Unit IV — Triangles

Appendix D - Theorems and Theorem Corollaries

Theorem 1 (WT- p. 228)

"If a given point lies outside a given line, then exactly one plane contains the given line and the given point."

Theorem 2 (WT- p. 231)

"If three different points are on a line, then at most one is between the other two."

Theorem 3 (WT- p. 234)

"If you have a given ray, then there is exactly one point at a given distance from the endpoint of the ray."

Theorem 4 (WT- p. 239)

"If you have a given line segment, then that segment has exactly one midpoint."

Theorem 5 (WT- p. 244)

"If two different lines intersect, then exactly one plane contains both lines."

Theorem 6 (WT- p. 247)

"If, in a plane, there is a point on a line, then there is exactly one perpendicular to the line, through that point."

Theorem 7 (WT- p. 250)

"If, in a half-plane, there is a ray in the edge of the half-plane, and a specific measure is given, then there is exactly one other ray in the half-plane, with the same endpoint as the endpoint of the given ray, such that the angle formed by the two rays has that given measure."

Theorem 8 (WT- p. 253)

"If, in a half-plane, there is a given angle, then that angle has exactly one bisector."

Theorem 9 (WT- p. 256)

"If two adjacent acute angles have their exterior sides in perpendicular lines, then the two angles are complementary."

Theorem 10 (WT- p. 258)

"If the exterior sides of two adjacent angles are opposite rays, then the two angles are supplementary."

Corollary 10a (WT- p. 258)

"If one angle of a linear pair is a right angle, then the other angle is a right angle."

Corollary 10b (WT- p. 258)

"If two angles are supplementary and congruent, then each angle is a right angle."

Corollary 10c (WT- p. 258)

"If two lines are perpendicular, then they form congruent adjacent angles."

Corollary 10d (WT- p. 258)

"If two congruent angles form a linear pair, then the intersecting lines forming the angles are perpendicular."

Theorem 11 (WT- p. 261)

"If you have right angles, then those right angles are congruent."

Theorem 12 (WT- p. 264)

"If you have straight angles, then those straight angles are congruent."

Theorem 13 (WT- p. 268)

"If two angles are complementary to the same angle or congruent angles, then they are congruent to each other."

Theorem 14 (WT- p. 271)

"If two angles are supplementary to the same angle or congruent angles, then they are congruent to each other."

Theorem 15 (WT- p. 274)

"If two lines intersect, then the vertical angles formed are congruent."

Theorem 16 (WT- p. 282)

"If two parallel lines are cut by a transversal, then alternate interior angles are congruent."

Corollary 16a (WT- p. 282)

"If two parallel lines are cut by a transversal, then alternate exterior angles are congruent."

Theorem 17 (WT- p. 285)

"If two parallel lines are cut by a transversal, then interior angles on the same side of the transversal are supplementary."

Corollary 17a (WT- p. 285)

"If two parallel lines are cut by a transversal, then exterior angles on the same side of the transversal are supplementary."

Theorem 18 (WT- p. 287)

"If a given line is perpendicular to one of two parallel lines, then it is perpendicular to the other."

Theorem 19 (WT- p. 291)

"If two lines are cut by a transversal so that corresponding angles are congruent, then the two lines are parallel."

Theorem 20 (WT- p. 294)

"If two lines are cut by a transversal so that alternate interior angles are congruent, then the two lines are parallel."

Corollary 20a (WT- p. 294)

"If two lines are cut by a transversal so that alternate exterior angles are congruent, then the two lines are parallel."

Theorem 21 (WT- p. 297)

"If two lines are cut by a transversal so that interior angles on the same side of the transversal are supplementary, then the two lines are parallel."

Corollary 21a (WT- p. 297)

"If two lines are cut by a transversal so that exterior angles on the same side of the transversal are supplementary, then the two lines are parallel."

Theorem 22 (WT- p. 301)

"If two lines are perpendicular to a third line, then the two lines are parallel."

Theorem 23 (WT- p. 304)

"If two lines are parallel to a third line, then the two lines are parallel to each other."

Theorem 24 (WT- p. 307)

"If two parallel planes are cut by a third plane, then the two lines of intersection are parallel."

Theorem 25 (WT- p. 324)

"If you have any given triangle, then the sum of the measures of its angles is 180."

Corollary 25a (WT- p. 324)

"If two angles of one triangle are congruent to two angles of another triangle, then the third pair of triangles is congruent."

Corollary 25b (WT- p. 324)

"If all of the angles of a triangle are congruent, then the measure of each angle is 60."

Corollary 25c (WT- p. 324)

"If you have a right triangle, then the sum of the measures of the two acute angles is 90."

Corollary 25d (WT- p. 324)

"If you have a triangle, then that triangle can have, at most, one non-acute angle."

Corollary 25e (WT- p. 324)

"If you have a quadrilateral, then the sum of the measures of its angles is 360."

Theorem 26 (WT- p. 327)

"If you have a given exterior angle of a triangle, then its measure is equal to the sum of the measures of the two remote interior angles."

Theorem 27 (WT- p. 341)

"If you have two similar polygons, then the ratio of the perimeters of the two polygons is equal to the ratio of the corresponding sides."

Theorem 28 (WT- p. 351)

"If a line is parallel to one side of a triangle, and intersects the other two sides in different points, then it divides the two sides proportionally."

Corollary 28a (WT- p. 351)

"If a line intersects two sides of a triangle in different points, in such a way that the two sides are divided proportionally, then the line is parallel to the third side of the triangle."

Corollary 28b (WT- p. 351)

"If three or more parallel lines intersect two transversals, then they divide the transversals into proportional segments."

Theorem 29 (WT- p. 358)

"If two triangles are similar, then their corresponding altitudes are in the same ratio as any pair of corresponding sides."

Corollary 29a (WT- p. 358)

"If two triangles are similar, then their corresponding angle bisectors are in the same ratio as any pair of corresponding sides."

Corollary 29b (WT- p. 358)

"If two triangles are similar, then their corresponding medians are in the same ratio as any pair of corresponding sides."

Theorem 30 (WT- p. 363)

"If the altitude is drawn to the hypotenuse of a right triangle, then the two triangles formed are similar to the original triangle, and to each other.

Corollary 30a (WT- p. 364)

"If you have a right triangle, then either leg is the geometric mean between the hypotenuse of the triangle, and the projection of that leg on that hypotenuse."

Corollary 30b (WT- p. 364)

"If you have a right triangle, then the altitude drawn to the hypotenuse of that triangle is the geometric mean between the segments of that hypotenuse, formed by drawing that altitude."

Corollary 30c (WT- p. 364)

"If you have an altitude drawn to the hypotenuse of a right triangle, then the product of the lengths of that altitude and the hypotenuse, is equal to the product of the lengths of the two legs."

Theorem 31 (WT- p. 369)

"If you have a right triangle, then the square of the measure of the hypotenuse, is equal to the sum of the squares of the measures of the two legs."
(The Pythagorean Theorem)

Corollary 31a (WT- p. 369)

"If you have a right triangle whose acute angles have measures of 30° and 60°, then the measure of the hypotenuse is twice the measure of the shorter leg, and the measure of the longer leg is times the measure of the shorter leg."

Corollary 31b (WT- p. 369)

"If you have a right triangle whose acute angles each have measures of 45°, then the measure of the hypotenuse is times the measure of either leg."

Theorem 32 (WT- p. 414)

"If two given triangles are both congruent to a third triangle, then the two given triangles are congruent to each other."

Theorem 33 (WT- p. 415)

"If two sides of a triangle are congruent, then the angles opposite them are congruent."

Corollary 33a (WT- p. 416)

"If a triangle is equilateral, then it is equiangular."

Corollary 33b (WT- p. 416)

"If a triangle is equilateral, then the measure of each of its angles is 60?."

Theorem 34 (WT- p. 422)

"If two angles of a triangle are congruent, then the sides opposite them are congruent."

Corollary 34a (WT- p. 423)

"If a triangle is equiangular, then it is equilateral."

Theorem 35 (WT- p. 426)

"If a ray bisects an angle of a triangle, then it intersects the opposite side in such a way that, the two segments formed are in the same ratio as the two other sides." (Angle Bisector Theorem)

Theorem 36 (WT- p. 430)

"If the sum of the squares of the measures of two sides of a triangle, is equal to the square of the measure of the third side, then the triangle is a right triangle." (The converse of the Pythagorean Theorem)

Theorem 37 (WT- p. 436)

"If you have a given exterior angle of a triangle, then the measure of that angle is greater than the measure of either remote interior angle." (The Exterior Angle Inequality Theorem)

Theorem 38 (WT- p. 442)

"In a given triangle, if two sides are not congruent, then the angles opposite those sides are not congruent."

Corollary 38a (WT-p. 442)

"In a given triangle, if the measure of one side is greater than the measure of a second side, then the measure of the angle opposite the longer side, is greater than the measure of the angle opposite the shorter side."

The "Hinge" Theorem – "Side-To-Angle" Version (WT- p. 443)

"If two sides of one triangle are congruent to two sides of a second triangle, and the length of the third side of the first triangle is greater than the length of the third side of the second triangle, then the measure of the angle opposite the third side of the first triangle, is greater than the measure of the angle opposite the third side of the second triangle."

Theorem 39 (WT- p. 446)

"In a given triangle, if two angles are not congruent, then the sides opposite those angles are not congruent."

Corollary 39a (WT- p. 446)

"In a given triangle, if the measure of one angle, is greater than the measure of a second angle, then the measure of the side opposite the first angle, is greater than the measure of the side opposite the second angle."

The "Hinge" Theorem – "Angle-To-Side" Version (WT- p. 447)

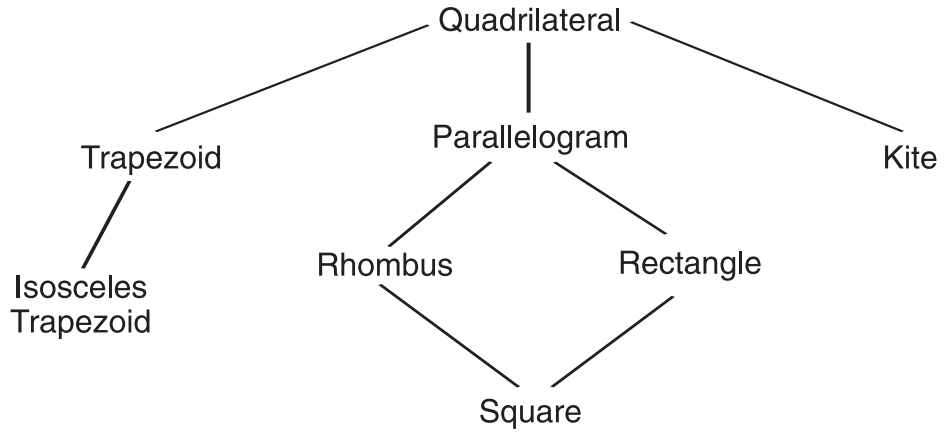
"If two sides of one triangle are congruent to two sides of a second triangle, and the measure of the included angle of the first triangle is greater than the measure of the included angle of the second triangle, then the length of the side opposite the included angle of the first triangle, is greater than the length of the side opposite the included angle of the second triangle."

Theorem 40 (WT- p. 451)

"In a given triangle, the sum of the lengths of any two sides, is greater than the length of the third side."

The Quadrilateral Hierarchy Theorem (WT-p.456, p.460, p.466, p.471, p.476)

"If a polygon is one of the seven types of quadrilaterals, then it is related to all other quadrilaterals, using the diagram below."



Theorem 41 (WT- p.460)

"If a quadrilateral is a parallelogram, then both pairs of opposite sides are congruent."

Theorem 42 (WT- p.461)

"If a quadrilateral is a parallelogram, then any pair of consecutive angles are supplementary."

Corollary 42a (WT- p. 461)

"If a quadrilateral is a parallelogram, then opposite angles are congruent."

Theorem 43 (WT- p. 461)

"If a quadrilateral is a parallelogram, then its diagonals bisect each other."

Theorem 44 (WT- p. 461)

"If, in a quadrilateral, both pairs of opposite sides are congruent, then the quadrilateral is a parallelogram."

Theorem 45 (WT- p. 461)

"If, in a quadrilateral, every pair of consecutive angles are supplementary, then the quadrilateral is a parallelogram."

Theorem 46 (WT- p. 461)

"If, in a quadrilateral, opposite angles are congruent, then the quadrilateral is a parallelogram."

Theorem 47 (WT- p. 461)

"If, in a quadrilateral, the diagonals bisect each other, then the quadrilateral is a parallelogram."

Theorem 48 (WT- p.466)

"If a quadrilateral is a rectangle, then it is a parallelogram."

Theorem 49 (WT- p.466)

"If a quadrilateral is a rectangle, then its diagonals are congruent."

Theorem 50 (WT- p.466)

"If a quadrilateral is a rhombus, then it is a parallelogram."

Theorem 51 (WT- p.466)

"If a quadrilateral is a rhombus, then its diagonals are perpendicular."

Theorem 52 (WT- p.466)

"If a quadrilateral is a rhombus, then the diagonals bisect the interior angles of the rhombus."

Theorem 53 (WT- p.471)

"If a quadrilateral is an isosceles trapezoid, then its base angles are congruent."

Corollary 53a (WT- p.471)

"If a quadrilateral is an isosceles trapezoid, then the diagonals are congruent."

Theorem 54 (WT-p.476)

"If a quadrilateral is a kite, then the pair of opposite angles, formed by the two pairs of non-congruent sides, are congruent."

Corollary 54a (WT-p.476)

"If a quadrilateral is a kite, then the symmetry diagonal bisects the angles to which it is drawn."

Corollary 54b (WT-p.476)

"If a quadrilateral is a kite, then the symmetry diagonal bisects the shorter diagonal."

Corollary 54c (WT-p.476)

"If a quadrilateral is a kite, then the diagonals are perpendicular to each other."

Theorem 55 (WT-p.480)

"If you have a triangle, then the midsegment joining two sides, is parallel to the third side, and is one-half the measure of that side."

Theorem 56 (WT-p.480)

"If you have a quadrilateral, then the four midsegments joining consecutive sides of that quadrilateral, form a parallelogram."

Theorem 57 (WT-p.489)

"If you have any polygon of n sides, then the sum of the measures of its interior angles is, $(n-2)\cdot 180°$."

Corollary 57a (WT-p.489)

"If you have any polygon of n sides, then the measure of each interior angle is $\frac{(n-2)\cdot 180°}{n}$

Theorem 58 (WT-p.501)

"If you have a right triangle, then the area inside the triangle is one-half the product of the legs."

Corollary 58a (WT-p.501)

"If you have a triangle, then the area inside the triangle is one-half the product of the measure of the base and the measure of the height."

Theorem 59 (WT-p.505)

"If you have a parallelogram, then the area inside the parallelogram is the product of the measures of any base and the corresponding altitude."

Theorem 60 (WT-p.508)

"If you have a trapezoid, then the area inside the trapezoid is one-half the product of the altitude and the sum of the lengths of its bases."

Theorem 61 (WT-p.512)

"If you have a regular n-gon, with sides of length s, and apothem of length a, the area A, inside the n-gon, is one-half the product of a and P, where P is the perimeter of the regular n-gon." As a formula, it would be written as follows:

$$A = \frac{1}{2}\cdot s\cdot a\cdot n \quad \text{or} \quad A = \frac{1}{2}\cdot a\cdot P$$

Theorem 62 (WT-p.514)

"If you have a median of a triangle, then that median separates the points inside the triangle into two polygonal regions with the same area."

Theorem 63 (WT-p.514)

"If you have a rhombus, then the area enclosed by that rhombus, is equal to one-half the product of the measures of the diagonals of the rhombus."

Theorem 64 (WT-p.512)

"If you have two similar polygons, then the ratio of the areas of the two polygons is equal to the square of the ratio of any pair of corresponding sides."

Theorem 66 (WT-p. 532)

"If, in the same circle, or in congruent circles, two minor arcs are congruent, then the central angles which intercept those minor arcs are congruent."

Theorem 67 (WT-p. 537)

"If you have an inscribed angle of a circle, then the measure of that angle is one-half the measure of its intercepted arc."

Corollary 67a (WT-p. 538)

"If you have an angle inscribed in a semicircle, then that angle must be a right angle."

Corollary 67b (WT-p. 538)

"If you have a quadrilateral inscribed in a circle, then its opposite angles are supplementary."

Corollary 67c (WT-p. 538)

"If, in the same circle, two inscribed angles intercept the same, or congruent arcs, then those angles are congruent."

Theorem 68 (WT-p. 541)

"If, in a circle, you have an angle formed by a secant ray and a tangent ray, both drawn from a point on the circle, then the measure of that angle, is one-half the measure of the intercepted arc."

Corollary 68a (WT-p. 542)

"If, in a circle, a diameter is drawn to a tangent line, at the point of tangency, then that diameter is perpendicular to the tangent line, at that point."

Theorem 69 (WT-p. 547)

"If, for a circle, two secant lines intersect inside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the sum of the measures of the arcs intercepted by the angle, and its vertical angle."

Theorem 70 (WT-p. 547)

"If, for a circle, two secant lines intersect outside the circle, then the measure of an angle formed by the two secant lines, (or its vertical angle), is equal to one-half the difference of the measures of the arcs intercepted by the angle."

Theorem 71 (WT-p. 551)
"If, for a circle, a secant line and a tangent line intersect outside a circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle."

Theorem 72 (WT-p. 551)
"If, for a circle, two tangent lines intersect outside the circle, then the measure of the angle formed, is equal to one-half the difference of the measures of the arcs intercepted by the angle."