \widehat{DB}$	1. Given 2. Theorem 15 - If two lines intersect, then the vertical angles are congruent. 3. Theorem 65 - In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent.

10. Eight equal arcs have eight equal central angles. 360° in a circle divided into 8 parts gives us 8 central angles of 45° each.

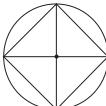

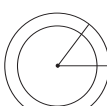
11.



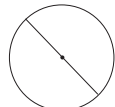
STATEMENT	REASONS
1. \overline{AQ} , \overline{CQ} and \overline{BQ} are radii of $\odot Q$ 2. Point C is the midpoint of \widehat{AB} 3. $\widehat{AC} \cong \widehat{CB}$ 4. $\angle AQC \cong \angle CQB$ 5. \overline{CQ} bisects $\angle AOB$	1. Given 2. Given 3. Definition of Midpoint 4. Theorem 66 - In a circle or in congruent circles, if two minor arcs are congruent, then their corresponding central angles are congruent. 5. Definition of Angle Bisector

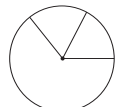

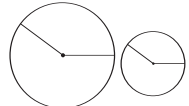
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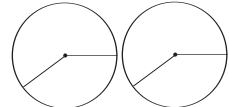
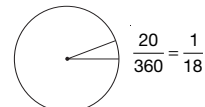
STATEMENT	REASONS
1. \overline{AQ} , \overline{CQ} and \overline{BQ} are radii of $\odot Q$ 2. \overline{CQ} bisects $\angle AOB$ 3. $\angle AQC \cong \angle BQC$ 4. $\widehat{AC} \cong \widehat{CB}$ 5. Point C is the midpoint of \widehat{AB}	1. Given 2. Given 3. Definition of Angle Bisector 4. Theorem 65 - In a circle or in congruent circles, if two central angles are congruent, then their corresponding intercepted arcs are congruent. 5. Definition of Midpoint of an Arc

13. In a circle, a radius bisects a central angle if and only if the endpoint of the radius is the midpoint of the arc intercepted by the central angle.

14. a) True  b) False  c) False 

d) True  e) False  f) True 

g) True  h) False  i) False 

j) True  k) True 

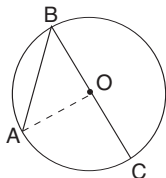
15. Midpoint of an Arc - The point on an arc which divides the arc into two equal or congruent parts.
 Center of an Arc - The point which is the center of the circle of which the arc is a part.

Unit VI — Circles

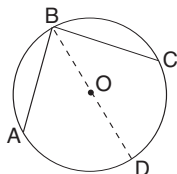
Part B — Angle and Arc Relationships

p. 537 – Lesson 2 — Theorem 67; Corollary 67a, 67b, 67c

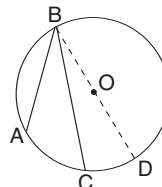
1. a) Theorem 67 - "If an angle is inscribed in a circle, then the measure of that angle is one-half the measure of the intercepted arc."
 b) To prove Theorem 67, you must consider three possible cases:



Case 1: The center lies on $\angle ABC$



Case 2: The center lies in the interior of $\angle ABC$



Case 3: The center lies in the exterior of $\angle ABC$

c) Given: $\angle ABC$ is inscribed in $\odot O$

d) Prove: $\angle ABC = \frac{1}{2} \cdot m\widehat{AC}$

e) Case 1: The center lies on $\angle ABC$

STATEMENT	REASONS
1. $\angle ABC$ is inscribed in $\odot O$	1. Given
2. Draw radius \overline{OA}	2. Postulate 2 - Assumption 1 - For any two different points, there is exactly one line (segment) containing them.
3. $\overline{OA} \cong \overline{OB}$	3. Radii of the same circle are congruent.
4. $\angle A \cong \angle B$	4. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite them are congruent.
5. $m\angle A = m\angle B$	5. Definition of Congruent Angles
6. $m\angle A + m\angle B = m\angle AOC$	6. Theorem 26 - If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
7. $m\angle B + m\angle B = m\angle AOC$	7. Substitution Principle
8. $2m\angle B = m\angle AOC$	8. Properties of Algebra
9. $m\angle B = \frac{1}{2} \cdot m\angle AOC$	9. Multiplication Property of Equality
10. $m\widehat{AC} = m\angle AOC$	10. Definition of Arc Measure
11. $m\angle B = \frac{1}{2} \cdot m\widehat{AC}$	11. Substitution

Case 2: The center of the circle lies in the interior of $\angle ABC$

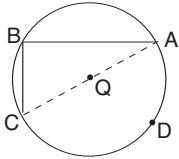
STATEMENT	REASONS
1. $\angle ABC$ is inscribed in $\odot O$	1. Given
2. Draw diameter \overline{BD}	2. Postulate 2 - Assumption 1 - For any two different points, there is exactly one line (segment) containing them.
3. $m\angle ABD = \frac{1}{2} \cdot m\widehat{AD}$	3. Case 1 of Theorem 67
4. $m\angle CBD = \frac{1}{2} \cdot m\widehat{CD}$	4. Case 1 of Theorem 67
5. $m\angle ABD + m\angle CBD = \frac{1}{2} m\widehat{AD} + \frac{1}{2} m\widehat{CD}$	5. Addition Property of Equality
6. $m\angle ABD + m\angle CBD = \frac{1}{2} (m\widehat{AD} + m\widehat{CD})$	6. Distributive Property of Multiplication over Addition
7. $m\angle ABD + m\angle CBD = m\angle ABC$	7. Postulate 7 - Protractor Postulate - Angle Addition Assumption
8. $m\angle ABC = \frac{1}{2} (m\widehat{AD} + m\widehat{CD})$	8. Substitution
9. $m\widehat{AD} + m\widehat{CD} = m\widehat{AC}$	9. Postulate 8 - Circle - Arc Addition Assumption
10. $m\angle ABC = \frac{1}{2} \cdot m\widehat{AC}$	10. Definition of Arc Measure

Case 3: The center of the circle lies in the exterior of $\angle ABC$

STATEMENT	REASONS
1. $\angle ABC$ is inscribed in $\odot O$	1. Given
2. Draw diameter \overline{BD}	2. Postulate 2 - Assumption 1 - For any two different points, there is exactly one line (segment) containing them.
3. $m\angle ABD = \frac{1}{2} \cdot m\widehat{AD}$	3. Case 1 of Theorem 67
4. $m\angle CBD = \frac{1}{2} \cdot m\widehat{CD}$	4. Case 1 of Theorem 67
5. $m\angle ABC + m\angle CBD = m\angle ABD$	5. Postulate 7 - Protractor Postulate - Angle Addition Assumption
6. $m\widehat{AC} + m\widehat{CD} = m\widehat{AD}$	6. Postulate 8 - Circle - Arc Addition Assumption
7. $m\angle ABC + m\angle CBD = \frac{1}{2} (m\widehat{AC} + m\widehat{CD})$	7. Substitution
8. $m\angle ABC + m\angle CBD = \frac{1}{2} m\widehat{AC} + \frac{1}{2} m\widehat{CD}$	8. Distributive Property of Multiplication over Addition
9. $m\angle ABC = \frac{1}{2} \cdot m\widehat{AC}$	9. Subtraction Property of Equality (8 subtract 4)

2. a) Corollary 67a - "If an angle inscribed in a semi-circle, then the angle is a right angle."

b)



c) Given: $\angle ABC$ is inscribed in semicircle \widehat{ADC} in $\odot Q$

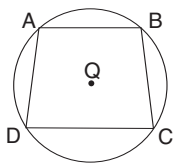
d) Prove: $\angle B$ is a right angle

e)

STATEMENT	REASONS
1. $\angle ABC$ is inscribed in semicircle \widehat{ADC}	1. Given
2. $m\angle ABC = \frac{1}{2} \cdot m\widehat{ADC}$	2. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one-half the measure of the intercepted arc.
3. \widehat{ADC} is a semicircle	3. Definition of an arc of a semicircle
4. $m\widehat{ADC} = 180^\circ$	4. Definition of Semicircle
5. $m\angle ABC = \frac{1}{2} \cdot 180$	5. Substitution
6. $m\angle ABC = 90$	6. Properties of Arithmetic
7. $\angle ABC$ is a right angle	7. Definition of Right Angle

3. a) Corollary 67b - "If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary."

b)



c) Given: ABCD is a quadrilateral inscribed in circle Q

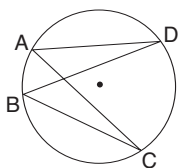
d) Prove: $\angle A$ and $\angle C$ are supplementary; $\angle B$ and $\angle D$ are supplementary

e)

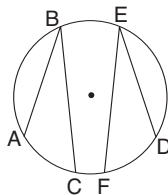
STATEMENT	REASONS
1. ABCD is a quadrilateral inscribed in circle Q	1. Given
2. $m\angle A = \frac{1}{2} \cdot m\widehat{BCD}$	2. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one-half the measure of the intercepted arc.
3. $m\widehat{BC} + m\widehat{CD} = m\widehat{BCD}$	3. Postulate 8 - 4th Assumption - Arc Addition Assumption
4. $m\angle A = \frac{1}{2} (m\widehat{BC} + m\widehat{CD})$	4. Substitution
5. $m\angle C = \frac{1}{2} \cdot m\widehat{DAB}$	5. Theorem 67
6. $m\widehat{DA} + m\widehat{AB} = m\widehat{DAB}$	6. Postulate 8 - 4th Assumption - Arc Addition Assumption
7. $m\angle C = \frac{1}{2} (m\widehat{DA} + m\widehat{AB})$	7. Substitution
8. $m\angle A + m\angle C = \frac{1}{2} (m\widehat{BC} + m\widehat{CD}) + \frac{1}{2} (m\widehat{DA} + m\widehat{AB})$	8. Addition Property of Equality
9. $m\angle A + m\angle C = \frac{1}{2} (m\widehat{BC} + m\widehat{CD} + m\widehat{DA} + m\widehat{AB})$	9. Distributive Property of Multiplication over Addition
10. $m\widehat{BC} + m\widehat{CD} + m\widehat{DA} + m\widehat{AB} = 360$	10. Postulate 8 - 4th Assumption - Arc Addition Assumption
11. $m\angle A + m\angle C = \frac{1}{2} \cdot 360$	11. Substitution
12. $m\angle A + m\angle C = 180$	12. Properties of Arithmetic
13. $\angle A$ and $\angle C$ are supplementary angles	13. Definition of Supplementary Angles
14. $m\angle B = \frac{1}{2} \cdot m\widehat{DAB}$	14. Theorem 67
15. $m\widehat{AD} + m\widehat{DC} = m\widehat{ADC}$	15. Postulate 8 - 4th Assumption - Arc Addition Assumption
16. $m\angle B = \frac{1}{2} (m\widehat{AD} + m\widehat{DC})$	16. Substitution
17. $m\angle D = \frac{1}{2} \cdot m\widehat{ABC}$	17. Theorem 67
18. $m\widehat{AB} + m\widehat{BC} = m\widehat{ABC}$	18. Postulate 8 - 4th Assumption - Arc Addition Assumption
19. $m\angle D = \frac{1}{2} (m\widehat{AB} + m\widehat{BC})$	19. Substitution
20. $m\angle B + m\angle D = \frac{1}{2} (m\widehat{AD} + m\widehat{DC}) + \frac{1}{2} (m\widehat{AB} + m\widehat{BC})$	20. Addition Property of Equality
21. $m\angle B + m\angle D = \frac{1}{2} (m\widehat{AD} + m\widehat{DC} + m\widehat{AB} + m\widehat{BC})$	21. Distributive Property of Multiplication over Addition
22. $m\widehat{AD} + m\widehat{DC} + m\widehat{AB} + m\widehat{BC} = 360$	22. Postulate 8 - 4th Assumption - Arc Addition Assumption
23. $m\angle B + m\angle D = \frac{1}{2} \cdot 360$	23. Substitution
24. $m\angle B + m\angle D = 180$	24. Principles of Arithmetic
25. $\angle B$ and $\angle D$ are supplementary angles	25. Definition of Supplementary Angles

4. a) Corollary 67c - "If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent."

b) To prove Corollary 67c, you must consider two possible cases:



Case 1: Angles intercept the same arc



Case 2: Angles intercept congruent arcs; $\widehat{AC} \cong \widehat{DF}$

c) Case 1 - Given: $\angle DAC$ intercepts \widehat{CD} ; $\angle CBD$ intercepts \widehat{CD}

Case 2 - Given: $\angle ABC$ intercepts \widehat{AC} ; $\angle DEF$ intercepts \widehat{DF} ; $\widehat{AC} \cong \widehat{DF}$

d) Case 1 - Prove: $\angle DAC \cong \angle CBD$

Case 2 - Prove: $\angle ABC \cong \angle DEF$

e) Case 1:

STATEMENT	REASONS
1. $\angle DAC$ intercepts \widehat{CD} ; $\angle CBD$ intercepts \widehat{CD} 2. $m\angle DAC = \frac{1}{2} \cdot m\widehat{CD}$; $m\angle CBD = \frac{1}{2} \cdot m\widehat{CD}$	1. Given 2. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one-half the measure of the intercepted arc.
3. $m\angle DAC = m\angle CBD$	3. Substitution
4. $\angle DAC \cong \angle CBD$	4. Definition of Congruent Angles

Case 2:

STATEMENT	REASONS
1. $\angle ABC$ intercepts \widehat{AC} ; $\angle DEF$ intercepts \widehat{DF} ; $\widehat{AC} \cong \widehat{DF}$ 2. $m\widehat{AC} = m\widehat{DF}$ 3. $\frac{1}{2} \cdot m\widehat{AC} = \frac{1}{2} \cdot m\widehat{DF}$	1. Given 2. Definition of Congruent Arcs 3. Multiplication Property of Equality
4. $m\angle ABC = \frac{1}{2} \cdot m\widehat{AC}$	4. Theorem 67
5. $m\angle DEF = \frac{1}{2} \cdot m\widehat{DF}$	5. Theorem 67
6. $m\angle ABC = m\angle DEF$	6. Substitution
7. $\angle ABC \cong \angle DEF$	7. Definition of Congruent Angles

5. Given: \overline{WZ} is a diameter of circle Q; $m\widehat{WX} = m\widehat{XY} = n$
 Prove: $m\angle Z = n$

STATEMENT	REASONS
1. \overline{WZ} is a diameter of circle Q	1. Given
2. $m\widehat{WX} = m\widehat{XY} = n$	2. Given
3. $m\widehat{WX} + m\widehat{XY} = m\widehat{WY}$	3. Postulate 8 - 4th Assumption - Arc Addition
4. $n + n = m\widehat{WY}$	4. Substitution
5. $2n = m\widehat{WY}$	5. Properties of Algebra
6. $m\angle Z = \frac{1}{2} \cdot m\widehat{WY}$	6. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one-half the measure of the intercepted arc.
7. $m\angle Z = \frac{1}{2} \cdot 2n$	7. Substitution
8. $m\angle Z = n$	8. Properties of Algebra

6. Corollary 67b - "If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary."

7. a) $\widehat{XZ} = 180 - 85 = 95$ b) \widehat{NP} c) $\widehat{AC} = 180 - 110 = 70$ d) \widehat{OMQ} (or \widehat{ONQ}) e) $\widehat{XZ} = 95$ f) \widehat{NP}

8. a) $m\angle Y = 70^\circ$ b) $m\angle O = 40^\circ$ c) $m\angle B = 55^\circ$ d) $m\angle MNQ = 40^\circ$ e) $m\angle W = 70^\circ$ f) $m\angle QPM = 40^\circ$

9. $\angle MNQ, \angle MOQ, \angle MPQ$

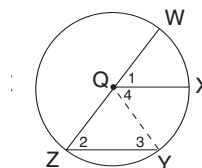
10. The angles are congruent by Corollary 67c: "If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent."

11. $\angle YXW, \angle WZY$

12. The angles are congruent by Corollary 67c: "If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent."

13. $m\angle U = 20^\circ, m\angle V = 30^\circ, m\angle UXV = 180 - (20 + 30) = 130^\circ$

14. Given: \overline{WZ} is a diameter of $\odot Q$; $\overline{QX} \parallel \overline{ZY}$
Prove: $\widehat{WX} \cong \widehat{XY}$



STATEMENT	REASONS
1. \overline{WZ} is a diameter of $\odot Q$	1. Given
2. $\overline{QX} \parallel \overline{ZY}$	2. Given
3. Draw \overline{QY}	3. Postulate 2 - 1st Assumption - For any two different points, there is exactly one line containing them.
4. $\angle 1 \cong \angle 2$	4. Postulate 11 - If two parallel lines are cut by a transversal, then corresponding angles are congruent.
5. $\overline{QZ} \cong \overline{QY}$	5. Radii of the same circle are congruent.
6. $\angle 2 \cong \angle 3$	6. Theorem 33 - If two sides of a triangle are congruent then the angles opposite them are congruent.
7. $\angle 3 \cong \angle 4$	7. Theorem 16 - If two parallel lines are cut by a transversal, the alternate interior angles are congruent.
8. $\angle 1 \cong \angle 4$	8. Transitive Property of Angle Congruence
9. $\widehat{WX} \cong \widehat{XY}$	9. Theorem 65 - If two central angles are congruent, then their intercepted arcs are congruent.

15. $m\widehat{RS} + m\widehat{RT} + m\widehat{ST} = 360$
 $(2x + 8) + (4x - 20) + (3x + 12) = 360$

$9x = 360$	$m\widehat{RS} = 2x + 8$	$m\widehat{RT} = 4x - 20$	$m\widehat{ST} = 3x + 12$
$x = 40$	$= 2(40) + 8$	$= 4(40) - 20$	$= 3(40) + 12$
	$= 80 + 8$	$= 160 - 20$	$= 120 + 12$
	$= 88$	$= 140$	$= 132$

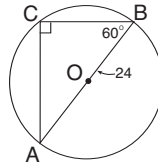
$m\angle R = \frac{1}{2} \cdot m\widehat{ST}$	$m\angle S = \frac{1}{2} \cdot m\widehat{RT}$	$m\angle T = \frac{1}{2} \cdot m\widehat{RS}$
$= \frac{1}{2} \cdot 132$	$= \frac{1}{2} \cdot 140$	$= \frac{1}{2} \cdot 88$
$= 66$	$= 70$	$= 44$

16.

STATEMENT	REASONS
1. \overline{AB} , \overline{AD} , \overline{AC} , and \overline{BD} are chords in $\odot Q$ 2. $\angle AEC \cong \angle BED$ 3. $\angle A \cong \angle B$ 4. $\angle C \cong \angle D$ 5. $\triangle ACE \sim \triangle BDE$	1. Given 2. Theorem 15 - If two lines intersect, then vertical angles formed are congruent. 3. Corollary 67c - If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent. 4. Corollary 67c 5. Postulate 12 - First Assumption - If the three angles of one triangle are congruent to the three corresponding angles of another triangle, then the two triangles are similar. (AAA Similarity Assumption)

17. \overline{BC} is the short leg of a 30-60-90 triangle.

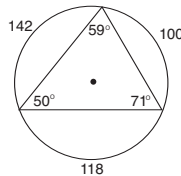
$$\begin{aligned} BC &= \frac{1}{2} \cdot BA \text{ or } \frac{BA}{2} \\ &= \frac{1}{2} \cdot 24 \\ &= 12 \end{aligned}$$



\overline{AC} is the long leg of a 30-60-90 triangle.

$$\begin{aligned} AC &= AB \cdot \frac{\sqrt{3}}{2} \text{ or } \frac{AB\sqrt{3}}{2} \\ &= \frac{24 \cdot \sqrt{3}}{2} \\ &= \cancel{2} \cdot 12 \cdot \sqrt{3} \\ &= 12\sqrt{3} \end{aligned}$$

18. $360 - (142 + 100)$
 $360 - 242$
 118



The angles measure 50, 59, and 71 degrees.

19. $\angle ABC \cong \angle DEF$

Corollary 67c - If two inscribed angles intercept the same arc or congruent arcs, then the angles are congruent.

20. $m\angle X = 90$, $m\angle Y = 90$, $m\angle Z = 90$

Corollary 67a - If an angle is inscribed in a semicircle, then the angle is a right angle.

21. Less than 180°

Unit VI — Circles

Part B — Angle and Arc Relationships

p. 541 – Lesson 3 — Theorem 68

1. $\angle ABG, \angle CBG$
 $\angle ABF, \angle CBF$
 $\angle ABE, \angle CBE$

2. $\widehat{BC}, \widehat{BEG}$
 $\widehat{BF}, \widehat{BEF}$
 $\widehat{BFE}, \widehat{BE}$

$$3. \quad m\angle VYW = \frac{1}{2} \cdot m\widehat{WY} = \frac{1}{2} \cdot 82 = \frac{1 \cdot 41 \cdot \cancel{2}}{\cancel{2}} = 41$$

$$m\angle VYX = \frac{1}{2} \cdot m\widehat{XY} = \frac{1}{2} \cdot [m(\widehat{XW} + \widehat{WY})] = \frac{1}{2} \cdot [96 + 82] = \frac{1 \cdot 69 \cdot \cancel{2}}{\cancel{2}} = 69$$

$$m\angle WYX = \frac{1}{2} \cdot m\widehat{XW} = \frac{1}{2} \cdot 96 = \frac{1 \cdot 48 \cdot \cancel{2}}{\cancel{2}} = 48$$

$$m\angle UYX = \frac{1}{2} \cdot m\widehat{XZY} = \frac{1}{2} \cdot [360 - (m\widehat{XW} + m\widehat{WY})] = \frac{1}{2} \cdot [360 - (82 + 96)] = \frac{1}{2} \cdot [360 - 178] = \frac{1 \cdot 91 \cdot \cancel{2}}{\cancel{2}} = 91$$

$$4. \quad \text{a) } m\angle CAD = \frac{1}{2} \cdot m\widehat{AC} = \frac{1}{2} \cdot 102 = \frac{1 \cdot 51 \cdot \cancel{2}}{\cancel{2}} = 51$$

$$\text{b) } m\widehat{BC} = 360 - m\widehat{AC} - m\widehat{AEB} = 360 - 102 - 200 = 58$$

$$\text{c) } m\angle ABC = \frac{1}{2} \cdot m\widehat{AC} = \frac{1}{2} \cdot 102 = \frac{1 \cdot 51 \cdot \cancel{2}}{\cancel{2}} = 51$$

$$\text{d) } m\angle BAF = \frac{1}{2} \cdot m\widehat{BEA} = \frac{1}{2} \cdot 200 = \frac{1 \cdot 100 \cdot \cancel{2}}{\cancel{2}} = 100$$

$$\text{e) } m\angle BAC = \frac{1}{2} \cdot m\widehat{BC} = \frac{1}{2} \cdot 58 = \frac{1 \cdot 29 \cdot \cancel{2}}{\cancel{2}} = 29$$

$$\text{f) } m\angle ACB = \frac{1}{2} \cdot m\widehat{BEA} = \frac{1}{2} \cdot 200 = \frac{1 \cdot 100 \cdot \cancel{2}}{\cancel{2}} = 100$$

$$5. \quad \text{a) } m\angle DAC = \frac{1}{2} \cdot m\widehat{AD} = \frac{1}{2} \cdot 80 = \frac{1 \cdot 40 \cdot \cancel{2}}{\cancel{2}} = 40$$

$$\text{b) } m\angle AFD = \frac{1}{2} \cdot m\widehat{AD} = \frac{1}{2} \cdot 80 = \frac{1 \cdot 40 \cdot \cancel{2}}{\cancel{2}} = 40$$

$$\text{c) } m\angle DEF = \frac{1}{2} \cdot m\widehat{DAF} = \frac{1}{2} \cdot 180 = \frac{1 \cdot 90 \cdot \cancel{2}}{\cancel{2}} = 90$$

$$\text{d) } m\angle DFE = \frac{1}{2} \cdot m\widehat{DE} = \frac{1}{2} \cdot 70 = \frac{1 \cdot 35 \cdot \cancel{2}}{\cancel{2}} = 35$$

$$\text{e) } m\widehat{AF} = 180 - m\widehat{AD} = 180 - 80 = 100$$

$$\text{f) } m\angle BAF = \frac{1}{2} \cdot m\widehat{AF} = \frac{1}{2} \cdot 100 = \frac{1 \cdot 50 \cdot \cancel{2}}{\cancel{2}} = 50$$

$$\text{g) } m\angle EDF = \frac{1}{2} \cdot m\widehat{EF} = \frac{1}{2} \cdot (180 - 70) = \frac{1}{2} \cdot 110 = \frac{1 \cdot 55 \cdot \cancel{2}}{\cancel{2}} = 55$$

6.

STATEMENT	REASONS
1. \overline{RT} is a common internal tangent to $\odot P$ and $\odot Q$.	1. Given
2. $\overline{PR} \perp \overline{RT}$	2. Corollary 68a - If you have a tangent to a circle, then the diameter (radius) drawn to the point of tangency is perpendicular to that tangent.
3. $\overline{QT} \perp \overline{RT}$	3. Corollary 68a
4. $\angle PRU$ is a right angle	4. Definition of Perpendicular Lines
5. $\angle QTU$ is a right angle	5. Definition of Perpendicular Lines
6. $\triangle PRU$ is a right triangle	6. Definition of Right Triangle
7. $\triangle QTU$ is a right triangle	7. Definition of Right Triangle
8. $\angle PUR \cong \angle QUT$	8. Theorem 15 - If two lines intersect, then the vertical angles formed are congruent.
9. $\triangle PUR \sim \triangle QTU$	9. Postulate Corollary 12b - If an acute angle of one right triangle is congruent to an acute angle of another right triangle, then the two triangles are similar.
10. $\frac{PU}{QU} = \frac{RU}{TU}$	10. If two triangles are similar, then corresponding angles are congruent and corresponding sides are proportional. (Definition of Similar Polygons)

7. $\overline{QT} \perp \overline{SR}$, so $\triangle QTR$ is a right triangle.

$$\begin{array}{l}
 (QT)^2 + (TR)^2 = (QR)^2 \\
 (6)^2 + (8)^2 = (QR)^2 \\
 36 + 64 = (QR)^2 \\
 100 = (QR)^2 \\
 10 = QR
 \end{array}
 \quad
 \begin{array}{l}
 PR - QR = PQ \\
 30 - 10 = PQ \\
 20 = PQ \\
 \\
 PQ = PS \\
 20 = PS
 \end{array}
 \quad
 \begin{array}{l}
 (PS)^2 + (SR)^2 = (PR)^2 \\
 (20)^2 + (SR)^2 = (30)^2 \\
 400 + (SR)^2 = 900 \\
 (SR)^2 = 500 \\
 SR = \sqrt{500} \\
 SR = \sqrt{5 \cdot 100} \\
 SR = 10\sqrt{5} \\
 \\
 SR - RT = ST \\
 10\sqrt{5} - 8 = ST
 \end{array}$$

8. a) QS, PS

b) QP, QS

c)

$$\begin{array}{l}
 \frac{QS}{TS} = \frac{TS}{PS} \\
 \frac{6}{TS} = \frac{TS}{24} \\
 (6)(24) = (TS)(TS) \\
 144 = TS^2 \\
 12 = TS
 \end{array}
 \quad
 \begin{array}{l}
 \frac{QP}{TP} = \frac{TP}{PS} \\
 \frac{PS + SQ}{TP} = \frac{TP}{24} \\
 \frac{6 + 24}{TP} = \frac{TP}{24} \\
 \frac{30}{TP} = \frac{TP}{24} \\
 (30)(24) = (TP)(TP) \\
 720 = TP^2 \\
 \sqrt{720} = TP \\
 \sqrt{144 \cdot 5} = TP \\
 12\sqrt{5} = TP
 \end{array}$$

9. $RQ = TQ$ since radii of the same circle have the same measure.

Let $RQ = TQ = x$ and $SQ = x + 7$. $\overline{RQ} \perp \overline{SR}$ by Corollary 68a, so $\triangle QRS$ is a right triangle.

$$(RQ)^2 + (SR)^2 = (SQ)^2$$

$$x^2 + 17^2 = (x + 7)^2$$

$$x^2 + 289 = (x + 7)(x + 7)$$

$$x^2 + 289 = x^2 + 14x + 49$$

$$240 = 14x$$

$$\frac{240}{14} = x$$

$$\cancel{2} \cdot \frac{120}{\cancel{2} \cdot 7} = x$$

$$\frac{120}{7} = x$$

$$17\frac{1}{7} = x$$

10.

STATEMENT	REASONS
1. $\overline{BQ} \perp \overleftrightarrow{ED}$ at point Q 2. $\angle BQD$ is a right angle 3. $m\angle BQD = 90$ 4. $m\widehat{BD} = 90$	1. Given 2. Definition of Perpendicular Lines 3. Definition of Right Angles 4. Definition of Arc Measure - The measure of an arc of a circle is equal to the measure of its corresponding central angle.

11.

STATEMENT	REASONS
1. \overleftrightarrow{AC} is tangent to $\odot P$ and $\odot Q$ at point B. 2. $m\angle DQB = m\widehat{BD}$ 3. $m\angle DBC = \frac{1}{2} \cdot m\widehat{BD}$ 4. $m\angle DBC = \frac{1}{2} \cdot m\angle DQB$	1. Given 2. Definition of Arc Measure - The measure of an arc of a circle is equal to the measure of its corresponding central angle. 3. Theorem 67 - If you have an inscribed angle of a circle, then the measure of the angle is one-half the measure of the intercepted arc. 4. Substitution

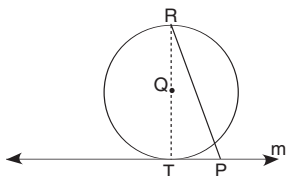
12. $m\angle USP = \frac{1}{2} \cdot m\widehat{SU}$ $m\angle RST = \frac{1}{2} \cdot m\widehat{SR}$
 $85 = \frac{1}{2} \cdot m\widehat{SU}$ $45 = \frac{1}{2} \cdot m\widehat{SR}$
 $170 = m\widehat{SU}$ $90 = m\widehat{SR}$

$$\begin{array}{l}
 m\angle SUR = \frac{1}{2} \cdot m\widehat{SR} \\
 = \frac{1}{2} \cdot 90 \\
 = \frac{90}{2} \\
 = \cancel{2} \cdot 45 \\
 = 45
 \end{array}
 \qquad
 \begin{array}{l}
 m\angle SRU = \frac{1}{2} \cdot m\widehat{SU} \\
 = \frac{1}{2} \cdot 170 \\
 = \frac{170}{2} \\
 = \cancel{2} \cdot 85 \\
 = 85
 \end{array}$$

$$\begin{array}{l}
 m\angle SUR + m\angle SRU + m\angle USR = 180 \\
 45 + 85 + m\angle USR = 180 \\
 130 + m\angle USR = 180 \\
 m\angle USR = 50
 \end{array}$$

13. a) Corollary 68a - "If you have a tangent to a circle, then the diameter drawn to the point of tangency is perpendicular to that tangent.

b)



- c) Given: Line m is a tangent line to circle Q at point T .

\overline{RT} is a diameter of circle Q

- d) Prove: $\overline{RT} \perp m$

- e) Indirect Proof - Assume temporarily that \overline{RT} is **not** perpendicular to line m . Then there must exist a line segment, say \overline{RP} , that is perpendicular to line m . (Postulate 10 - Uniqueness of Perpendicular Lines - In a plane, through a point on a given line, there is exactly one line perpendicular to the given line.) Since the perpendicular segment from a point to a line is the shortest distance from the point to the line, $RP < RT$. Because tangent line m intersects circle Q only in point T , point P would have to lie outside circle Q and $RP > RT$. The statements $RP < RT$ and $RP > RT$ are contradictory. Thus, the temporary assumption is false and $\overline{RT} \perp m$.

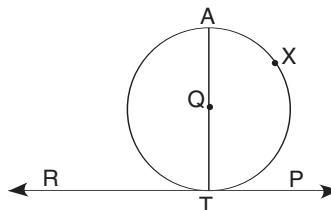
14. Case 1:

If you have an angle formed by a secant and a tangent at the point of tangency, then the measure of that angle is one-half the measure of its intercepted arc.

Given: $\odot Q$ with chord \overline{TA} intersecting tangent \overleftrightarrow{RP} at point T .

Prove: $m\angle ATP = \frac{1}{2} \cdot m\widehat{AXT}$

Case 1: Point Q , the center of $\odot Q$, lies on the angle.



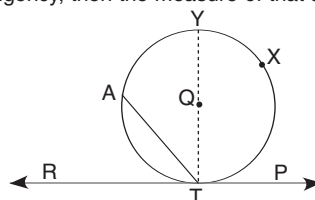
STATEMENT	REASONS
1. $\odot Q$ with chord \overline{TA} intersecting tangent \overleftrightarrow{RP} at point T .	1. Given
2. \overline{TA} is a diameter of $\odot Q$	2. Definition of diameter
3. $\overline{TA} \perp \overleftrightarrow{RP}$	3. Corollary 68a - If you have a tangent to a circle, then the diameter drawn to the point of tangency is perpendicular to that tangent.
4. $\angle ATP$ is a right angle	4. Definition of Perpendicular
5. $m\angle ATP = 90$	5. Definition of Right Angle
6. \widehat{AXT} is a semicircle	6. Definition of Semi Circle - An arc of a circle whose endpoints are the endpoints of a diameter of that circle.
7. $m\widehat{AXT} = 180$	7. The measure of a semicircle is 180.
8. $\frac{1}{2} \cdot 180 = 90$	8. Arithmetic Fact
9. $m\angle ATP = \frac{1}{2} \cdot 180$	9. Substitution
10. $m\angle ATP = \frac{1}{2} \cdot m\widehat{AXT}$	10. Substitution.

14. Case 2:

If you have an angle formed by a secant and a tangent at the point of tangency, then the measure of that angle is one-half the measure of its intercepted arc.

Given: $\odot Q$ with chord \overline{TA} intersecting tangent \overleftrightarrow{RP} at point T.
 Prove: $m\angle ATP = \frac{1}{2} \cdot m\widehat{AXT}$

Case 2: Point Q, the center of $\odot Q$, lies in the interior of $\angle ATP$.



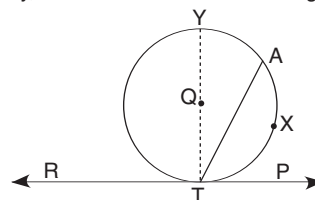
STATEMENT	REASONS
1. $\odot Q$ with chord \overline{TA} intersecting tangent \overleftrightarrow{RP} at point T.	1. Given
2. Draw \overline{YT} through point Q, the center of circle Q.	2. Postulate 2 - For any two different points, there is exactly one line containing them.
3. \overline{YT} is a diameter of $\odot Q$	3. Definition of diameter
4. $\overline{YT} \perp \overline{RP}$	4. Corollary 68a - If you have a tangent to a circle, then the diameter drawn to the point of tangency is perpendicular to that tangent.
5. $m\angle YTP = \frac{1}{2} \cdot m\widehat{YXT}$	5. Proof in case 1
6. $m\angle ATY = \frac{1}{2} \cdot m\widehat{AY}$	6. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one half the measure of the intercepted arc.
7. $m\angle ATP = m\angle ATY + m\angle YTP$	7. Postulate 7 - Protractor Postulate - Angle Addition Assumption - Fourth Assumption.
8. $m\widehat{AXT} = m\widehat{AY} + m\widehat{YXT}$	8. Postulate 8 - Arc Addition Assumption - Fourth Assumption.
9. $m\angle ATP = \frac{1}{2} \cdot m\widehat{AY} + \frac{1}{2} \cdot m\widehat{YXT}$	9. Substitution
10. $m\angle ATP = \frac{1}{2} (m\widehat{AY} + m\widehat{YXT})$	10. Distributive Property of Multiplication over Addition
11. $m\angle ATP = \frac{1}{2} m\widehat{AXT}$	11. Substitution

Case 3:

If you have an angle formed by a secant and a tangent at the point of tangency, then the measure of that angle is one-half the measure of its intercepted arc.

Given: $\odot Q$ with chord \overline{TA} intersecting tangent \overleftrightarrow{RP} at point T.
 Prove: $m\angle ATP = \frac{1}{2} \cdot m\widehat{AXT}$

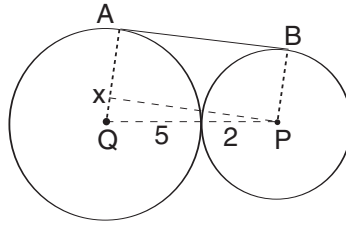
Case 3: Point Q, the center of $\odot Q$, lies in the exterior of $\angle ATP$.



STATEMENT	REASONS
1. $\odot Q$ with chord \overline{TA} intersecting tangent \overleftrightarrow{RP} at point T.	1. Given
2. Draw \overline{YT} through point Q, the center of circle Q.	2. Postulate 2 - For any two different points, there is exactly one line containing them.
3. \overline{YT} is a diameter of $\odot Q$	3. Definition of diameter
4. $\overline{YT} \perp \overline{RP}$	4. Corollary 68a - If you have a tangent to a circle, then the diameter drawn to the point of tangency is perpendicular to that tangent.
5. $m\angle YTP = \frac{1}{2} \cdot m\widehat{YXT}$	5. Proof in case 1
6. $m\angle ATY = \frac{1}{2} \cdot m\widehat{AY}$	6. Theorem 67 - If an angle is inscribed in a circle, then the measure of that angle is one half the measure of the intercepted arc.
7. $m\angle YTP - m\angle ATY = \frac{1}{2} \cdot m\widehat{YXT} - \frac{1}{2} \cdot m\widehat{AY}$	7. Subtraction Property of Equality
8. $m\angle ATY + m\angle ATP = m\angle YTP$	8. Postulate 7 - Protractor Postulate - Angle Addition Assumption - Fourth Assumption.
9. $m\angle ATP = m\angle YTP - m\angle ATY$	9. Subtraction Property of Equality
10. $m\angle ATP = \frac{1}{2} \cdot m\widehat{YXT} - \frac{1}{2} \cdot m\widehat{AY}$	10. Substitution.
11. $m\angle ATP = \frac{1}{2} (m\widehat{YXT} - m\widehat{AY})$	11. Distributive Property of Multiplication over Addition (Subtraction)
12. $m\widehat{AY} + m\widehat{AXT} = m\widehat{YXT}$	12. Postulate 8 - Arc Addition Assumption - Fourth Assumption
13. $m\widehat{AXT} = m\widehat{YXT} - m\widehat{AY}$	13. Subtraction Property of Equality
14. $m\angle ATP = \frac{1}{2} \cdot m\widehat{AXT}$	14. Substitution

$$\begin{aligned}
 15. \quad & (XQ)^2 + (XP)^2 = (PQ)^2 \\
 & (AQ - AX)^2 + (XP)^2 = (PQ)^2 \\
 & (5 - 2)^2 + (XP)^2 = (5 + 2)^2 \\
 & (3)^2 + (XP)^2 = (7)^2 \\
 & 9 + (XP)^2 = 49 \\
 & (XP)^2 = 40
 \end{aligned}$$

$$XP = \sqrt{40} = \sqrt{4 \cdot 10} = 2\sqrt{10}$$



Unit VI — Circles

Part B — Angle and Arc Relationships

p. 547 – Lesson 4 — Theorem 69 and 70

$$\begin{aligned}
 1. \quad m\angle 1 &= \frac{1}{2} \cdot (50 + 20) \\
 &= \frac{1}{2} \cdot 70 \\
 &= \frac{\cancel{2} \cdot 35}{\cancel{2}} \\
 &= 35
 \end{aligned}$$

$$\begin{aligned}
 m\angle 2 &= 180 - 35 \\
 &= 145
 \end{aligned}$$

$$\begin{aligned}
 2. \quad m\angle 1 &= 180 - \left[\frac{1}{2} (130 + 100) \right] \\
 &= 180 - \left[\frac{1}{2} \cdot 230 \right] \\
 &= 180 - \left[\frac{\cancel{2} \cdot 115}{\cancel{2}} \right] \\
 &= 180 - 115 \\
 &= 65
 \end{aligned}$$

$$\begin{aligned}
 3. \quad m\angle 1 &= \frac{1}{2} (120 - 40) \\
 &= \frac{1}{2} (80) \\
 &= \frac{\cancel{2} \cdot 40}{\cancel{2}} \\
 &= 40
 \end{aligned}$$

$$\begin{aligned}
 4. \quad 33 &= \frac{1}{2} (108 - x) \\
 66 &= 108 - x \\
 -42 &= -x \\
 42 &= x
 \end{aligned}$$

$$\begin{aligned}
 5. \quad 85 &= \frac{1}{2} (x + 100) \\
 170 &= x + 100 \\
 70 &= x
 \end{aligned}$$

$$\begin{aligned}
 6. \quad 20 &= \frac{1}{2} (x - 30) \\
 40 &= x - 30 \\
 70 &= x
 \end{aligned}$$

$$\begin{aligned}
 7. \quad x &= 135 \\
 m\angle 1 &= 180 - 135 \\
 &= 45
 \end{aligned}$$

$$\begin{aligned}
 8. \quad m\angle B &= \frac{1}{2} (m\widehat{AC} - m\widehat{DE}) \\
 90 - a &= \frac{1}{2} (x - (90 + a)) \\
 180 - 2a &= x - (90 + a) \\
 180 - 2a &= x - 90 - a \\
 270 - a &= x
 \end{aligned}$$

$$\begin{aligned}
 9. \quad m\angle B &= \frac{1}{2} (m\widehat{AC} - m\widehat{DE}) & 10. \quad m\angle 1 &= 50 \\
 \frac{3a}{4} &= \frac{1}{2} (x - a) & & x = 180 - 36 \\
 3a &= 2(x - a) & & = 144 \\
 3a &= 2x - 2a \\
 5a &= 2x \\
 \frac{5a}{2} &= x
 \end{aligned}$$

11.

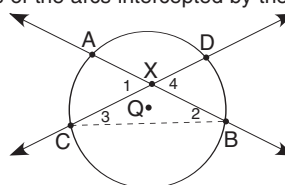
STATEMENT	REASONS
1. $\overline{RP} \perp \overline{YT}$	1. Given
2. $\angle PXT$ is a right angle	2. Definition of Perpendicular
3. $\overline{RT} \perp \overline{YS}$	3. Given
4. $\angle SVT$ is a right angle	4. Definition of Perpendicular
5. $\angle PXT \cong \angle SVT$	5. Theorem 11 - If you have right angles, then those right angles are congruent.
6. $m\angle PXT = m\angle SVT$	6. Definition of Congruent Angles
7. $m\angle PXT = \frac{1}{2} \cdot (m\widehat{PT} + m\widehat{RY})$	7. Theorem 69 - If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
8. $m\angle SVT = \frac{1}{2} \cdot (m\widehat{ST} + m\widehat{RY})$	8. Theorem 69
9. $\frac{1}{2} (m\widehat{PT} + m\widehat{RY}) = \frac{1}{2} (m\widehat{ST} + m\widehat{RY})$	9. Substitution
10. $m\widehat{PT} + m\widehat{RY} = m\widehat{ST} + m\widehat{RY}$	10. Multiplication Property of Equality
11. $m\widehat{PT} = m\widehat{ST}$	11. Addition (Subtraction) Property of Equality

12.

STATEMENT	REASONS
1. $m\widehat{PT} = m\widehat{ST}$	1. Given
2. $m\widehat{RY} = m\widehat{RY}$	2. Reflexive Property of Equality
3. $m\widehat{PT} + m\widehat{RY} = m\widehat{ST} + m\widehat{RY}$	3. Addition Property of Equality
4. $\frac{1}{2} (m\widehat{PT} + m\widehat{RY}) = \frac{1}{2} (m\widehat{ST} + m\widehat{RY})$	4. Multiplication Property of Equality
5. $\frac{1}{2} (m\widehat{PT} + m\widehat{RY}) = m\angle PXT$	5. Theorem 69 - If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the sum of the measures of the arcs intercepted by the angle and its vertical angle.
6. $\frac{1}{2} (m\widehat{ST} + m\widehat{RY}) = m\angle SVT$	6. Theorem 69
7. $m\angle PXT = m\angle SVT$	7. Substitution

13. Theorem 69 - If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle) is equal to one-half the sum of the measures of the arcs intercepted by the angle, and its vertical angle.

Given: Secant Lines, \overleftrightarrow{AB} and \overleftrightarrow{CD} , intersect inside $\odot Q$ at point X.



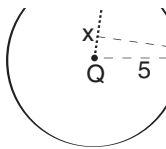
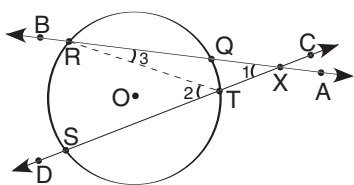
Part 1 Prove: $m\angle 1 = \frac{1}{2} \cdot (m\widehat{AC} + m\widehat{BD})$

STATEMENT	REASONS
1. Secant Lines, \overleftrightarrow{AB} and \overleftrightarrow{CD} , intersect inside $\odot Q$ at point X.	1. Given
2. Draw \overline{CB} forming $\triangle CXB$	2. Postulate 2 - For any two different points, there is exactly one line containing them.
3. $m\angle 1 = m\angle 2 + m\angle 3$	3. Theorem 26 - If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two interior angles.
4. $m\angle 2 = \frac{1}{2} m\widehat{AC}$	4. Theorem 67 - If an angle is inscribed in a circle, then the measure of the angle is one-half the measure of the intercepted arcs.
5. $m\angle 3 = \frac{1}{2} m\widehat{BD}$	5. Theorem 67
6. $m\angle 1 = \frac{1}{2} m\widehat{AC} + \frac{1}{2} m\widehat{BD}$	6. Substitution
7. $m\angle 1 = \frac{1}{2} (m\widehat{AC} + m\widehat{BD})$	7. Distributive Property of Multiplication over Addition

Part 2 Prove: $m\angle 4 = \frac{1}{2} \cdot (m\widehat{BD} + m\widehat{AC})$

STATEMENT	REASONS
1. Secant Lines, \overleftrightarrow{AB} and \overleftrightarrow{CD} , intersect inside $\odot Q$ at point x.	1. Given
2. Draw \overline{BC}	2. Postulate 2 - For any two different points, there is exactly one line containing them.
3. $m\angle 4 = m\angle 3 + m\angle 2$	3. Theorem 26 - If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two interior angles.
4. $m\angle 3 = \frac{1}{2} m\widehat{BD}$	4. Theorem 67 - If an angle is inscribed in a circle, then the measure of the angle is one-half the measure of the intercepted arcs.
5. $m\angle 2 = \frac{1}{2} m\widehat{AC}$	5. Theorem 67
6. $m\angle 4 = \frac{1}{2} m\widehat{BD} + \frac{1}{2} m\widehat{AC}$	6. Substitution
7. $m\angle 4 = \frac{1}{2} (m\widehat{BD} + m\widehat{AC})$	7. Distributive Property of Multiplication over Addition

14. Theorem 70 - If, for a circle, two secant lines intersect outside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the difference of the measure of the arcs intercepted by the angle.



Given: Secant Lines, \overleftrightarrow{AB} and \overleftrightarrow{CD} , intersect outside $\odot O$ at point X.

Prove: $m\angle 1 = \frac{1}{2} \cdot (m\widehat{SR} - m\widehat{TQ})$

STATEMENT	REASONS
1. Secant Lines, \overleftrightarrow{AB} and \overleftrightarrow{CD} , intersect outside $\odot O$ at point X.	1. Given
2. Draw \overline{RT} forming $\triangle RTX$	2. Postulate 2 - For any two different points, there is exactly one line containing them.
3. $m\angle 2 = m\angle 1 + m\angle 3$	3. Theorem 26 - If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two interior angles.
4. $m\angle 2 = \frac{1}{2} m\widehat{SR}$	4. Theorem 67 - If an angle is inscribed in a circle, then the measure of the angle is one-half the measure of the intercepted arc.
5. $m\angle 3 = \frac{1}{2} m\widehat{TQ}$	5. Theorem 67
6. $\frac{1}{2} m\widehat{SR} = m\angle 1 + \frac{1}{2} m\widehat{TQ}$	6. Substitution
7. $\frac{1}{2} m\widehat{SR} - \frac{1}{2} m\widehat{TQ} = m\angle 1$	7. Addition (Subtraction) Property of Equality
8. $\frac{1}{2} (m\widehat{SR} - m\widehat{TQ}) = m\angle 1$	8. Distributive Property of Multiplication over Subtraction

15.

STATEMENT	REASONS
1. $\widehat{NS} \cong \widehat{RT}$	1. Given
2. $\widehat{SNM} \cong \widehat{TRM}$	2. Given
3. $m\widehat{NS} = m\widehat{RT}$	3. Definition of Congruent Arcs
4. $m\widehat{SNM} = m\widehat{TRM}$	4. Definition of Congruent Arcs
5. $m\widehat{SNM} = m\widehat{NS} + m\widehat{MN}$	5. Postulate 8 - Fourth Assumption - Arc Addition Assumption
6. $m\widehat{TRM} = m\widehat{RT} + m\widehat{MR}$	6. Postulate 8
7. $m\widehat{NS} + m\widehat{MN} = m\widehat{RT} + m\widehat{MR}$	7. Substitution
8. $m\widehat{RT} + m\widehat{MN} = m\widehat{NS} + m\widehat{MR}$	8. Substitution
9. $\frac{1}{2} (m\widehat{RT} + m\widehat{MN}) = \frac{1}{2} (m\widehat{NS} + m\widehat{MR})$	9. Multiplication Property of Equality
10. $\frac{1}{2} (m\widehat{RT} + m\widehat{MN}) = m\angle PUM$	10. Theorem 69 - If you have two secants (chords) intersecting inside a circle, then the measure of the angle formed is equal to one-half the sum of the measures of the intercepted arcs.
11. $\frac{1}{2} (m\widehat{NS} + m\widehat{MR}) = m\angle UPM$	11. Theorem 69
12. $m\angle PUM = m\angle UPM$	12. Substitution
13. $\angle PUM \cong \angle UPM$	13. Definition of Congruent Angles
14. $\overline{MP} \cong \overline{MU}$	14. Theorem 34 - If two angles of a triangle are congruent, the sides opposite them are congruent.
15. $\triangle MPU$ is isosceles	15. A triangle is isosceles if and only if the triangle has at least two congruent sides.

Unit VI — Circles

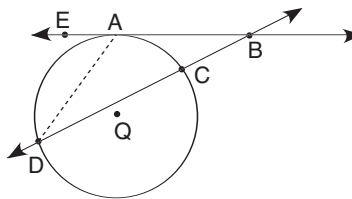
Part B — Angle and Arc Relationships

p. 551 – Lesson 5 — Theorem 71 & 72

1. Theorem 71 - "If, for a circle, a secant line and a tangent line intersect outside a circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle."

Given: \overleftrightarrow{AB} is tangent to $\odot Q$ at point A.
 \overleftrightarrow{BD} is a secant of $\odot Q$.

Prove: $m\angle ABD = \frac{1}{2} (m\widehat{AD} - m\widehat{AC})$

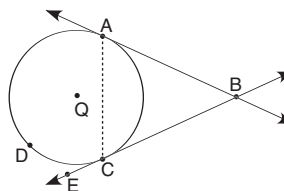


STATEMENT	REASONS
1. \overleftrightarrow{AB} is tangent to $\odot Q$ at point A.	1. Given
2. \overleftrightarrow{BD} is a secant of $\odot Q$	2. Given
3. Draw AD	3. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
4. $m\angle EAD = m\angle ADB + m\angle ABD$	4. Theorem 26 - If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
5. $m\angle EAD = \frac{1}{2} m\widehat{AD}$	5. Theorem 68 - If you have an angle formed by a secant and a tangent at the point of tangency, then the measure of that angle is one-half the measure of its intercepted arc.
6. $m\angle ADB = \frac{1}{2} m\widehat{AC}$	6. Theorem 67 - If an angle is inscribed in a circle, then the measure of the angle is one-half the measures of the intercepted arc.
7. $\frac{1}{2} m\widehat{AD} = \frac{1}{2} m\widehat{AC} + m\angle ABD$	7. Substitution
8. $\frac{1}{2} m\widehat{AD} - \frac{1}{2} m\widehat{AC} = m\angle ABD$	8. Addition (Subtraction) Property of Equality
9. $\frac{1}{2} (m\widehat{AD} - m\widehat{AC}) = m\angle ABD$	9. Disctibutive Property of Multiplication over Subtraction
10. $m\angle ABD = \frac{1}{2} (m\widehat{AD} - m\widehat{AC})$	10. Symmetric Property of Equality

2. Theorem 72 - "If, for a circle, two tangent lines intersect outside the circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle."

Given: \overleftrightarrow{AB} is tangent to $\odot Q$ at point A.
 \overleftrightarrow{BC} is tangent to $\odot Q$ at point C.

Prove: $m\angle ABC = \frac{1}{2} (m\widehat{ADC} - m\widehat{AC})$



STATEMENT	REASONS
1. \overleftrightarrow{AB} is tangent to $\odot Q$ at point A.	1. Given
2. \overleftrightarrow{BC} is tangent to $\odot Q$ at point C.	2. Given
3. Draw AC	3. Postulate 2 - For any two different points, there is exactly one line (segment) containing them.
4. $m\angle BAC = \frac{1}{2} m\widehat{AC}$	4. Theorem 68 - If you have an angle formed by a secant and a tangent at the point of tangency, then the measure of that angle is one-half the measure of its intercepted arc.
5. $m\angle ACE = \frac{1}{2} m\widehat{ADC}$	5. Theorem 68
6. $m\angle ACE = m\angle BAC + m\angle ABC$	6. Theorem 26 - If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles.
7. $\frac{1}{2} m\widehat{ADC} = \frac{1}{2} m\widehat{AC} + m\angle ABC$	7. Substitution
8. $\frac{1}{2} m\widehat{ADC} - \frac{1}{2} m\widehat{AC} = m\angle ABC$	8. Addition (Subtraction) Property of Equality
9. $\frac{1}{2} (m\widehat{ADC} - m\widehat{AC}) = m\angle ABC$	9. Disctibutive Property of Multiplication over Subtraction
10. $m\angle ABC = \frac{1}{2} (m\widehat{ADC} - m\widehat{AC})$	10. Symmetric Property of Equality

$$\begin{aligned}
 3. \quad x &= \frac{1}{2}(230 - 60) \\
 &= \frac{1}{2} \cdot 170 \\
 &= \frac{\cancel{2} \cdot 85}{\cancel{2}} \\
 &= 85
 \end{aligned}$$

$$\begin{aligned}
 4. \quad y &= 360 - 90 \\
 &= 270 \\
 x &= \frac{1}{2}(270 - 90) \\
 &= \frac{1}{2} \cdot 180 \\
 &= \frac{\cancel{2} \cdot 90}{\cancel{2}} \\
 &= 90
 \end{aligned}$$

$$\begin{aligned}
 5. \quad 35 &= \frac{1}{2}[(360 - 90 - x) - x] \\
 35 &= \frac{1}{2}[270 - 2x] \\
 35 &= \frac{\cancel{2} \cdot 135}{\cancel{2}} - \frac{\cancel{2} \cdot x}{\cancel{2}} \\
 35 &= 135 - x \\
 -100 &= -x \\
 100 &= x
 \end{aligned}$$

$$\begin{aligned}
 6. \quad x &= \frac{1}{2}(250 - 110) \\
 &= \frac{1}{2} \cdot 140 \\
 &= \frac{\cancel{2} \cdot 70}{\cancel{2}} \\
 &= 70
 \end{aligned}$$

$$\begin{aligned}
 7. \quad 30 &= \frac{1}{2}(x - 80) \\
 30 &= \frac{1}{2}x - \frac{\cancel{2} \cdot 40}{\cancel{2}} \\
 30 &= \frac{1}{2}x - 40 \\
 70 &= \frac{1}{2}x \\
 140 &= x
 \end{aligned}$$

$$\begin{aligned}
 8. \quad 90 - x &= \frac{1}{2}(x - (90 - a)) \\
 90 - x &= \frac{1}{2}(x - 90 + a) \\
 180 - 2x &= x - 90 + a \\
 270 &= 3x + a \\
 270 - a &= 3x \\
 90 - \frac{1}{3}a &= x
 \end{aligned}$$

$$\begin{aligned}
 9. \quad x &= \frac{1}{2}(280 - 80) \\
 &= \frac{1}{2} \cdot 200 \\
 &= \frac{\cancel{2} \cdot 100}{\cancel{2}} \\
 &= 100
 \end{aligned}$$

$$\begin{aligned}
 10. \quad y &= 360 - x \\
 44 &= \frac{1}{2}(y - x) \\
 44 &= \frac{1}{2}((360 - x) - x) \\
 88 &= 360 - 2x \\
 -272 &= -2x \\
 136 &= x \\
 y &= 360 - 136 \\
 y &= 224
 \end{aligned}$$

$$\begin{aligned}
 11. \quad 100 + a + m\widehat{BC} &= 360 \\
 m\widehat{BC} &= 360 - 100 - a \\
 &= 260 - a \\
 x &= \frac{1}{2}[(260 - a) - a] \\
 &= \frac{1}{2} \cdot (260 - 2a) \\
 &= \frac{\cancel{2} \cdot 130}{\cancel{2}} - \frac{\cancel{2} \cdot a}{\cancel{2}} \\
 &= 130 - a
 \end{aligned}$$

$$\begin{aligned}
 12. \quad m\widehat{AB} &= 180 - 80 \\
 &= 100
 \end{aligned}$$

$$\begin{aligned}
 13. \quad m\angle ACE &= \frac{1}{2}(m\widehat{AE} - m\widehat{AD}) \\
 18 &= \frac{1}{2}(92 - m\widehat{AD}) \\
 36 &= 92 - m\widehat{AD} \\
 -56 &= -m\widehat{AD} \\
 56 &= m\widehat{AD}
 \end{aligned}$$

$$\begin{aligned}
 14. \quad m\widehat{BD} &= 180 - m\widehat{AD} \\
 &= 180 - 56 \\
 &= 124
 \end{aligned}$$

$$\begin{aligned}
 x &= \frac{1}{2}(100 - 80) \\
 &= \frac{1}{2} \cdot (20) \\
 &= \frac{\cancel{2} \cdot 10}{\cancel{2}} \\
 &= 10
 \end{aligned}$$

$$\begin{aligned}
 15. \quad m\widehat{BE} &= 180 - 92 \\
 &= 88 \\
 m\angle EFB &= \frac{1}{2}(m\widehat{AD} + m\widehat{BE}) \\
 &= \frac{1}{2} \cdot (56 + 88) \\
 &= \frac{1}{2} \cdot (144) \\
 &= \frac{\cancel{2} \cdot 72}{\cancel{2}} \\
 &= 72
 \end{aligned}$$

$$\begin{aligned}
 16. \quad m\angle DAC &= \frac{1}{2}m\widehat{BC} \\
 &= \frac{1}{2} \cdot (56) \\
 &= \frac{\cancel{2} \cdot 28}{\cancel{2}} \\
 &= 28
 \end{aligned}$$

$$\begin{aligned}
 17. \quad m\angle ADC &= 180 - m\angle ACD - m\angle DAB \\
 &= 180 - 18 - 28 \\
 &= 180 - 46 \\
 &= 134
 \end{aligned}$$

$$\begin{aligned}
 18. \quad m\angle ADE &= 180 - m\angle ADC \\
 &= 180 - 134 \\
 &= 46
 \end{aligned}$$

$$\begin{aligned}
 &\text{or} \\
 m\angle ADE &= m\angle ACD + m\angle DAC \\
 &= 18 + 28 \\
 &= 46
 \end{aligned}$$

$$\begin{aligned}
 19. \quad m\angle AEC &= m\angle AED \\
 &= \frac{1}{2} \cdot m\widehat{AD} \\
 &= \frac{1}{2} \cdot 56 \\
 &= \frac{\cancel{2} \cdot 28}{\cancel{2}} \\
 &= 28
 \end{aligned}$$

20. Parallel

$$21. \quad m\angle ADB = 90$$

$$22. \quad m\angle CAB = 90$$

$$\begin{aligned}
 23. \quad m\angle ABE &= \frac{1}{2} m\widehat{AE} \\
 &= \frac{1}{2} \cdot (92) \\
 &= \frac{\cancel{2} \cdot 46}{\cancel{2}} \\
 &= 46
 \end{aligned}$$

$$24. \quad m\angle BEA = 90$$

$$\begin{aligned}
 25. \quad m\angle ABD &= \frac{1}{2} m\widehat{AB} \\
 &= \frac{1}{2} \cdot (56) \\
 &= \frac{\cancel{2} \cdot 28}{\cancel{2}} \\
 &= 28
 \end{aligned}$$