

Unit VI - Circles

Part C - Line and Segment Relationships

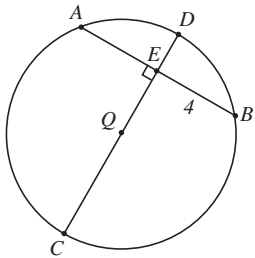
Lesson 1 - Theorem 73 - "If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord."

Lesson 2 - Theorem 74 - "If a diameter of a circle bisects a chord of the circle which is not a diameter of the circle, then that diameter is perpendicular to that chord."

Theorem 75 - "If a chord of a circle is a perpendicular bisector of another chord of that circle, then the original chord must be a diameter of the circle."

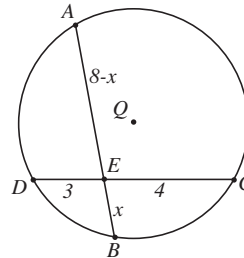
Lesson 3 - Theorem 76 - "If two chords intersect within a circle, then the product of the lengths of the segments of one chord, is equal to the product of the lengths of the segments of the other chord."

1. Find AE in $\odot Q$.



AE = _____

2. Find AE and BE in $\odot Q$.



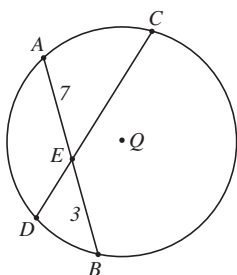
AE = _____

BE = _____

3. Find CD in $\odot Q$.

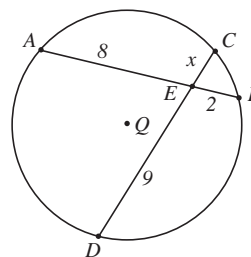
Given: $CE = 2x + 7$

$DE = x - 2$



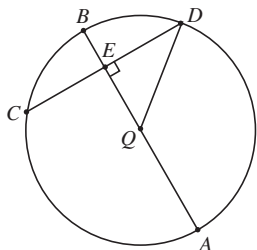
CD = _____

4. Find CD in $\odot Q$.



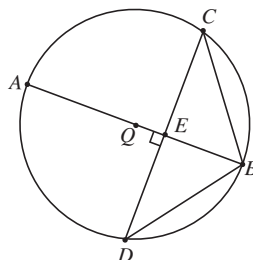
CD = _____

5. Find EQ in $\odot Q$,
 if $CD = 10$ and $DQ = 9$.



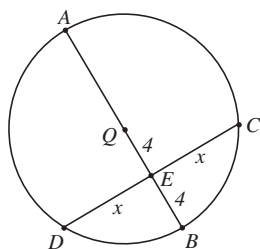
$EQ =$ _____

6. Find $m\widehat{CB}$ in $\odot Q$,
 if $m\widehat{CD} = 96^\circ$



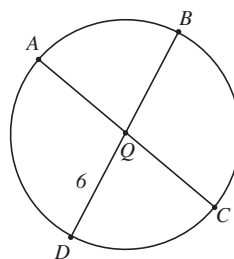
$m\widehat{CB} =$ _____

7. Find DC in $\odot Q$.



$DC =$ _____

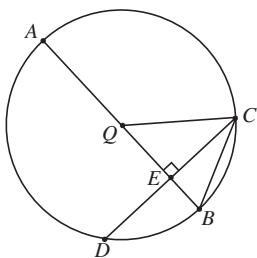
8. Find BD and AC in $\odot Q$.



$BD =$ _____

$AC =$ _____

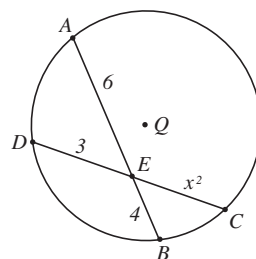
9. Find $m\widehat{CB}$ in $\odot Q$, given
 that $CD = 16$, $AQ = 9$
 and $EQ = 5$



$CD =$ _____

10. Find CE in $\odot Q$.

$CE =$ _____



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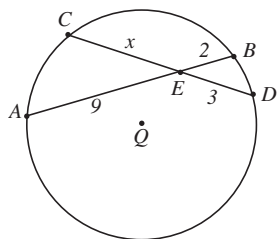
Lesson 1 - Theorem 73 - "If a diameter of a circle is perpendicular to a chord of that circle, then that diameter bisects that chord."

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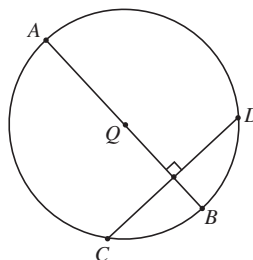
Lesson 3 - Theorem 76 - "If two chords intersect within a circle, then the product of the lengths of the segments of one chord, is equal to the product of the lengths of the segments of the other chord."

1. Find CE in $\odot Q$.



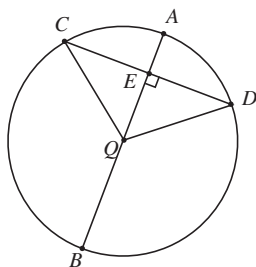
CE = _____

2. Find $m\widehat{BD}$ in $\odot Q$,
if $m\widehat{CD} = 110$



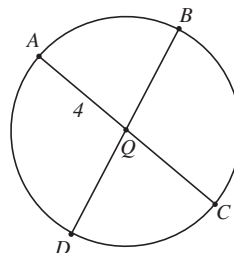
$m\widehat{BD} =$ _____

3. Find CD in $\odot Q$,
if $QD = 10$ and $QE = 4$

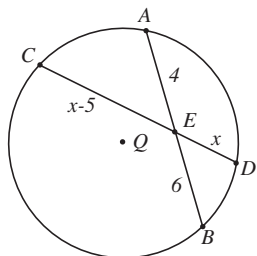


CD = _____

4. If $AQ = 4$, find BD in $\odot Q$. BD = _____

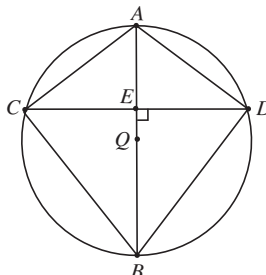


5. Find CD in $\odot Q$.

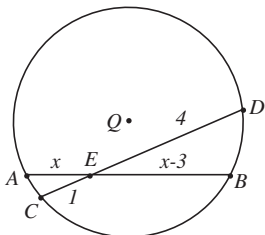


$CD =$ _____

6. Find BD in $\odot Q$, if $CD = 10$, $BD =$ _____
 $AE = 2$, and $QB = 8$.



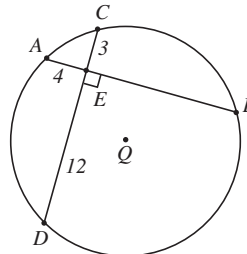
7. Find AE in $\odot Q$.



$AE =$ _____

8. Find BE in $\odot Q$.

$BE =$ _____

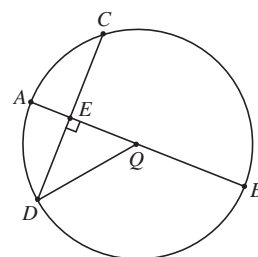
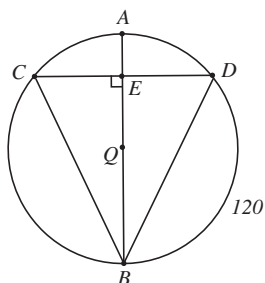


9. Find $m\widehat{CD}$ in $\odot Q$.

$m\widehat{CD} =$ _____

10. Find DQ in $\odot Q$, if
 $QE = 2$ and $CE = 4$.

$CE =$ _____



Unit VI - Circles

Part C - Line and Segment Relationships

Lesson 4 - Theorem 77 - "If two secant segments are drawn to a circle from a single point outside the circle, the product of the lengths of one secant segment and its external segment, is equal to the product of the lengths of the other secant segment and its external segment."

Theorem 78 - "If a secant segment and a tangent segment are drawn to a circle, from a single point outside the circle, then the length of that tangent segment is the mean proportional between the length of the secant segment, and the length of its external segment."

Lesson 5 - Theorem 79 - "If a line is perpendicular to a diameter of a circle at one of its endpoints, then the line must be tangent to the circle, at that endpoint."

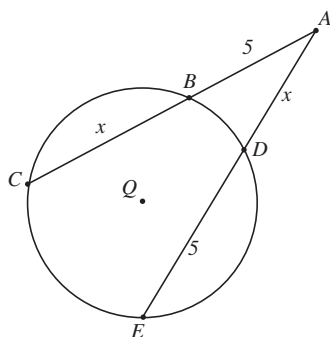
Lesson 6 - Theorem 80 - "If two tangent segments are drawn to a circle from the same point outside the circle, then those tangent segments are congruent."

Lesson 7 - Theorem 81 - "If two chords of a circle are congruent, then their intercepted minor arcs are congruent."

Theorem 82 - "If two minor arcs of a circle are congruent, then the chords which intercept them are congruent."

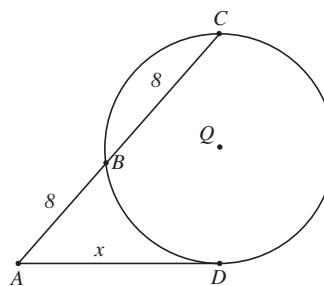
1. Find AC in $\odot Q$.

AC = _____



2. Find AD in $\odot Q$.

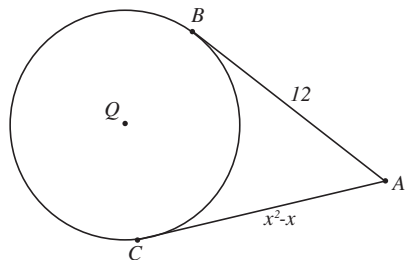
AD = _____



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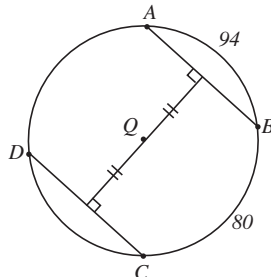
3. Find x in $\odot Q$.

$x =$ _____



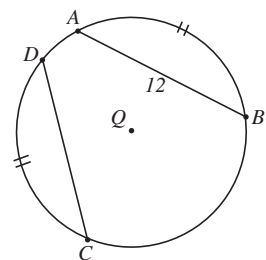
4. Find $m\widehat{AD}$ in $\odot Q$.

$m\widehat{AD} =$ _____



5. Find CD in $\odot Q$.

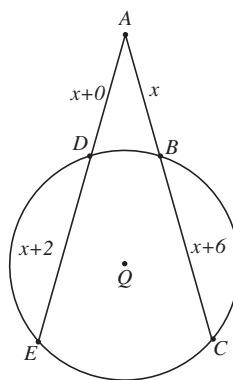
$CD =$ _____



6. Find AC and AE in $\odot Q$.

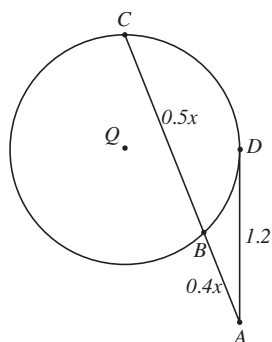
$AC =$ _____

$AE =$ _____



7. Find AB and BC in $\odot Q$. $AB =$ _____

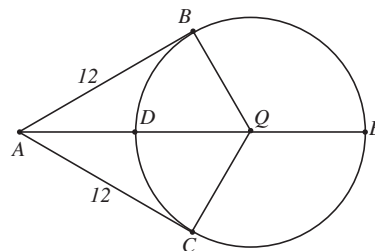
$BC =$ _____



8. AB and AC are tangents to $\odot Q$, and $m\angle BAC = 42$.

$m\angle BAE =$ _____

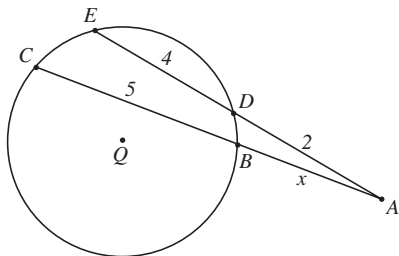
Find $m\angle BAE$.



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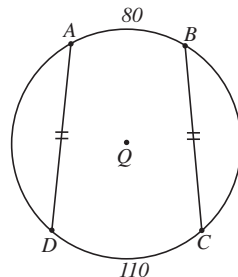
9. Find AB in $\odot Q$.

$AB =$ _____



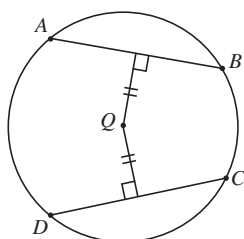
10. Find $m\widehat{BC}$ in $\odot Q$.

$m\widehat{BC} =$ _____



11. If $AB = 7$, find CD in $\odot Q$.

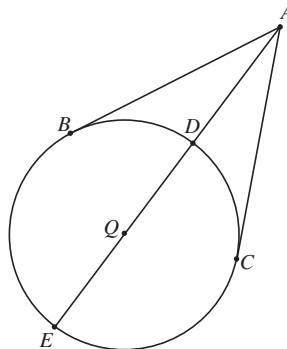
$CD =$ _____



12. Find AB and AC in $\odot Q$ if $AD = 8$ and $DE = 12$

$AB =$ _____

$AC =$ _____



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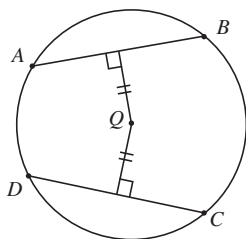
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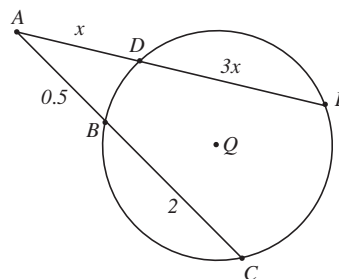
Theorem 82 - "If two minor arcs of a circle are congruent, then the chords which intercept them are congruent."

1. In $\odot Q$, if $AB = 9$, why is $CD = 9$?



2. Find DE in $\odot Q$.

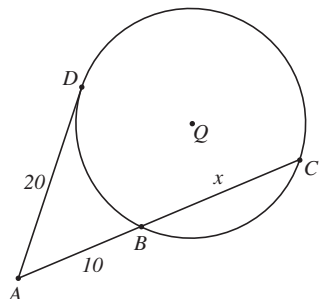
$DE =$ _____



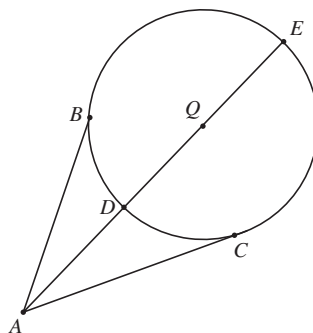
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3. Find BC in $\odot Q$.

BC = _____

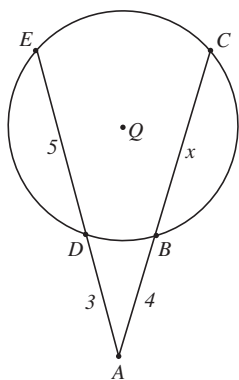


4. If, in $\odot Q$, $m\angle BAC = 52$, $m\angle CAE =$ _____
find $m\angle CAE$.

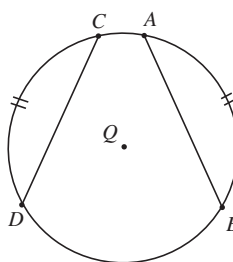


5. Find AC in $\odot Q$.

AC = _____



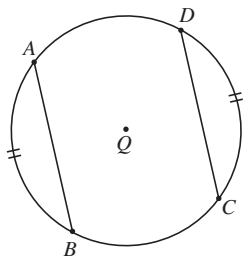
6. If, in $\odot Q$, $AB = 8$, why is $CD = 8$? _____



7. Find AB and CD in $\odot Q$,
given that $DC = x^2 + 2x$
and $AB = 2x^2 - 8$.

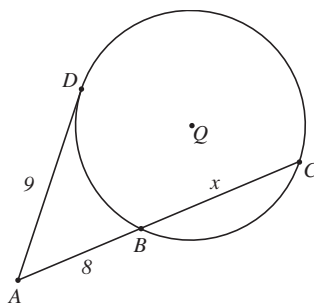
AB = _____

CD = _____



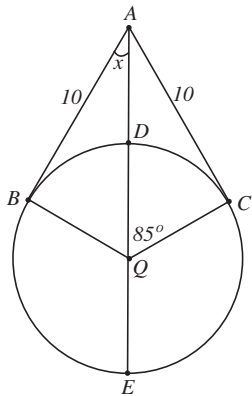
8. Find AC in $\odot Q$.

AC = _____

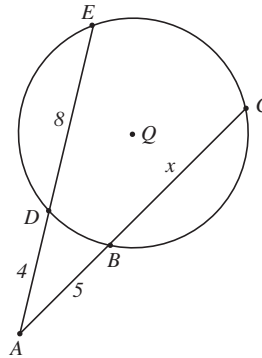


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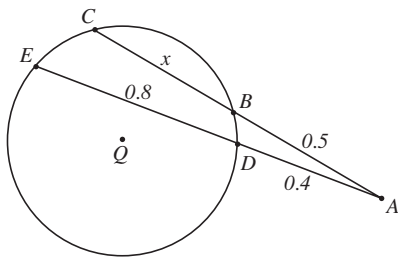
9. If, in $\odot Q$, \overline{AB} and \overline{AC} are tangent segments, find x .



10. Find AC in $\odot Q$. AC = _____

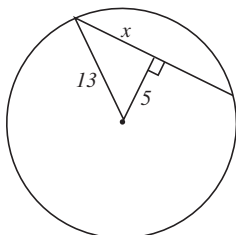


11. Find BC in $\odot Q$. BC = _____



12. A chord is located 5 inches from the center of a circle with radius 13. Find the length of the chord.

chord = _____



Unit VI - Circles

Part D - Circle Concurrency

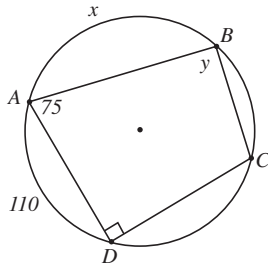
Lesson 1 - Theorem 83 - "If you have a triangle, then that triangle is cyclic."

Lesson 2 - Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic."

1. Quadrilateral ABCD is cyclic. Find x and y .

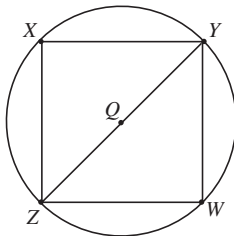
$x =$ _____

$y =$ _____



2. Quadrilateral (Kite) ABCD is cyclic. Find $m\widehat{AB}$.

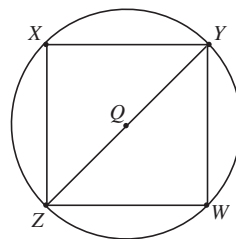
$\widehat{AB} =$ _____



Quadrilateral ABCD is a kite, so $CD \cong CB$, $BC \cong DC$ (Theorem 81), and $m\angle C = 134$.

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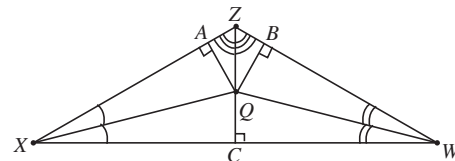
3. Given: \square Quadrilateral $XYWZ$ is cyclic.
 \overline{ZY} is a diameter of $\odot Q$.
 $\widehat{XY} \cong \widehat{WZ}$



Prove: $\angle XYZ \cong \angle WZY$

STATEMENT	REASON

4. The angle bisectors of the angles of $\triangle XYZ$ meet at point Q .
 $QX = 75$ and $QC = 20$. Find QB . Explain your answer.



$QB =$ _____

Complete the following statements by choosing “sometimes”, “always”, or “never”.

- Rectangles are _____ cyclic quadrilaterals.
- Irregular quadrilaterals are _____ cyclic.
- Regular polygons are _____ cyclic.
- A kite is _____ a cyclic quadrilateral.
- Opposite angles of a cyclic quadrilateral _____ add up to 180 degrees.
- Isosceles trapezoids are _____ cyclic quadrilaterals.

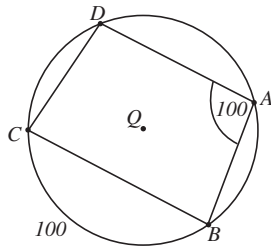
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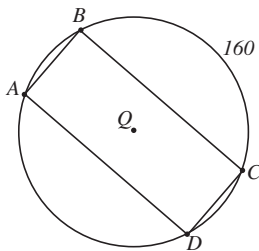
Lesson 1 - Theorem 83 - "If you have a triangle, then that triangle is cyclic."

Lesson 2 - Theorem 84 - "If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic."

1. Quadrilateral ABCD is cyclic with the measures shown below. Find $m\widehat{CD}$. $\widehat{CD} = \underline{\hspace{2cm}}$

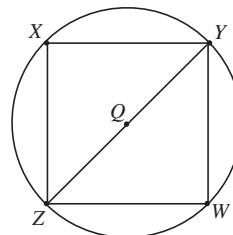


2. Quadrilateral ABCD is a cyclic rectangle, with the measures shown below. $\widehat{CD} = \underline{\hspace{2cm}}$
Find $m\widehat{CD}$.



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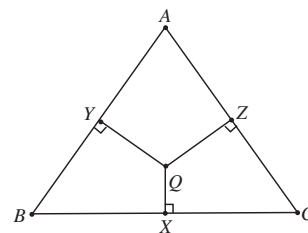
3. Given: Quadrilateral $XYWZ$ is cyclic, with diameter \overline{ZY} .
 $\widehat{XY} \cong \widehat{WZ}$



Prove: $\overline{XY} \parallel \overline{WZ}$

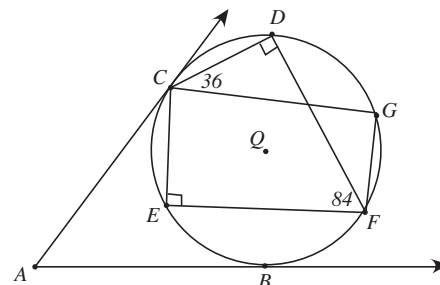
STATEMENT	REASON

4. The perpendicular bisectors of the sides of $\triangle ABC$ meet at point Q . $QX = 2$ and $QA = 7$. Find QC . Explain your answer.



$QC =$ _____

Use the figure to the right for problems 5 - 10.



5. Find $m\angle CGF$. _____
6. Find $m\widehat{ED}$. _____
7. Find $m\widehat{CBF}$. _____
8. Find $m\angle GFD$. _____
9. Find $m\widehat{DG}$. _____
10. Find $m\angle ECG$. _____

Unit VI - Circles

Match each term in Column I with a phrase in Column II which best fits that term.

- | | | |
|-------|---------------------------------------|---|
| _____ | (A-1)
1. Tangent | a) A line that intersects a circle in two points |
| _____ | (A-2)
2. Central Angle | b) A line segment whose endpoints are the center of a circle and a point on the circle. |
| _____ | (A-2)
3. Major Arc | c) A line segment whose endpoints are on the circle. |
| _____ | (A-2)
4. Intercepted Arc | d) The set of all points of a circle determined by the endpoints of a diameter. |
| _____ | (A-1)
5. Chord | e) A line that intersects a circle in exactly one point. |
| _____ | (A-1)
6. Diameter | f) The set of points in a plane such that all points are the same distance from a given point. |
| _____ | (A-2)
7. Minor Arc | g) An angle whose vertex is the center of a circle. |
| _____ | (A-1)
8. Secant | h) A line segment whose endpoints are on the circle and that contains the center of the circle. |
| _____ | (A-1)
9. Circle | i) The set of all points of a circle on or inside a central angle. |
| _____ | (A-3)
10. Inscribed Polygon | j) A polygon whose sides are chords of a circle. |
| _____ | (A-2)
11. Semicircle | k) The set of all points of a circle on or outside a central angle. |
| _____ | (A-3)
12. Radius | l) The set of all points of a circle determined by the points of intersection of the circle and the sides of an angle of the circle. |

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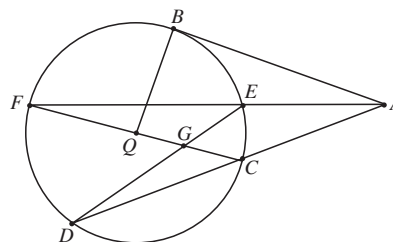
Determine whether each of the following is always, sometimes, or never true.

- _____ ^(C-7) **13.** Congruent chords of different circles intercept congruent arcs.
- _____ ^(B-2) **14.** An angle inscribed in a semicircle is a right angle.
- _____ ^(A-1) **15.** Two circles are congruent if their radii are congruent.
- _____ ^(A-3) **16.** Two externally tangent circles have two common tangents.
- _____ ^(A-1) **17.** A radius is a segment that joins two points on a circle.
- _____ ^(A-3) **18.** A polygon inscribed in a circle is a regular polygon.
- _____ ^(A-1) **19.** A secant is a line that lies in the plane of a circle, and contains a
_____ ^(B-2) chord of the circle.
- _____ **20.** The opposite angles of an inscribed quadrilateral are supplementary.
- _____ ^(Postulate 8 - p198) **21.** If point X is on \widehat{AB} , then $m\widehat{AX} + m\widehat{XB} = m\widehat{AXB}$.
- _____ ^(A-3) **22.** The common tangent segments of two circles of unequal radii are congruent.
- _____ ^(A-3) **23.** Tangent segments from an external point to two different circles
_____ ^(D-2) are congruent.
- _____ **24.** Cyclic quadrilaterals are congruent.
- _____ ^(A-3) **25.** If two circles are internally tangent, then the circles have three
common tangents.

—Continued—

Use the given figure to answer problems 26 to 35.

(Note: \overline{AB} is tangent to $\odot Q$ at point B)



(B-2) **26.** If $m\widehat{DF} = 96$, find $m\angle DEF$.

$m\angle DEF =$ _____

(B-4) **27.** If $m\widehat{CD} = 62$ and $m\angle EGF = 110$, find $m\widehat{EF}$.

$m\widehat{EF} =$ _____

(B-4) **28.** If $m\widehat{DF} = 96$ and $m\widehat{CE} = 40$, find $m\angle FAD$.

$m\angle FAD =$ _____

(B-5) **29.** If $m\widehat{BD} = 170$ and $m\widehat{BC} = 110$, find $m\angle BAD$.

$m\angle BAD =$ _____

(B-3) **30.** Find $m\angle ABQ$.

$m\angle ABQ =$ _____

(B-4) **31.** If $m\angle ADE = 26$, find $m\widehat{CE}$.

$m\angle BAD =$ _____

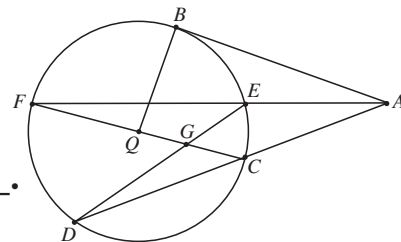
(B-2)

32. If $m\angle ADE = 26$, find $m\angle AFC$.

$m\angle AFC = \underline{\hspace{2cm}}$

(B-2)

33. $\angle DCF \cong \underline{\hspace{2cm}}$.



(B-5)

34. If $m\angle FAB = 18$ and $m\widehat{BE} = 80$, find $m\widehat{BF}$.

$m\widehat{BF} = \underline{\hspace{2cm}}$

(A-2)

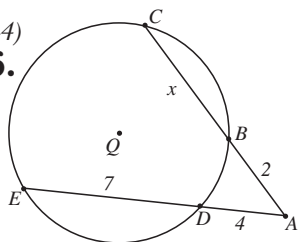
35. If $m\angle BQF = 90$, find $m\widehat{BF}$.

$m\widehat{BF} = \underline{\hspace{2cm}}$

For problems 36 to 41, find the value of x .

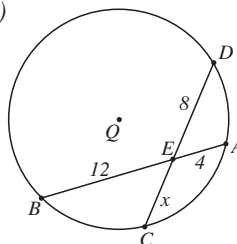
(C-4)

36. $x = \underline{\hspace{2cm}}$



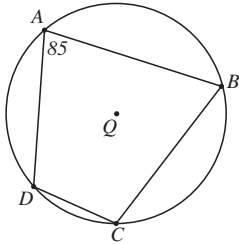
(C-3)

37. $x = \underline{\hspace{2cm}}$



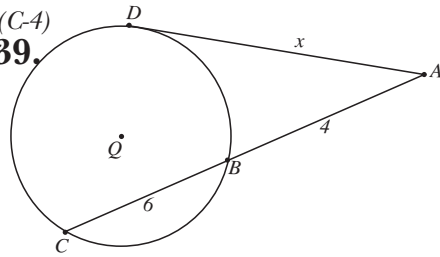
—Continued—

(B-2)
38. Find $m\angle C$

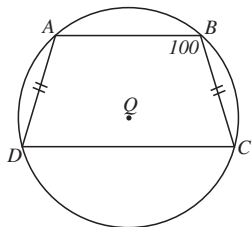


$m\angle C =$ _____

(C-4)
39. $x =$ _____

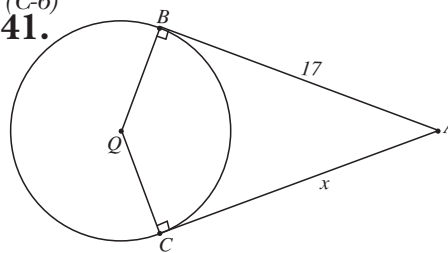


(B-2)
40. Find $m\angle C$

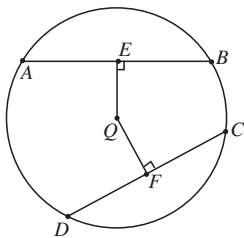


$m\angle D =$ _____

(C-6)
41. $x =$ _____

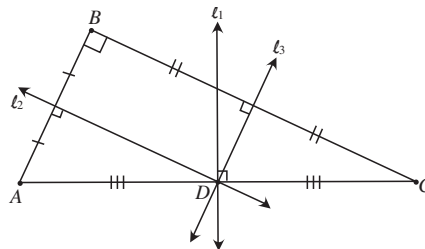


(C-1)
42. Find QE



QE = _____

(D-1)
43. Find BD. BD = _____



Given: $\overline{AB} \cong \overline{CD}$, $DC = 8$,
the radius of $\odot Q$ is 5.

Given: l_1 and l_3 are perpendicular
bisectors of the sides of $\triangle ABC$.
 $AC = 20$.

Unit VI - Circles

Match each term in Column I with a phrase in Column II.

- | | | |
|-------|--|--|
| _____ | (A-3)
1. Common Internal Tangent | a) Circles in the same plane with the same center. |
| _____ | (A-3)
2. Tangent Circles | b) A line segment on a tangent of a circle one of the endpoints of which is the point of tangency. |
| _____ | (A-2)
3. Inscribed Angle | c) Measure of central angle AQB of $\odot Q$ |
| _____ | (A-3)
4. Circumscribed Polygon | d) A point on a tangent line of a circle where the line is tangent to the circle. |
| _____ | (A-3)
5. Externally Tangent Circles | e) Any common tangent which intersects the line segment joining the centers of two circles |
| _____ | (A-2)
6. Measure of \widehat{AB} of $\odot Q$ | f) Two circles that intersect in exactly one point. |
| _____ | (D-2)
7. Cyclic Quadrilateral | g) A common tangent of two circles that does not intersect the line segment joining the centers of the two circles. |
| _____ | (A-3)
8. Concentric Circles | h) An angle whose vertex is on a circle and whose sides intersect the circle in two other points. |
| _____ | (A-1)
9. Tangent Segment | i) A quadrilateral around which a circle can be drawn to pass through all its vertices. |
| _____ | (D-1)
10. Bisectors of the angles of a Triangle. | j) Two circles in the same plane that touch at only one point neither of which is inside the other. |
| _____ | (A-1)
11. Point of Tangency | k) A polygon whose sides are tangent segments of a circle. |
| _____ | (A-3)
12. Common External Tangent | l) Lines which are concurrent at a point equidistant from the three sides of a triangle. |

—Continued—

Determine whether each of the following is always, sometimes or never true.

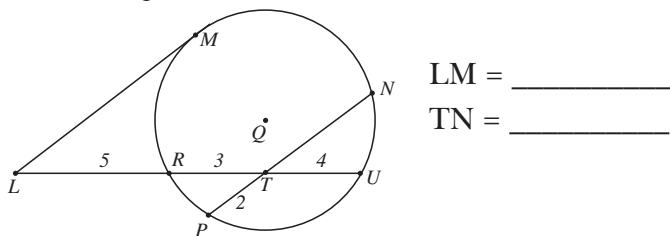
- _____ ^(A-1) **13.** The longest chord of a circle is a diameter.
- _____ ^(B-2) **14.** An angle inscribed in a semicircle has a measure of 180° .
- _____ ^(C-1) **15.** A line perpendicular to a chord of a circle bisects the chord.
- _____ ^(C-7) **16.** If the measure of an arc in one circle is equal to the measure of an arc in
_____ ^(B-2) another circle, the chords for these arcs are congruent.
- _____ **17.** If a quadrilateral is inscribed in a circle, then its opposite angles
_____ ^(C-5) are complimentary.
- _____ **18.** If a line is perpendicular to a radius of a circle at a point on the circle, then the
_____ ^(C-5) line is tangent to the circle.
- _____ **19.** A tangent to a circle at one endpoint of a chord is perpendicular to the chord.
- _____ ^(B-1) **20.** If two circles have unequal radii, but a central angle of one is congruent to a central
_____ ^(B-1) angle of the other, then the degree measures of their intercepted arcs are equal.
- _____ **21.** If the degree measure of an arc of one circle is equal to the degree measure of
_____ ^(C-7) an arc of another circle, then the arcs are congruent.
- _____ **22.** If a chord of one circle is congruent to a chord of a congruent circle, then the
_____ ^(D-1) chords are equidistant from the center of their respective circles.
- _____ **23.** Cyclic triangles are congruent.
- _____ ^(D-2) **24.** Regular polygons, triangles, and rectangles are cyclic.
- _____ ^(B-1, B-2) **25.** If an inscribed angle and a central angle of a circle intercept the same arc, the
_____ measure of the central angle is greater than the measure of the inscribed angle.

—Continued—

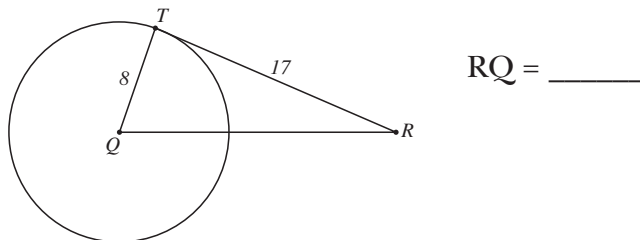
- (B-4) **32.** Find $m\angle HIE$ $m\angle HIE =$ _____ (B-2) **33.** Find $m\angle BDH$ $m\angle BDH =$ _____

- (B-2) **34.** Find $m\angle BHD$ $m\angle BHD =$ _____ (B-2) **35.** Find $m\angle HDQ$ $m\angle HDQ =$ _____

- (C-4) **36.** In the figure below \overline{LM} is tangent to $\odot Q$. Find LM and TN.

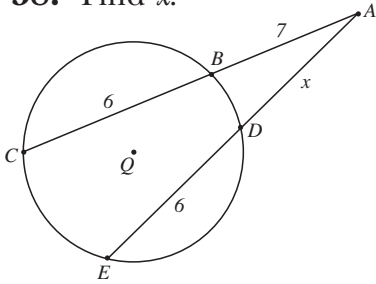


- (B-3) **37.** In the figure below, \overleftrightarrow{RT} is tangent to $\odot Q$ at point T. If $QT = 8$ and $RT = 17$. Find RQ.



(C-4)

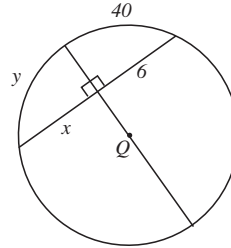
38. Find x . $x =$ _____



(C-1)

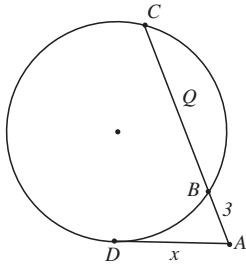
39. Find x and y . $x =$ _____

$y =$ _____



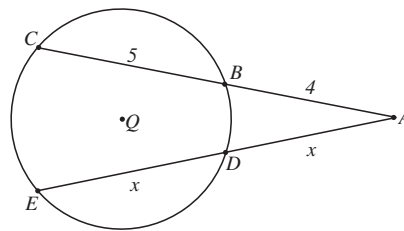
(C-4)

40. Find x . $x =$ _____



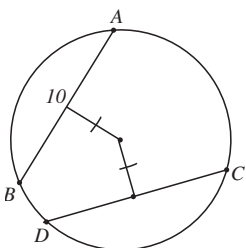
(C-4)

41. Find x . $x =$ _____



(C-7)

42. Find DC. DC = _____



(D-1)

43. Find DF. DF = _____

