Unit IV — Triangles

Appendix C - Postulates and Postulate Corollaries

Postulate 1 – Existence of Points (WT- p. 170)
- “Every line contains at least 2 different points.”
- “Every plane contains at least 3 different, non-collinear points.”
- “Space contains at least 4 different, non-coplanar points, no three of which are collinear.”

Postulate 2 – Uniqueness of Lines, Planes, and Spaces (WT- p. 175)
- “For any 2 different points, there is exactly 1 line containing them.”
- “For any 3 different, non-collinear points, there is exactly 1 plane containing them.”
- “For any 4 different, non-coplanar points, no 3 of which are collinear, there is exactly 1 space containing them.”

Postulate 3 – One, Two, and Three Dimensions (WT- p. 178)
- “For any 2 different points in a plane, the line containing them is in the plane.”
- “For any line in a plane, there is at least 1 point in the plane that is not on the line.”
- “For any plane in space, there is at least 1 point in space that is not on the plane.”

Postulate 4 – Separation of Lines, Planes, and Spaces (WT- p. 180)
- “A point separates a line into two non-empty sets called half-lines. If two points are in the same half-line, then the segment joining them does not contain the given point. If two points are in different half-lines, then the segment joining them does contain the given point.”
- “A line separates a plane into two non-empty sets called half-planes. If two points are in the same half-plane, then the segment joining them does not intersect the given line. If two points are in different half-planes, then the segment joining them does intersect the given line.”
- “A plane separates space into two non-empty sets called half-spaces. If two points are in the same half-space, then the segment joining them does not intersect the given plane. If two points are in different half-spaces, then the segment joining them does intersect the given plane.”

Postulate 5 – Intersection of Lines or Planes (WT- p. 184)
- “If 2 different lines intersect, the intersection is a unique point.”
- “If 2 different planes intersect, the intersection is a unique line.”

Postulate 6 – Ruler (WT- p. 187)

- "The set of all points on a line can be put into a one-to-one correspondence with the real numbers, so that any point may correspond to 0, and any other point may correspond to 1."
- "To every pair of points on a line, there corresponds exactly one real number, called the unique distance between the points."
- "The distance between any 2 points on a line is the absolute value of the difference between their coordinates."
- "If, on a line, point B lies between points A and C, then:
  mAB + mBC = mAC (Segment-Addition Assumption)

Postulate 7 – Protractor (WT- p. 194)

- "In a half-plane, the set of all rays with a common endpoint in the edge of the half-plane, can be put into a one-to-one correspondence with the real numbers from 0 to 180, inclusive, pairing either ray in the edge of the half-plane with 0."
- "To every pair of rays with a common endpoint in the edge of a half-plane, there corresponds exactly one real number from 0 to 180, inclusive, called the unique measure of the angle formed by the rays."
- "The measure of an angle is the absolute value of the difference between the coordinates of its rays."
- "If, in a half-plane, a ray OB lies between rays OA and OC, then:
  $m\angle AOB + m\angle BOC = m\angle AOC$ (Angle-Addition Assumption)

Postulate 8 – Circle (WT- p. 198, p. 525)

- "The set of all points on a circle can be put into a one-to-one correspondence with the real numbers from 0 to 360, inclusive, with the exception of any one point which may be paired with 0 and 360."
- "To every pair of points on a circle, there correspond exactly 2 real numbers whose sum is 360, each of which may be called the distance between the 2 points."
- "The distance between any 2 points on a circle is the absolute value of the difference between their coordinates."
- "If, on a circle, a point B lies between points A and C, then:
  mAB + mBC = mAC (Arc-Addition Assumption)

Postulate 9 – Uniqueness of Parallel Lines (WT- p. 202)

- "In a plane, through a point not on a given line, there is exactly one line parallel to the given line."

Postulate 10 – Uniqueness of Perpendicular Lines (WT- p. 207)

- "In a plane, through a point not on a given line, there is exactly one line perpendicular to the given line."
- "Through a point not in a given plane, there is exactly one line perpendicular to the given plane."

Postulate 11 – Corresponding Angles of Parallel Lines (WT- p. 277)

- "If two parallel lines are cut by a transversal, then corresponding angles are congruent."

Postulate 12 – Triangle Similarity (WT- p. 346)

- "If the three angles of one triangle are congruent to the three corresponding angles of another triangle, then the two triangles are similar."
  (AAA Similarity Assumption)
- "If the three sides of one triangle are in proportion to the three corresponding sides of another triangle, then the two triangles are similar."
  (SSS Similarity Assumption) Note: After Exercise 21 of Lesson IV-F-2, you may refer to this conditional as Postulate Corollary 12d.
- "If an angle of one triangle is congruent to an angle of another triangle, and the sides that include the first angle are in proportion to the sides that include the other angle, then the two triangles are similar." (SAS Similarity Assumption) Note: After Exercise 22 of Lesson IV-F-2, you may refer to this conditional as Postulate Corollary 12e.

Postulate Corollary 12a (WT- p. 346)

"If two angles of one triangle are congruent to the two corresponding angles of another triangle, then the two triangles are similar."
(AA Similarity Postulate Corollary)

Postulate Corollary 12b (WT- p. 346)

"If an acute angle of one right triangle is congruent to an acute angle of another right triangle, then the two right triangles are similar."

Postulate Corollary 12c (WT- p. 346)

"If two triangles are similar to a third triangle, then the two triangles are similar to each other."

Postulate 13 – Triangle Congruence (WT- p. 389)

- "If the three sides of one triangle are congruent to the three corresponding sides of another triangle, then the two triangles are congruent."
  (SSS Congruence Assumption)
- "If two sides and the included angle of one triangle are congruent to the two corresponding sides and the corresponding included angle of another triangle, then the two triangles are congruent." (SAS Congruence Assumption)
- "If two angles and the included side of one triangle are congruent to the two corresponding angles and the corresponding included side of another triangle, then the two triangles are congruent." (ASA Congruence Assumption)

Postulate Corollary 13a (WT- p. 389)

- "If two angles and a non-included side of one triangle are congruent to the corresponding two angles and non-included side of another triangle, then the two triangles are congruent." (AAS Congruence Postulate Corollary)

Postulate Corollary 13b (WT- p. 395)

- "If the hypotenuse and an acute angle of one right triangle are congruent to the hypotenuse and the corresponding acute angle of another right triangle, then the two right triangles are congruent." (HA Congruence Postulate Corollary)

Postulate Corollary 13c (WT- p. 395)

- "If a leg and an acute angle and of one right triangle are congruent to the corresponding leg and acute angle of another right triangle, then the two right triangles are congruent." (LA Congruence Postulate Corollary)

Postulate Corollary 13d (WT- p. 395)

- "If the two legs of a right triangle are congruent to the two corresponding legs of another right triangle, then the two right triangles are congruent." (LL Congruence Postulate Corollary)

Postulate Corollary 13e (WT- p. 395)

- "If the hypotenuse and one leg of one right triangle are congruent to the hypotenuse and the corresponding leg of another right triangle, then the two right triangles are congruent." (HL Congruence Postulate Corollary)

Postulate Corollary 14 - Area (WT- p. 494)

- "To every polygonal region, there corresponds a unique, positive real number, called its area, relative to a specific unit."
- "Congruent polygons have the same area."
- "If a plane figure can be separated into a number of non-overlapping, polygonal regions, then the area of that plane figure is the sum of the areas of those polygonal regions." (Area-Addition Assumption)
- "The area A, of a rectangular region, with dimensions b (for the base), and h (for the height), is b • h (or base • height)."