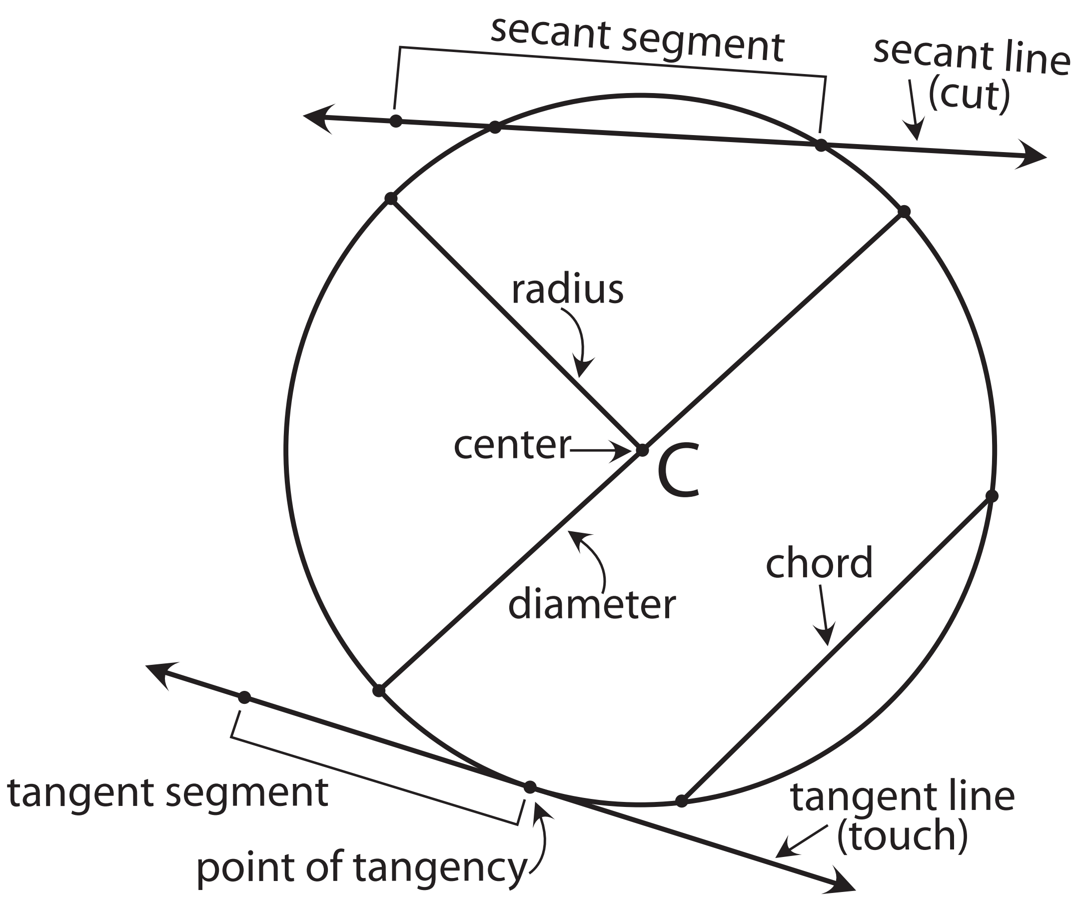
COURSE NOTE 272

LINES & LINE SEGMENTS RELATED TO A CIRCLE



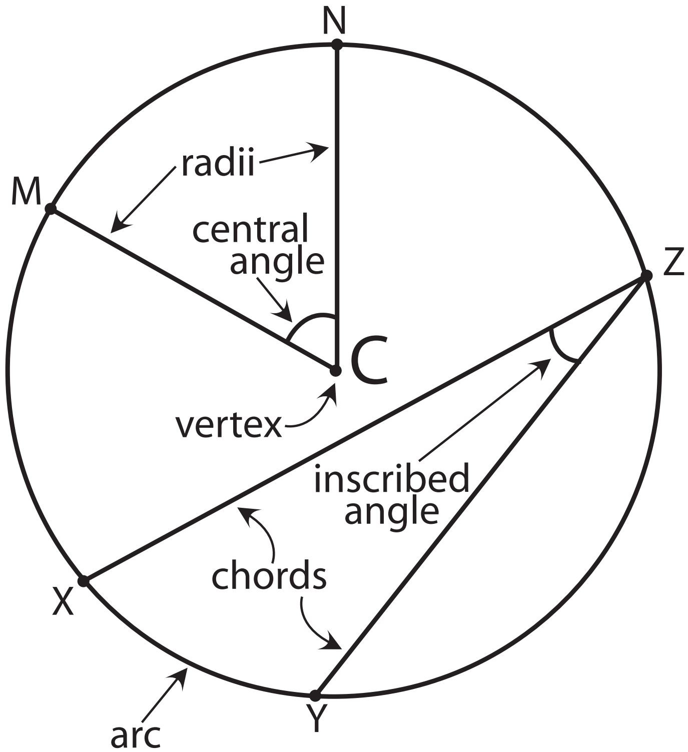
Circle C is symbolized by ⊙C

Secant Line - A line which intersects a circle in two distinct points.

Tangent Line - A line which intersects a circle in exactly one point, called the "point of tangency"

COURSE NOTE 273

ARCS AND ANGLES RELATED TO A CIRCLE



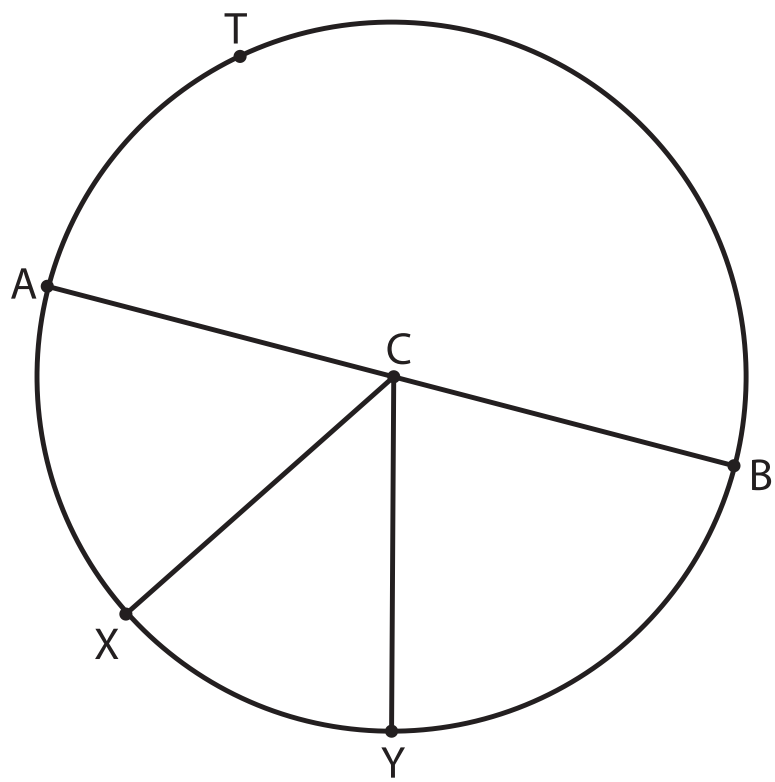
Central Angle of a Circle - An angle whose vertex is the center of the circle, and whose sides are radii of the circle (see $\angle$MCN above)

Inscribed Angle of a Circle - An angle whose vertex is on the circle, and whose sides are chords of the circle. (see $\angle$XZY above)

Arc of a Circle - Any set of continuous points on a circle. (see $\overset{\frown}{XY}$ above)

COURSE NOTE 274

ARCS AND ANGLES RELATED TO A CIRCLE



In $\odot C$ above,

$\overset{\frown}{XY}$ is a minor arc (measure $<180^{\circ}$)

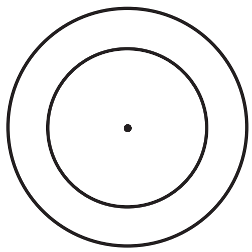
$\overset{\frown}{XTY}$ is a major arc (measure $>180^{\circ}$)

$m\overset{\frown}{XY} = m\angle XCY$

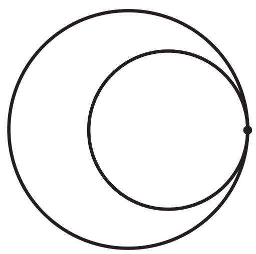
$m\overset{\frown}{XTY} = 360^{\circ} - m\angle XCY$

$m\overset{\frown}{ATB} = 180^{\circ}$ (a semicircle)

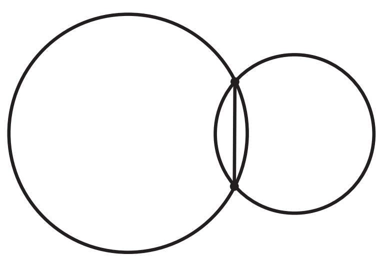
RELATIONSHIPS BETWEEN CIRCLES



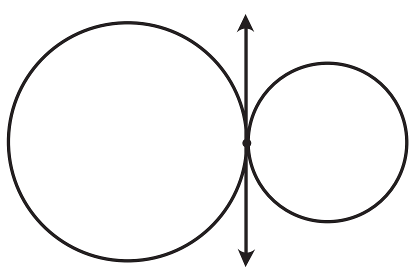
Concentric
(same center)



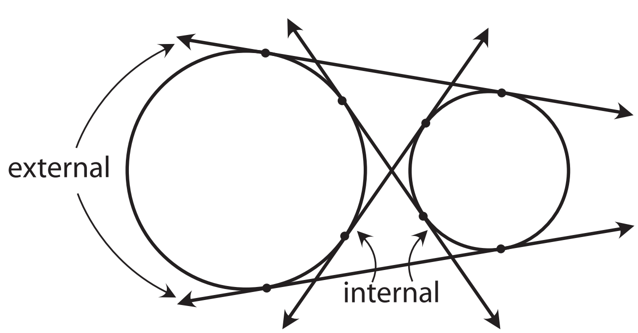
Internally Tangent



Common Chord



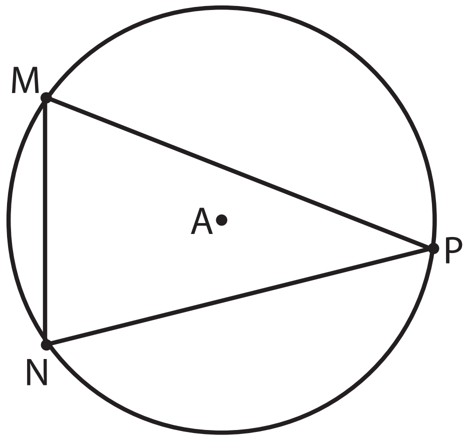
Externally Tangent



Common Tangents

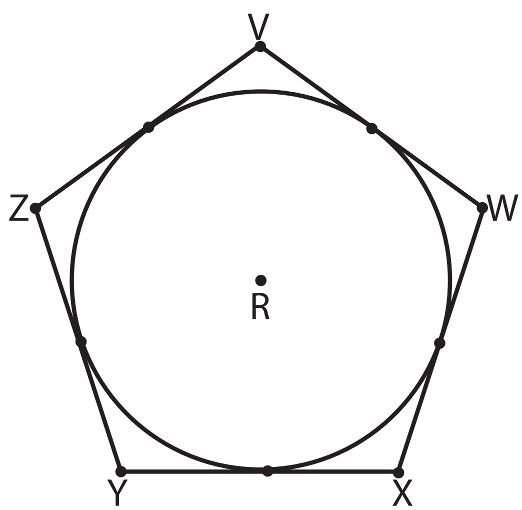
COURSE NOTE 276

POLYGON AND CIRCLE RELATIONSHIPS



⊙A is "circumscribed" about △MNP

△MNP is "inscribed" in ⊙A



⊙R is "inscribed" in Pentagon VWXYZ

Pentagon VWXYZ is "circumscribed" about ⊙R

COURSE NOTE 277

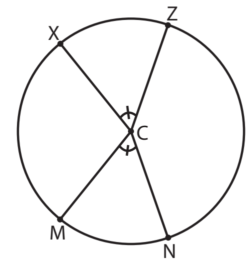
Theorem 65

1) "If, in the same circle, or congruent circles, two given central angles are congruent, then their intercepted minor arcs are congruent."

3) Given: In $\odot C$, $\angle XCZ \cong \angle NCM$

4) Prove: $\overset{\frown}{XZ} \cong \overset{\frown}{NM}$

2)



5) Analysis: Definition of a Central Angle, Definition of the Measure of a Central Angle, Definition of Congruent Angles

6) **STATEMENT**	**REASON**
1. In $\odot C$, $\angle XCZ \cong \angle NCM$	1. Given
2. $m\angle XCZ = m\angle NCM$	2. Definition of Congruent Angles
3. $m\angle XCZ = m\overset{\frown}{XZ}$	3. Definition of the Measure of a Central Angle
4. $m\angle NCM = m\overset{\frown}{NM}$	4. Definition of the Measure of a Central Angle
5. $m\overset{\frown}{XZ} = m\overset{\frown}{NM}$	5. Substitution (statements 2, 3, 4)
6. $\overset{\frown}{XZ} \cong \overset{\frown}{NM}$	6. Definition of Congruent Arcs (Q.E.D.)

Theorem 66

(the converse of Theorem 65)

"If, in the same circle, or congruent circles, two minor arcs are congruent, then the central angles which intercept those minor arcs are congruent."

COURSE NOTE 278

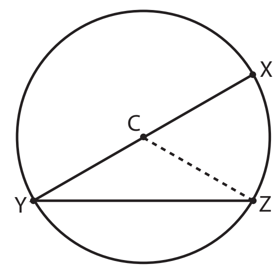
Theorem 67 - Case 1

1) "If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc"

3) Given: $\odot C$, with inscribed $\angle XYZ$

4) Prove: $m\angle XYZ = \frac{1}{2}\, m\angle \widehat{XZ}$

2)

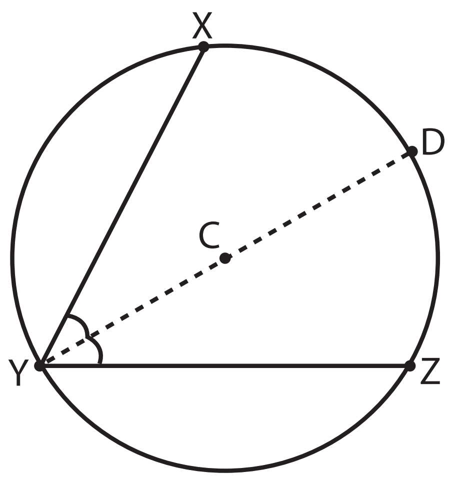


5) Analysis: Auxiliary Line, Theorem 26, Theorem 33

6) **STATEMENT**	**REASON**
1. $\odot C$, with inscribed $\angle XYZ$	1. Given
2. Draw $\overline{CZ}$	2. Postulate 2
3. $m\angle XCZ = m\angle XYZ + m\angle CZY$	3. Theorem 26 - The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.
4. $\overline{CY} \cong \overline{CZ}$	4. Definition of a Radius
5. $\angle XYZ \cong \angle CZY$	5. Theorem 33 - If two sides of a triangle are congruent, then the angles opposite those sides are congruent.
6. $m\angle XYZ = m\angle CZY$	6. Definition of Congruent Angles
7. $m\angle XCZ = m\angle XYZ + m\angle XYZ$	7. Substitution (statements 3 & 6)
8. $m\angle XCZ = 2m\angle XYZ$	8. Substitution
9. $\frac{1}{2}\, m\angle XCZ = m\angle XYZ$	9. Multiplication of Equality
10. $m\angle XCZ = m\widehat{XZ}$	10. Definition of the Measure of a Central Angle
11. $\frac{1}{2}\, m\widehat{XZ} = m\angle XYZ$	11. Substitution (statements 9 &10) Q.E.D.

COURSE NOTE 279

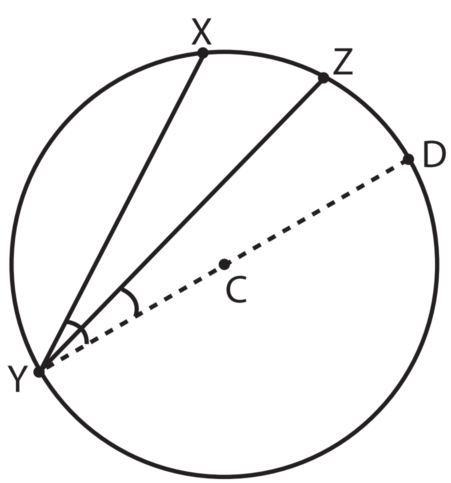
THEOREM 67 - CASE 2
(Supporting Argument)



Auxiliary Lines (segment $\overline{YD}$)
$m\angle XYD = \frac{1}{2} m\overset{\frown}{XD}$ (case 1)
$m\angle ZYD = \frac{1}{2} m\overset{\frown}{ZD}$ (case 1)

Postulate 7 - Protractor
(Angle-Addition)
Postulate 8 - Circle
(Arc-Addition)

THEOREM 67 - CASE 3
(Supporting Argument)

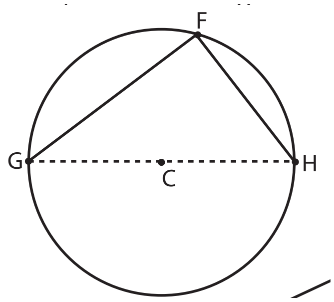


Auxiliary Line (segment $\overline{YD}$)
$m\angle XYD = \frac{1}{2} m\overset{\frown}{XD}$ (case 1)
$m\angle ZYD = \frac{1}{2} m\overset{\frown}{ZD}$ (case 1)

Postulate 7 - Protractor
(Angle-Addition [subtraction])
Postulate 8 - Circle
(Arc-Addition [subtraction])

COROLLARY 67a

"If you have an angle inscribed in a semicircle, then that angle must be a right angle"

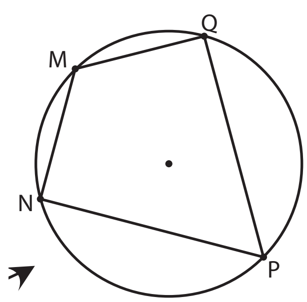


$m\overparen{GH} = 180^{O}$ (semicircle)

$m\angle F = 90^{O}$ (Th. 67)

COROLLARY 67b

"If you have a quadrilateral inscribed in a circle, then its opposite angles must be supplementary"



$m\angle M = \frac{1}{2}\, m\overparen{NPQ}$ (Th. 67)

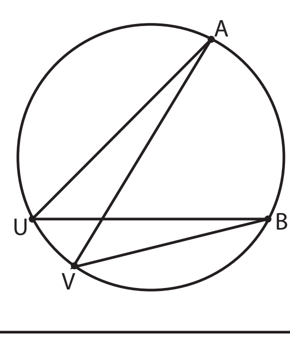
$m\angle P = \frac{1}{2}\, m\overparen{NMQ}$ (Th. 67)

$m\overparen{NPQ} + m\overparen{NMQ} = 360^{O}$

$m\angle M + m\angle P = 180^{O}$

COROLLARY 67c

"If in the same circle, two inscribed angles intercept the same or congruent arcs, then those angles are congruent.



$m\angle U = \frac{1}{2}\, m\overparen{AB}$ (Th. 67)

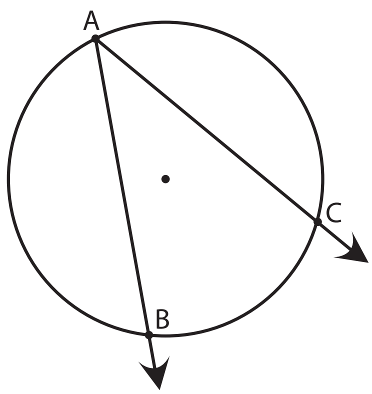
$m\angle V = \frac{1}{2}\, m\overparen{AB}$ (Th. 67)

$m\angle U = m\angle V$

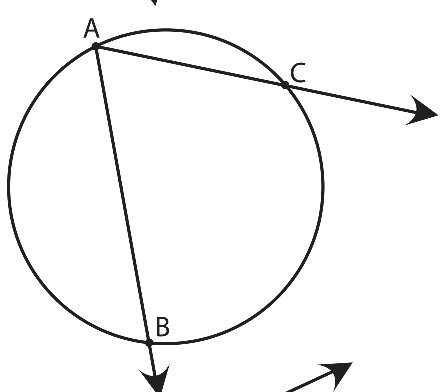
THEOREM 68

(Supporting Argument using Limits)

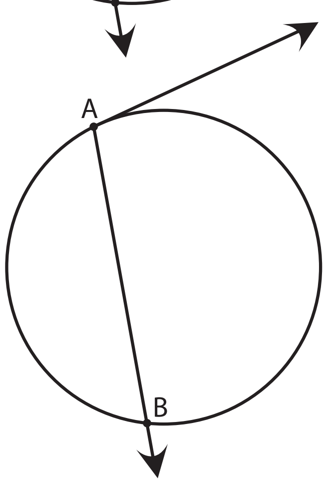
"If, in a circle, you have an angle formed by a secant ray and a tangent ray, both drawn from a point on the circle, then the measure of that angle is one-half the measure of the intercepted arc."



$m\angle BAC = {}^1\!/_2\, m\overset{\frown}{BC}$



$m\angle BAC = {}^1\!/_2\, m\overset{\frown}{BC}$

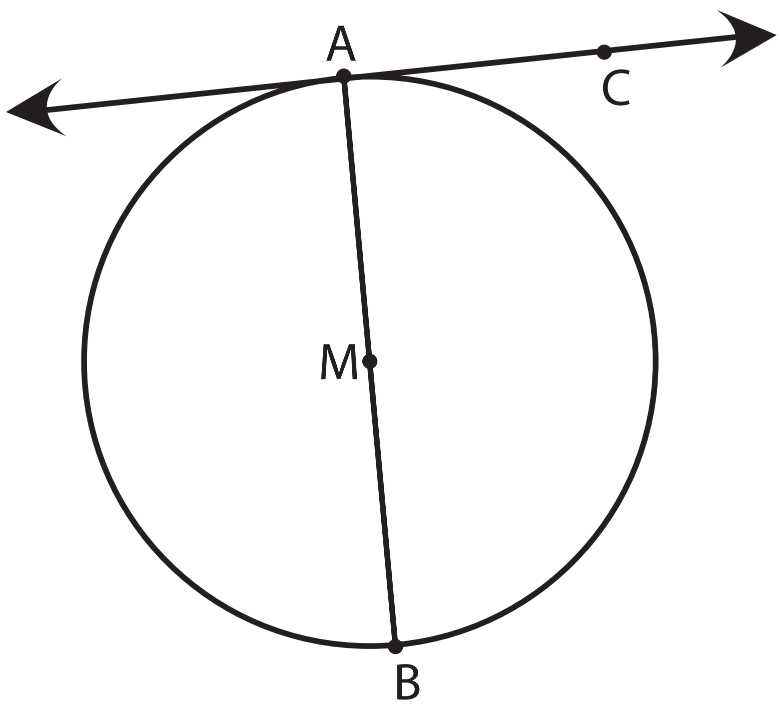


$m\angle A = {}^1\!/_2\, m\overset{\frown}{BA}$
(the Limit)

COROLLARY 68a

(Supporting Argument)

"If in a circle, a diameter is drawn to a tangent line, at the point of tangency, then the diameter is perpendicular to the tangent line, at that point."



$m\overset{\frown}{AB} = 180^{\circ}$ (definition of semicircle)

$m\angle BAC = 90^{\circ}$ (Theorem 68)

COURSE NOTE 283

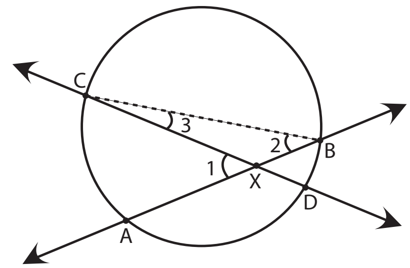
Theorem 69

1) "If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the sum of the measures of the arcs, intercepted by the angle and its vertical angle."

3) Given: $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at X, inside the circle

4) Prove:
$m\angle 1 = 1/2\ (m\overset{\frown}{AC} + m\overset{\frown}{BD})$

2)



5) Analysis: Auxiliary Line, Theorem 26, Theorem 67

6) STATEMENT	REASON
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at X, inside the circle	1. Given
2. Draw $\overline{CB}$	2. Postulate 2
3. $m\angle 1 = m\angle 2 + m\angle 3$	3. Theorem 26 - The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.
4. $m\angle 2 = 1/2\ m\overset{\frown}{AC}$	4. Theorem 67 - If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc.
5. $m\angle 3 = 1/2\ m\overset{\frown}{BD}$	5. Theorem 67 - If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc.
6. $m\angle 1 = 1/2\ m\overset{\frown}{AC} + 1/2\ m\overset{\frown}{BD}$	6. Substitution (statements 3, 4, 5)
7. $m\angle 1 = 1/2\ (m\overset{\frown}{AC} + m\overset{\frown}{BD})$	7. Distributive Property (Q.E.D.)

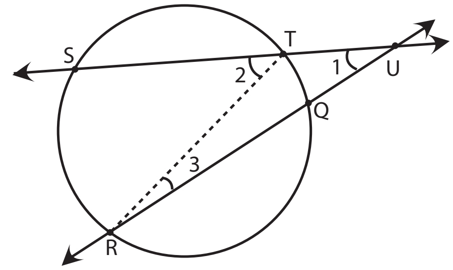
Theorem 70

1) "If, for a circle, two secant lines intersect outside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the difference of the measures of the arcs, intercepted by the angle."

3) Given: $\overleftrightarrow{SU}$ and $\overleftrightarrow{RU}$ intersect at U, outside the circle

4) Prove:
$m\angle 1 = 1/2\,(m\overset{\frown}{SR} - m\overset{\frown}{TQ})$

2)



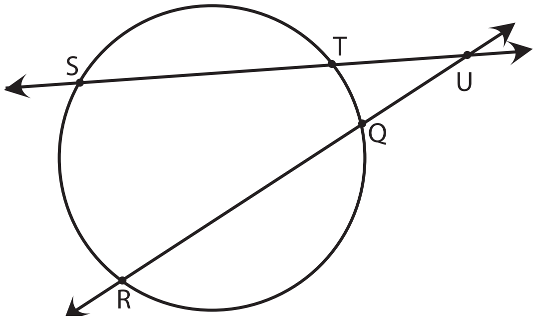
5) Analysis: Auxiliary Line, Theorem 26, Theorem 67

6) STATEMENT	**REASON**
1. $\overleftrightarrow{SU}$ and $\overleftrightarrow{RU}$ intersect at U, outside the circle.	1. Given
2. Draw $\overline{RT}$	2. Postulate 2
3. $m\angle 2 = m\angle 1 + m\angle 3$	3. Theorem 26 - The measure of an exterior angle of a triangle is equal to the sum of the measures of the two remote interior angles.
4. $m\angle 2 = 1/2\ m\overset{\frown}{SR}$	4. Theorem 67 - If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc.
5. $m\angle 3 = 1/2\ m\overset{\frown}{TQ}$	5. Theorem 67 - If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc.
6. $1/2m\overset{\frown}{SR} = m\angle 1 + 1/2\ m\overset{\frown}{TQ}$	6. Substitution (statements 3, 4, 5)
7. $1/2m\overset{\frown}{SR} - 1/2\ m\overset{\frown}{TQ} = m\angle 1$	7. Addition Property of Equality
8. $1/2(m\overset{\frown}{SR} - m\overset{\frown}{TQ}) = m\angle 1$	7. Distributive Property (Q.E.D.)

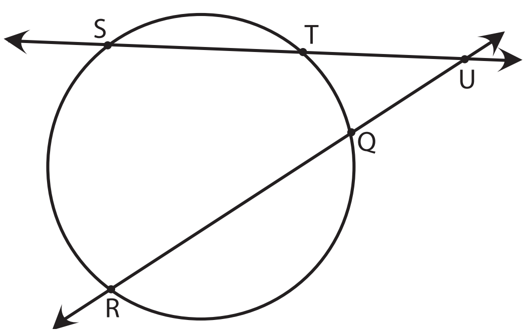
COURSE NOTE 285

THEOREM 71
(Supporting Argument using Limits)

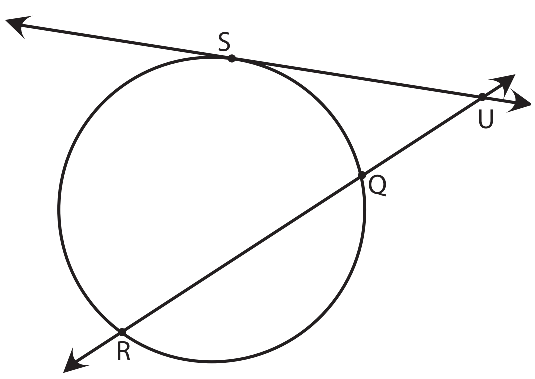
"If, for a circle, a secant line and a tangent line intersect outside the circle, then the measure of the angle formed is equal to one-half the difference of the measures of the arcs, intercepted by the angle."



$m\angle RUS = \frac{1}{2}(m\widehat{SR} - m\widehat{TQ})$
[Theorem 70]



$m\angle RUS = \frac{1}{2}(m\widehat{SR} - m\widehat{TQ})$
[Theorem 70]

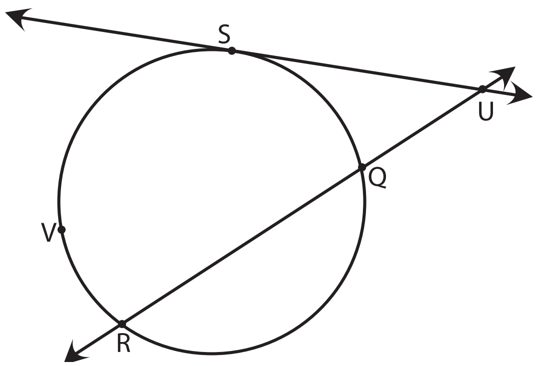


$m\angle RUS = \frac{1}{2}(m\widehat{SR} - m\widehat{SQ})$
[the Limit]

THEOREM 72

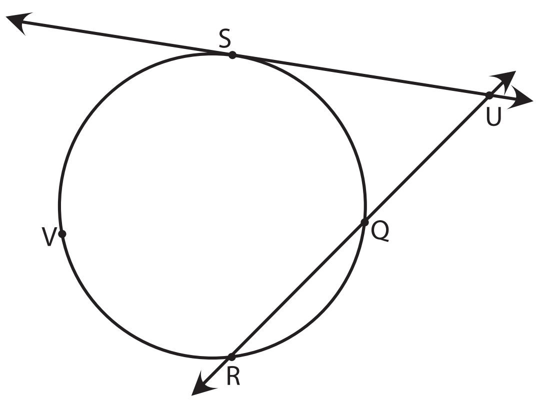
(Supporting Argument using Limits)

"If, for a circle, two tangent lines intersect outside the circle, then the measure of the angle formed is equal to one-half the difference of the measures of the arcs intercepted by the angle"



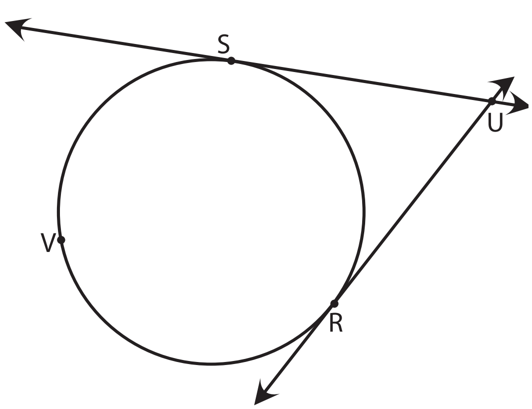
$m\angle RUS = \frac{1}{2}(m\overset{\frown}{SR} - m\overset{\frown}{SQ})$

[Theorem 71]



$m\angle RUS = \frac{1}{2}(m\overset{\frown}{SR} - m\overset{\frown}{SQ})$

[Theorem 71]



$m\angle RUS = \frac{1}{2}(m\overset{\frown}{SVR} - m\overset{\frown}{SR})$

[the Limit]